

Beyond PMNS with high-energy astrophysical neutrinos

Mauricio Bustamante

Niels Bohr Institute, University of Copenhagen

26th NuFact

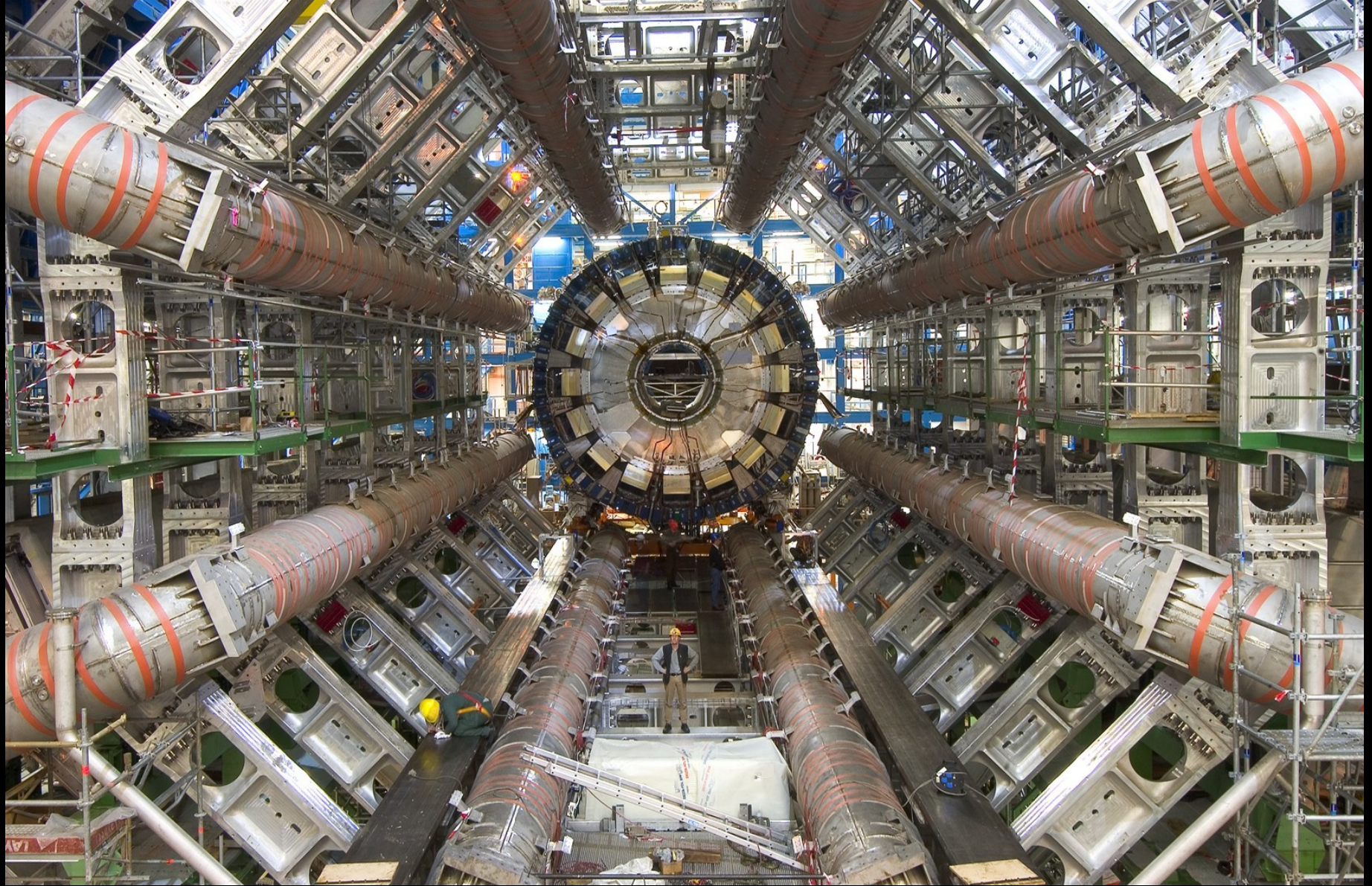
Liverpool, September 04, 2025

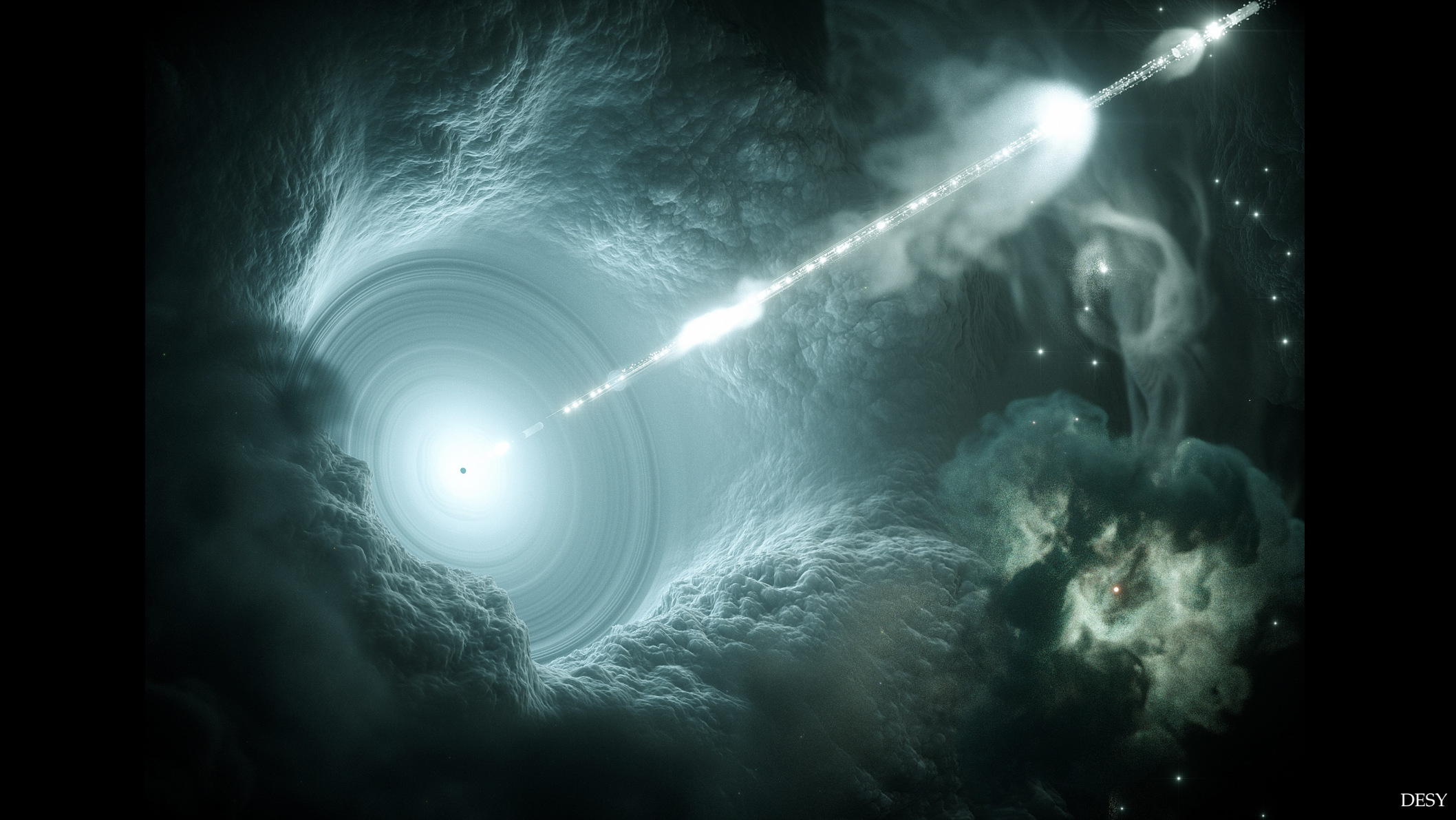
UNIVERSITY OF
COPENHAGEN



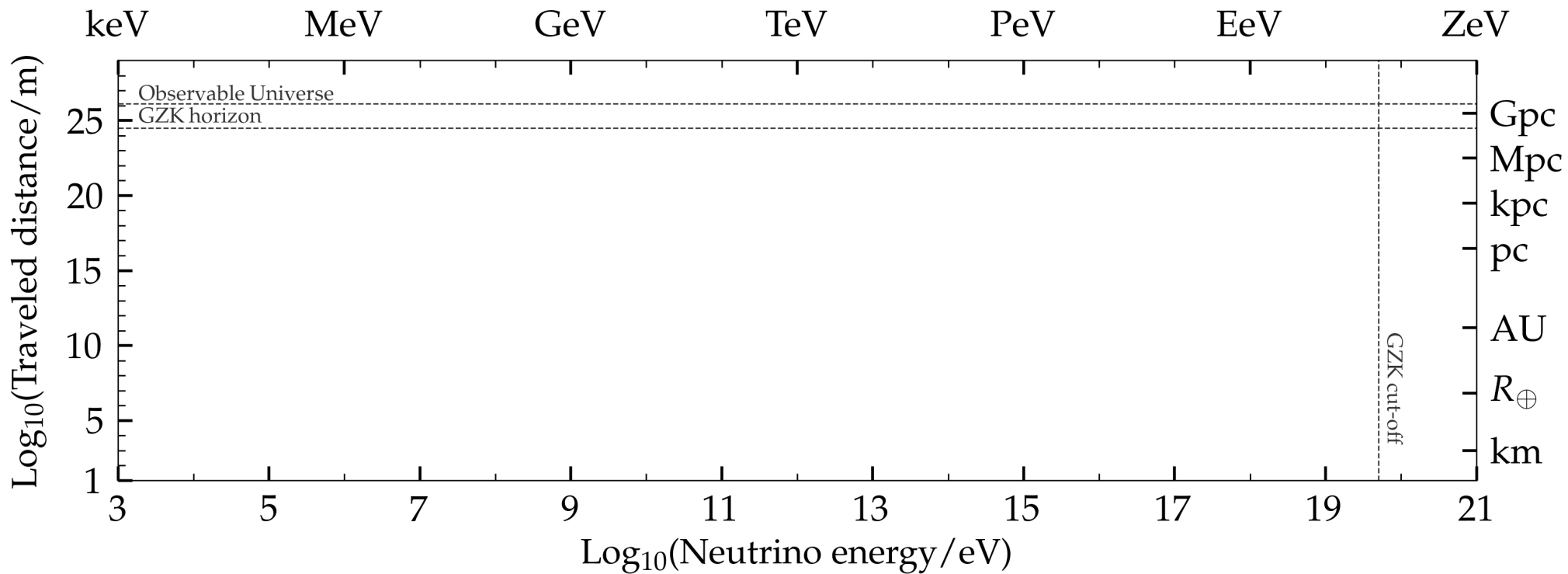
VILLUM FONDEN

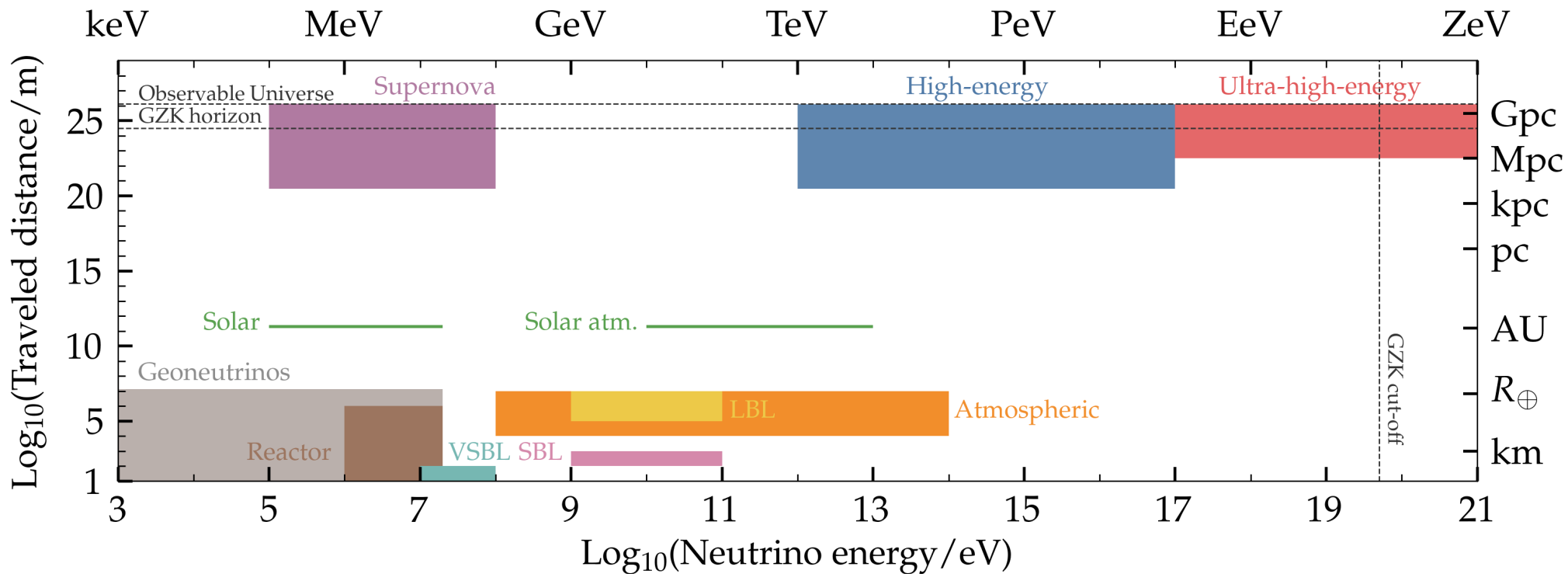




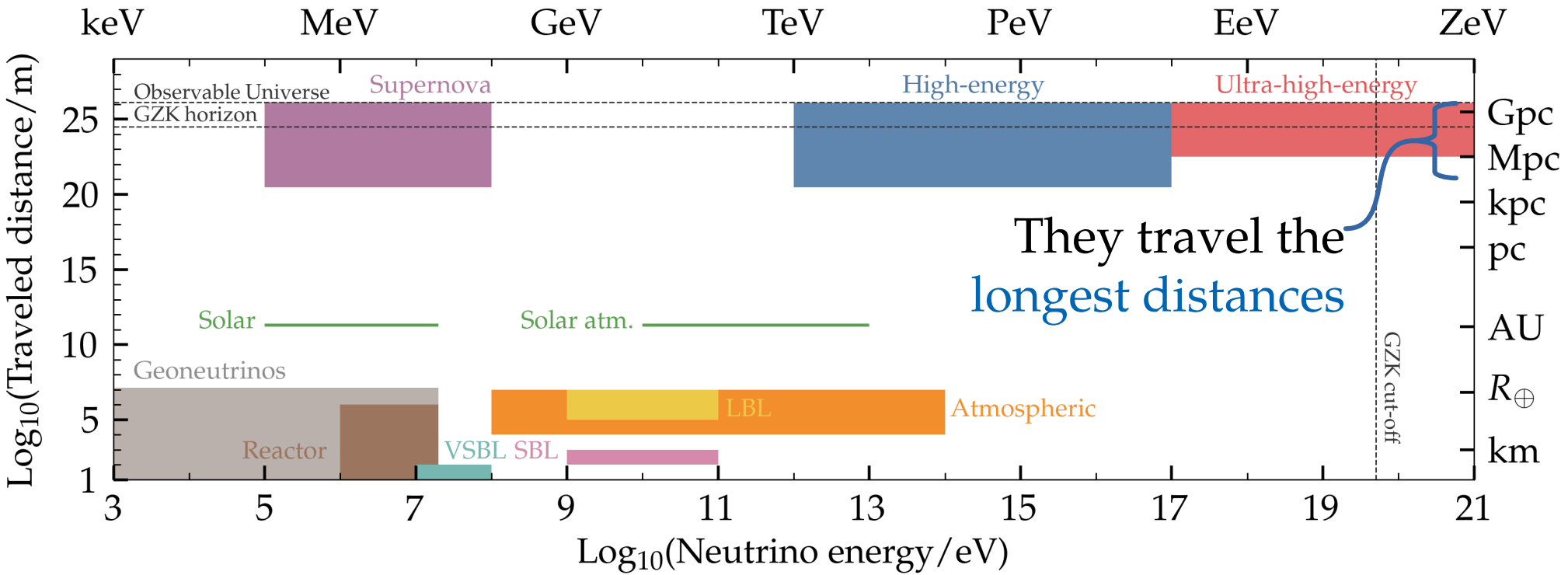


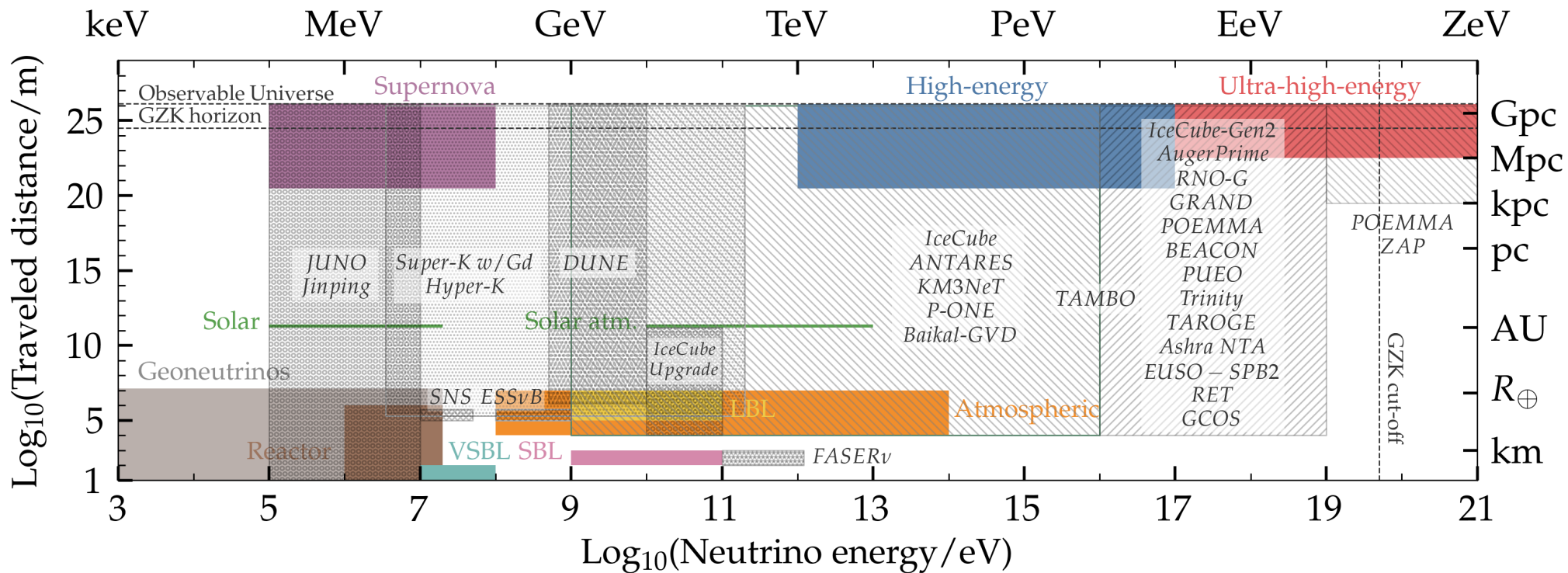


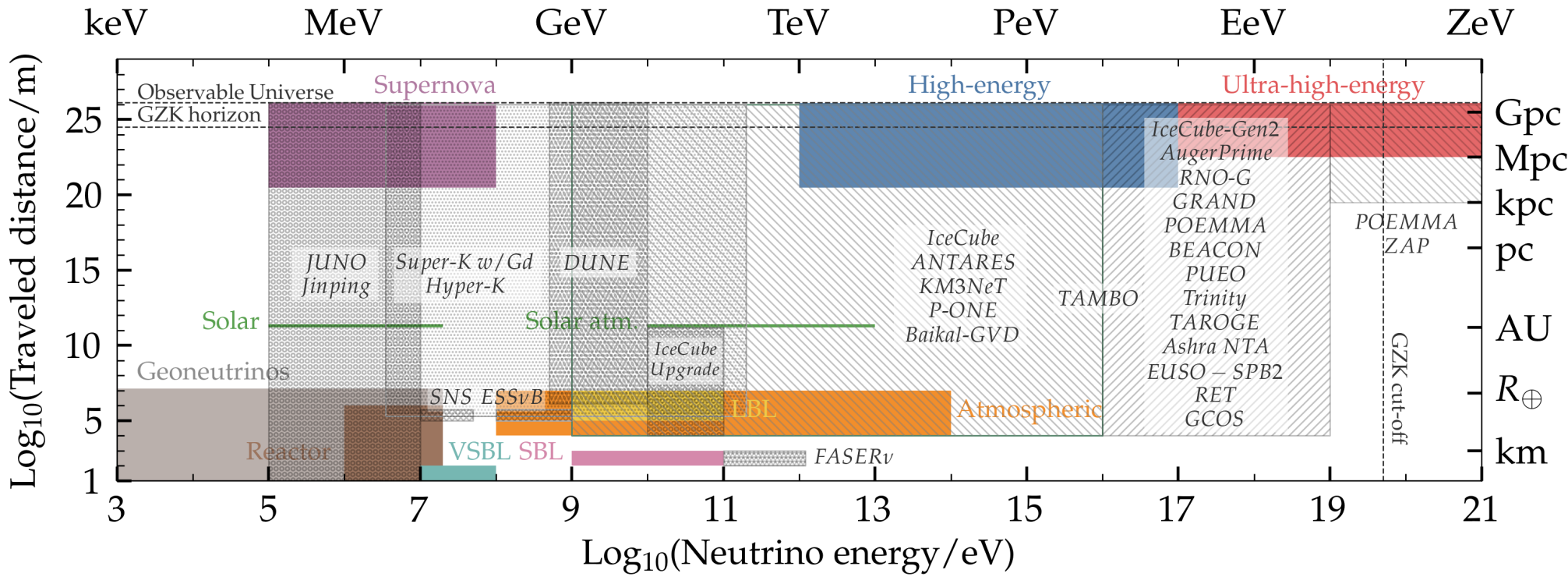




They have the **highest energies**

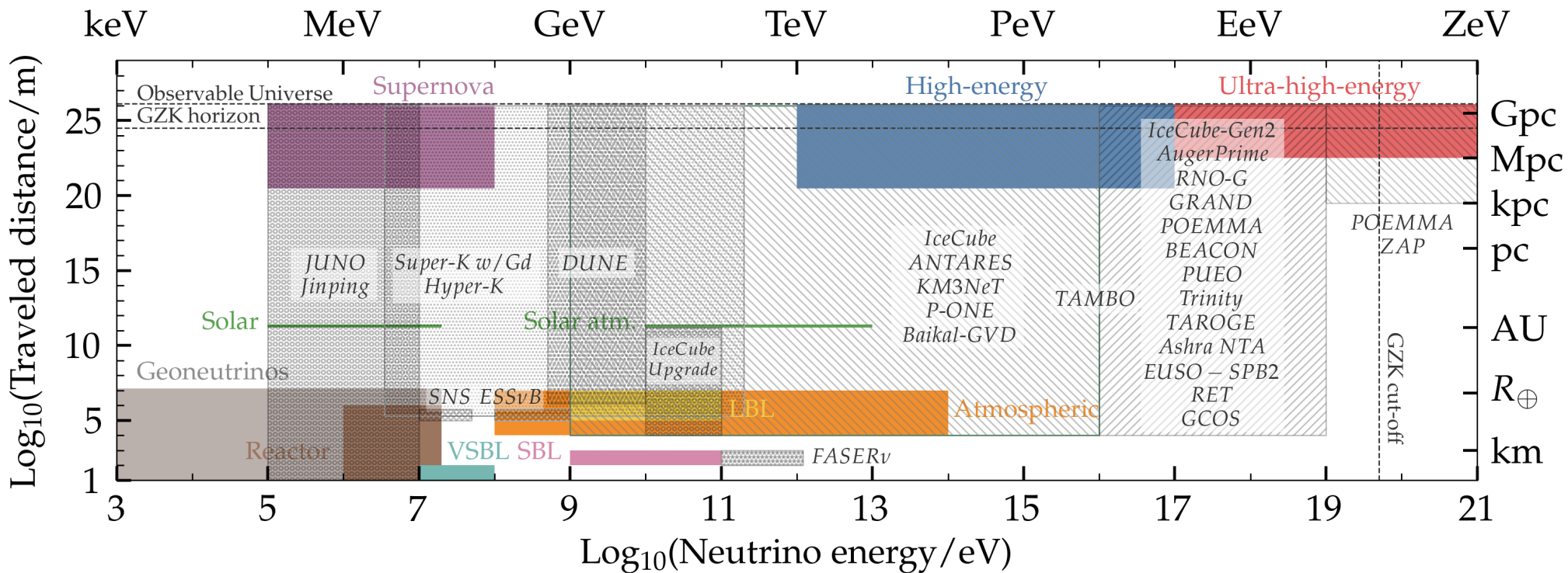






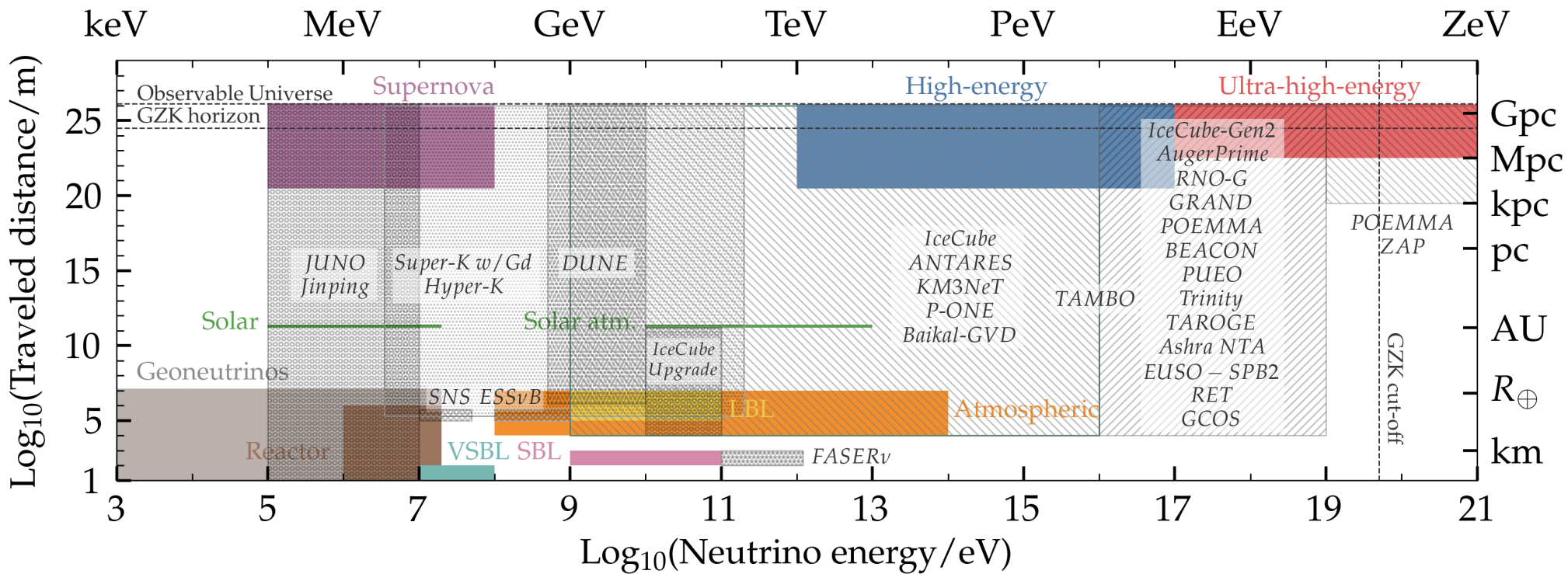
Synergies with lower energies

Discovered in 2013
by IceCube



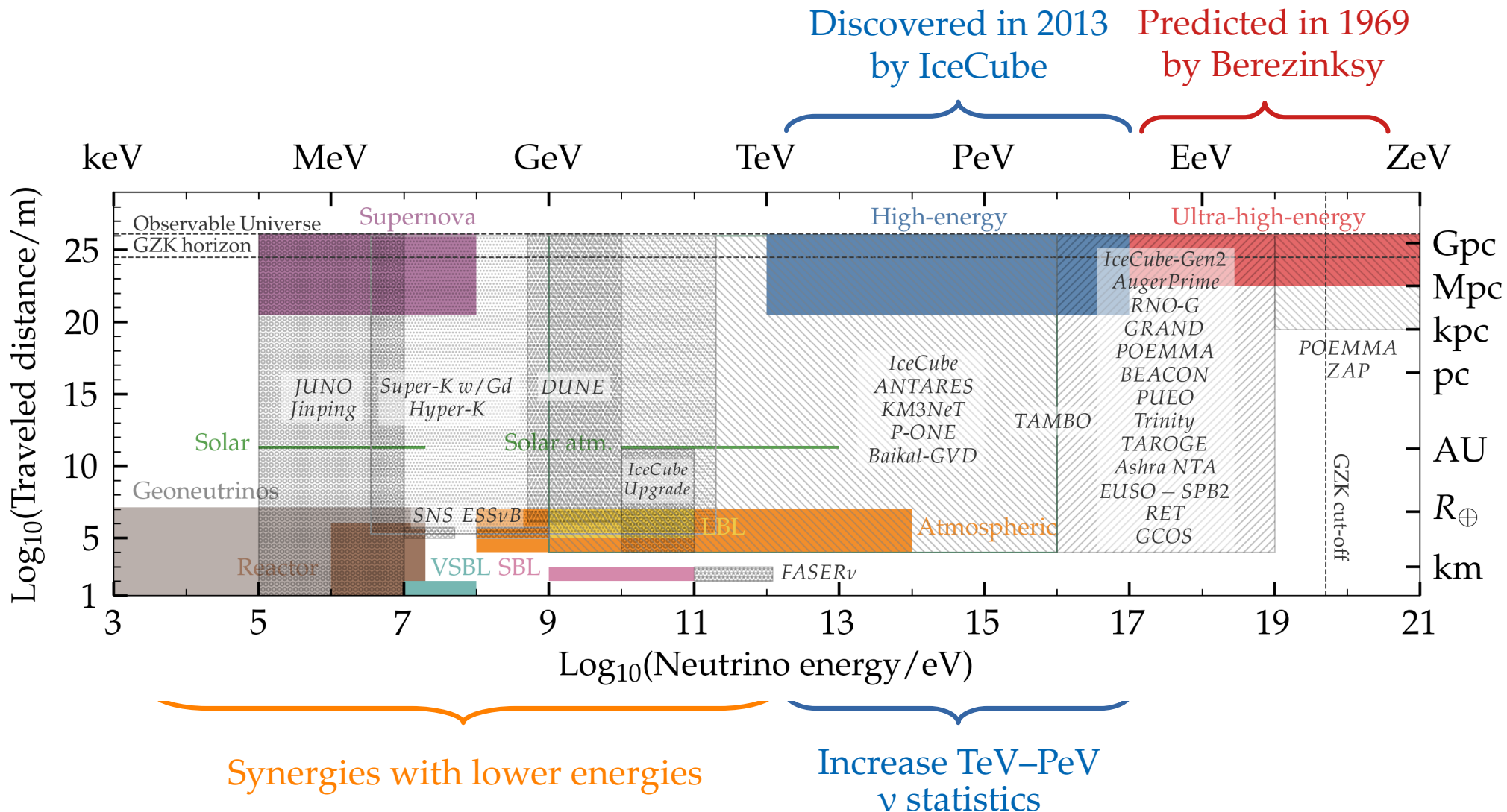
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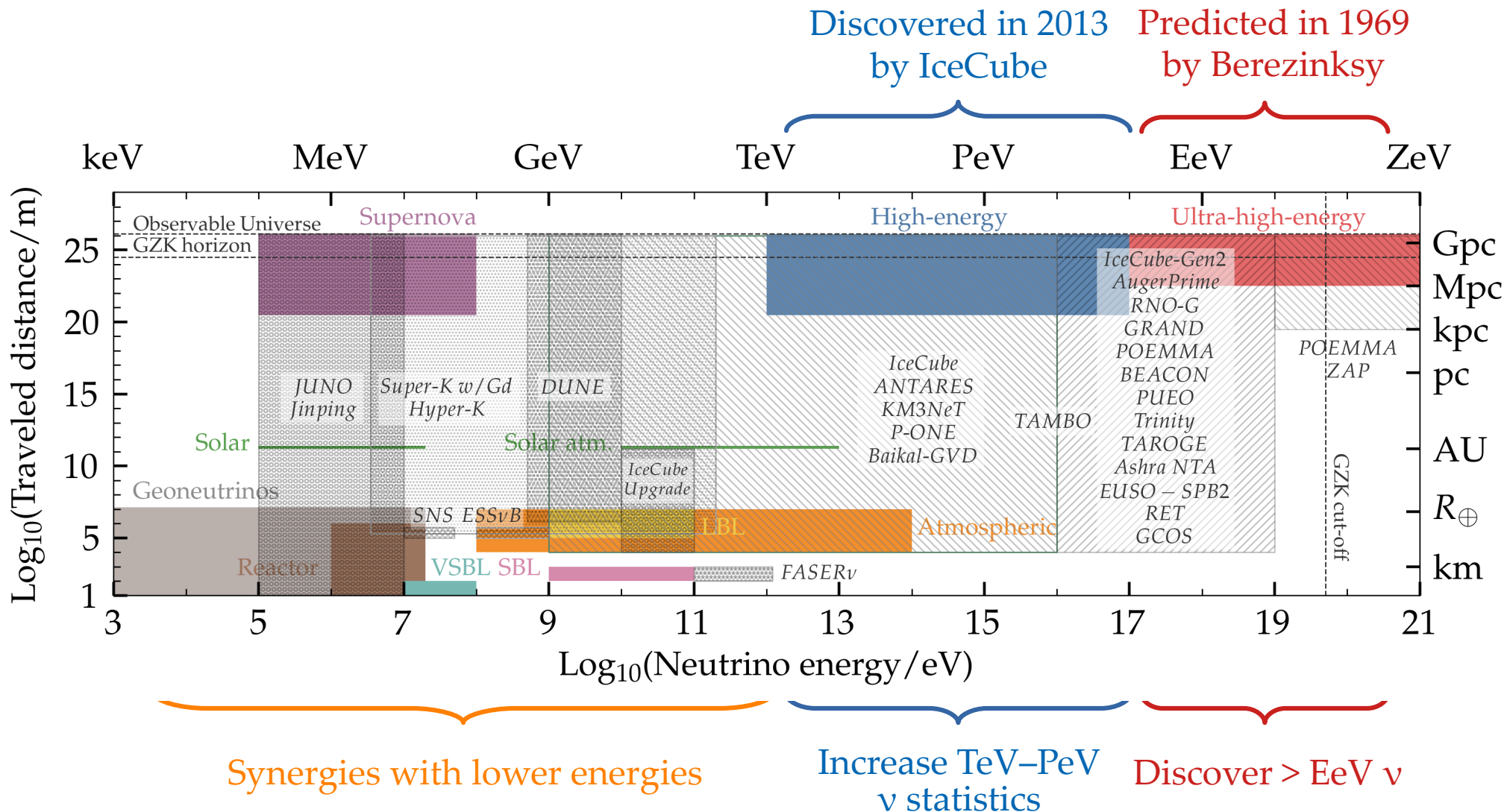
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Synergies with lower energies

Increase TeV-PeV
ν statistics





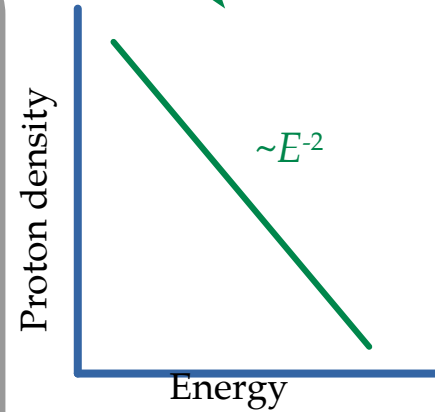
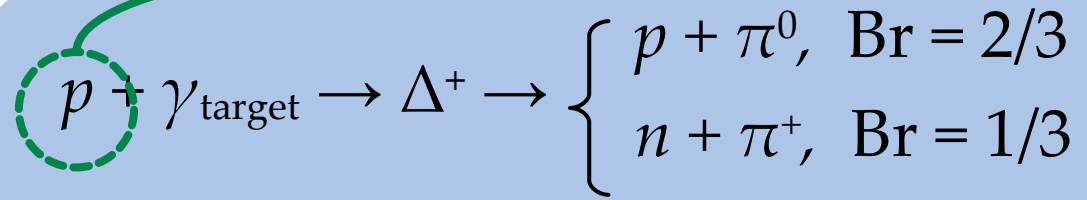
The multi-messenger connection: a simple picture

(or $p + p$)

$$p + \gamma_{\text{target}} \rightarrow \Delta^+ \rightarrow \begin{cases} p + \pi^0, & \text{Br} = 2/3 \\ n + \pi^+, & \text{Br} = 1/3 \end{cases}$$

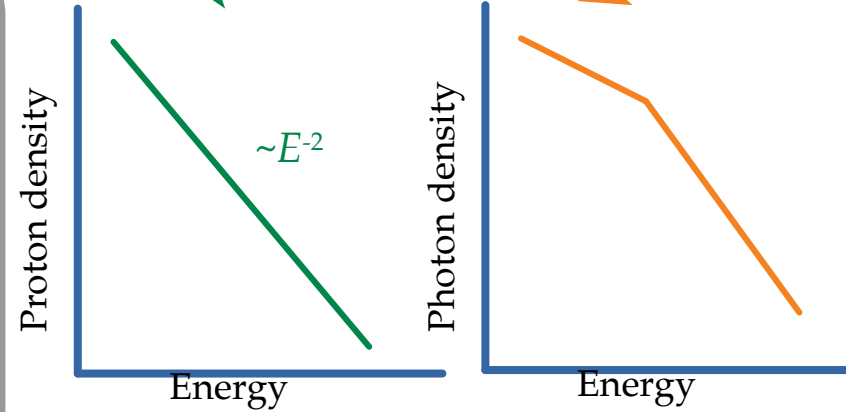
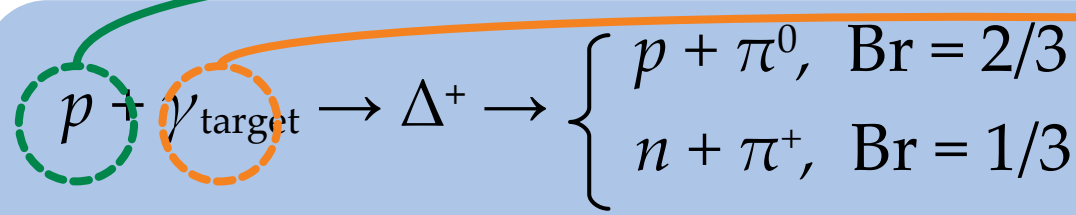
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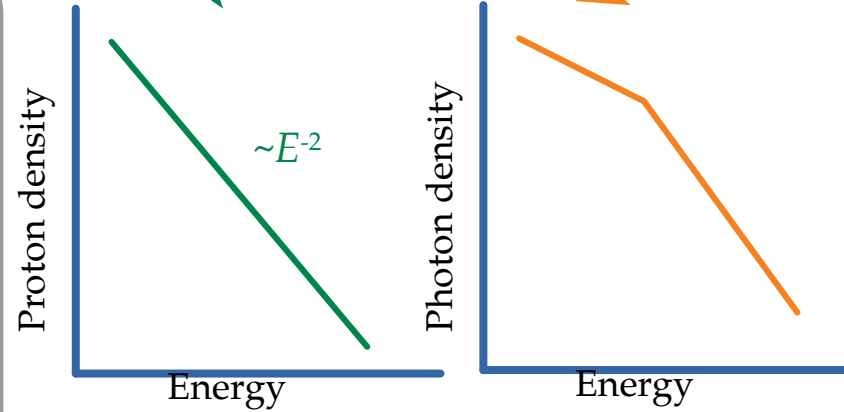
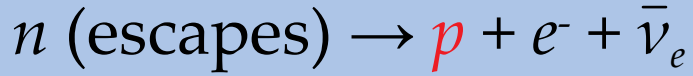
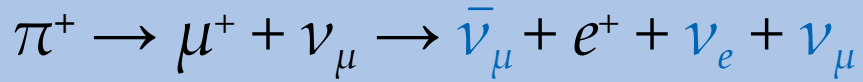
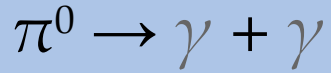
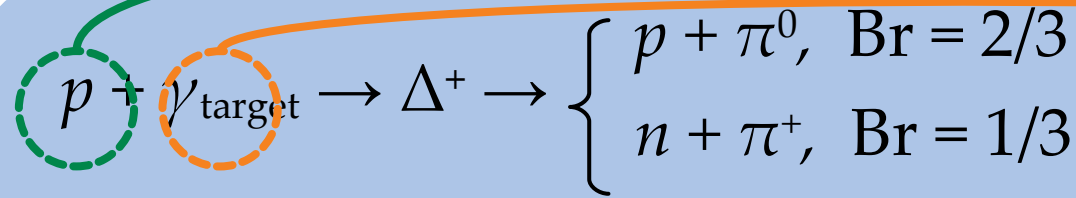
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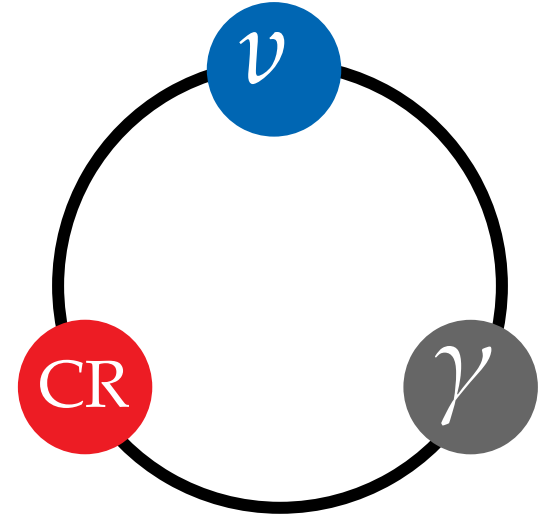
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$$\pi^+ \rightarrow \mu^+ + \nu_{\mu} \rightarrow \bar{\nu}_{\mu} + e^+ + \nu_e + \nu_{\mu}$$

$$n \text{ (escapes)} \rightarrow p + e^- + \bar{\nu}_e$$



Neutrino energy = Proton energy / 20

Gamma-ray energy = Proton energy / 10

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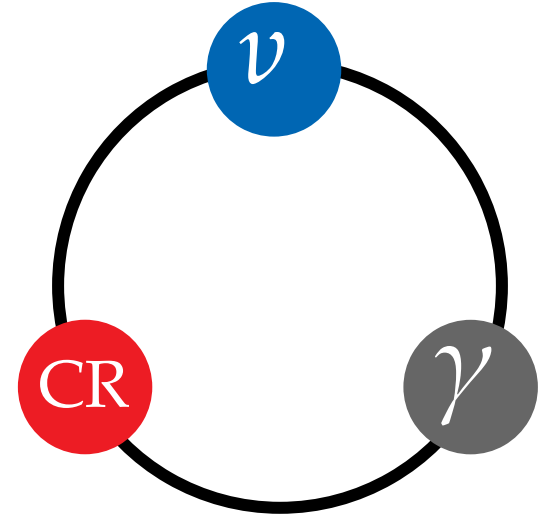
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1 PeV

20 PeV

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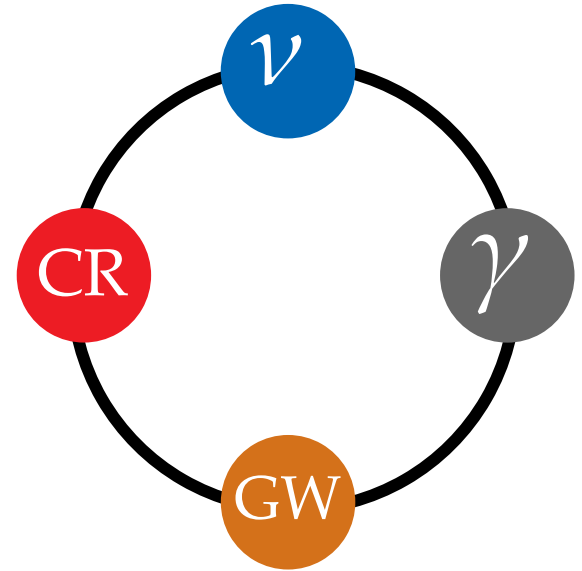
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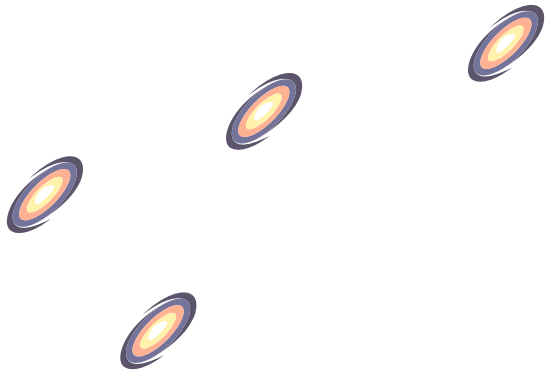
Gamma-ray energy = Proton energy / 10

Redshift



$z = 0$

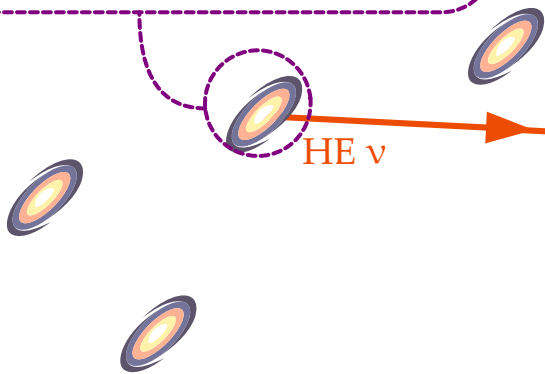
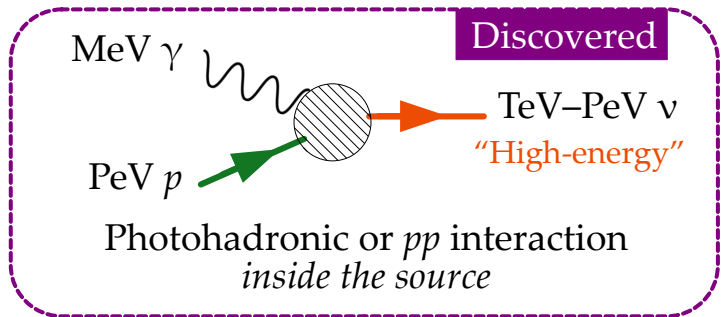
Note: ν sources can be steady-state or transient



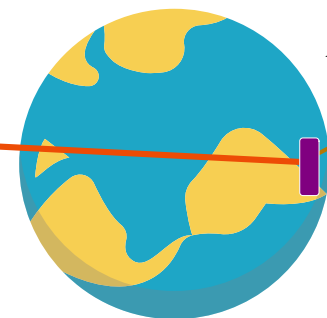
Redshift ←

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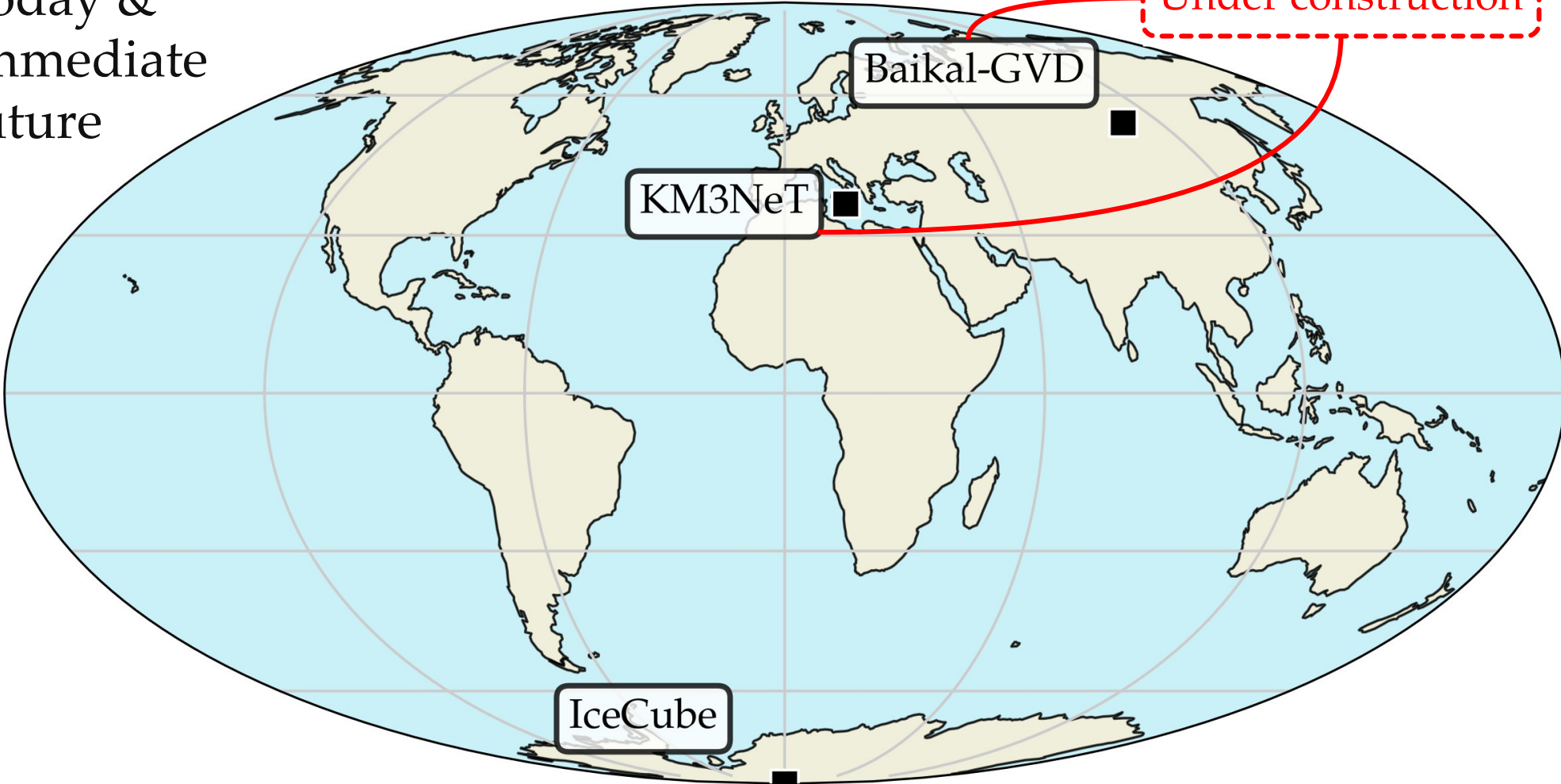


ν propagation
inside the Earth

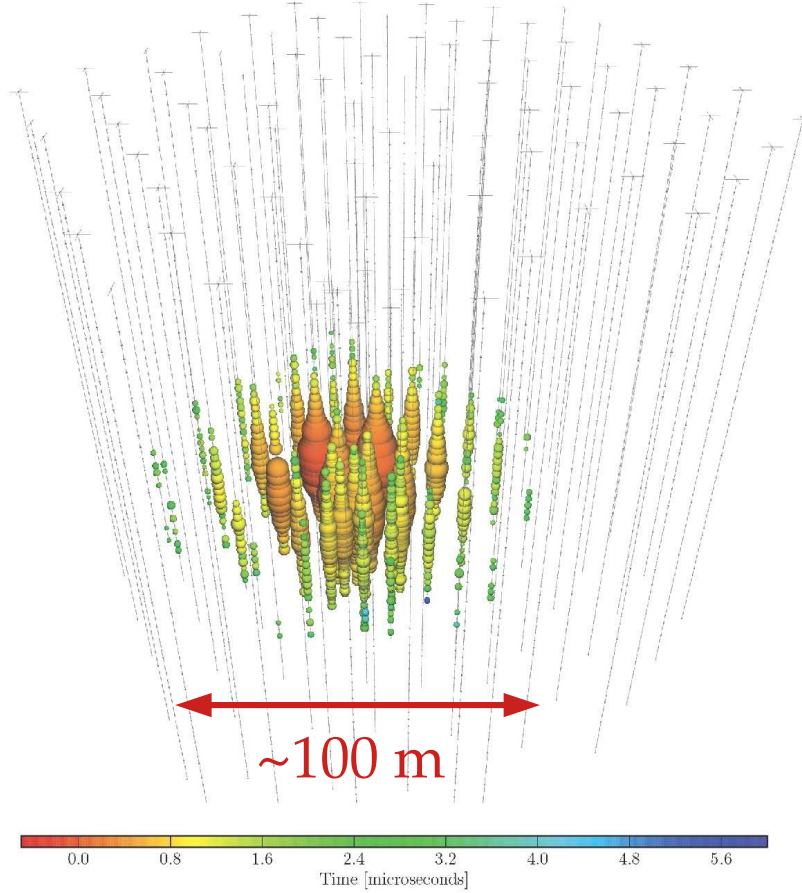


ν detection

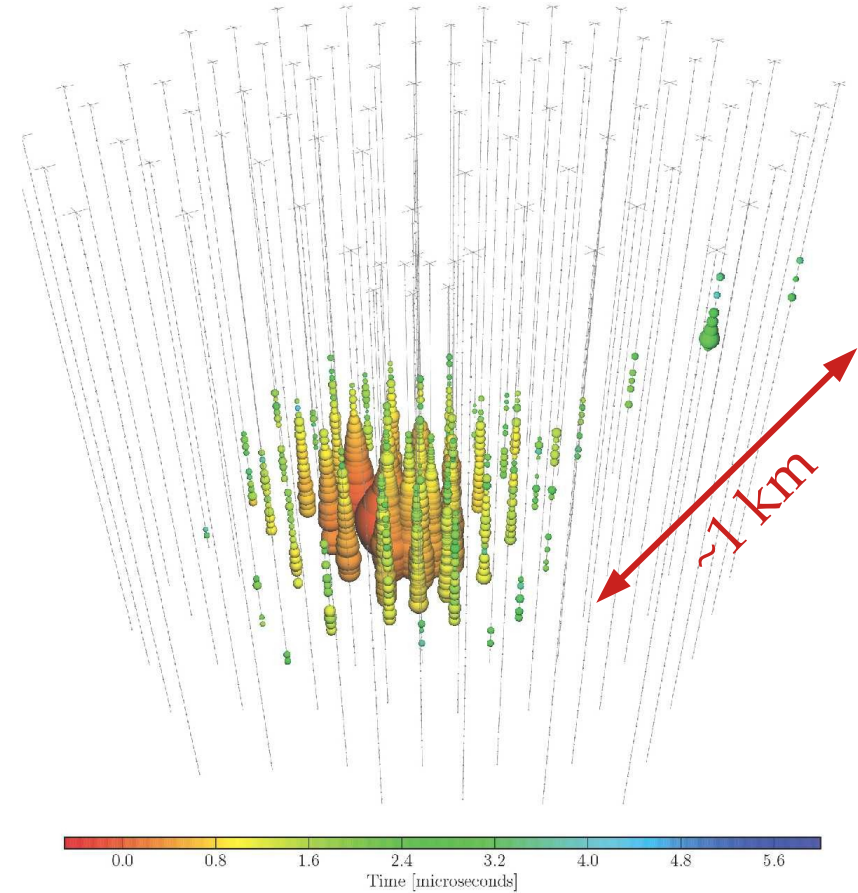
Today &
immediate
future



Shower (mainly from ν_e and ν_τ)

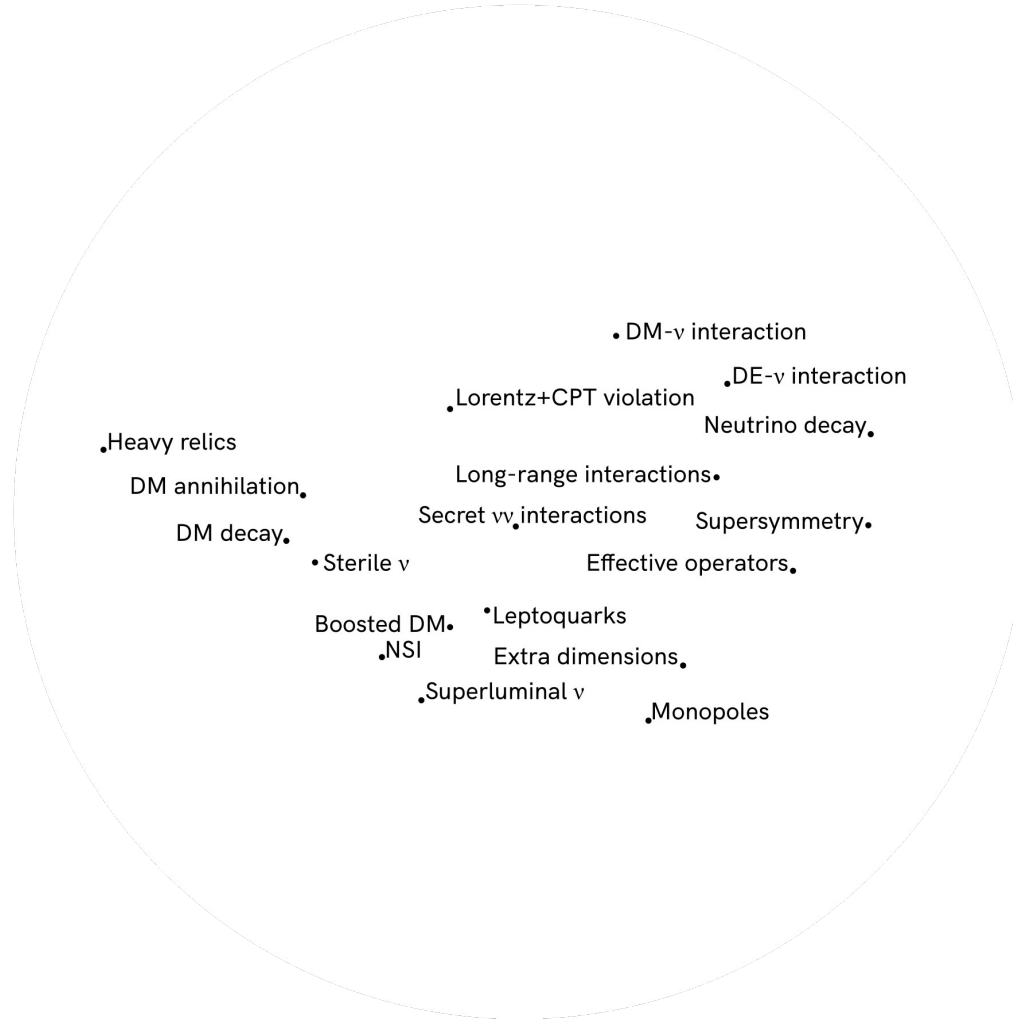


Track (mainly from ν_μ)



Poor angular resolution: $< 5^\circ$

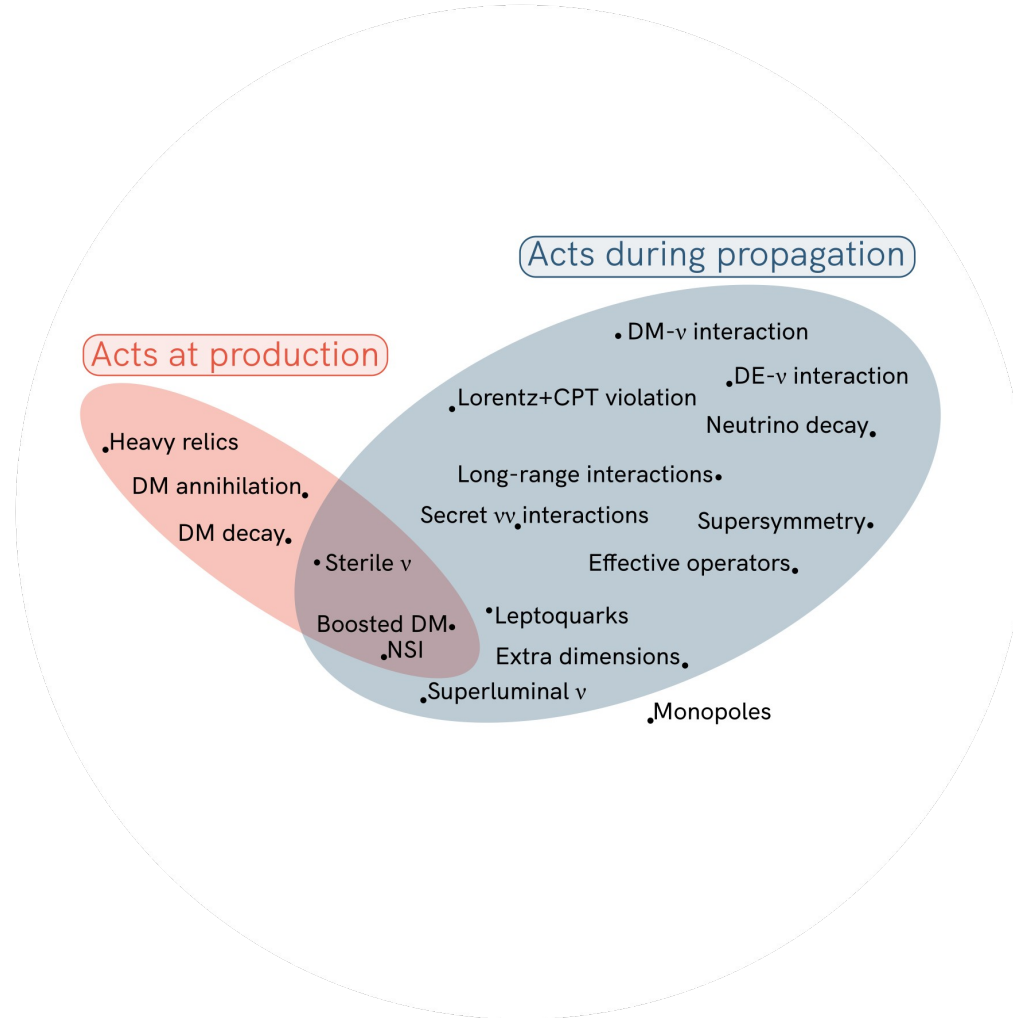
Angular resolution: $< 1^\circ$



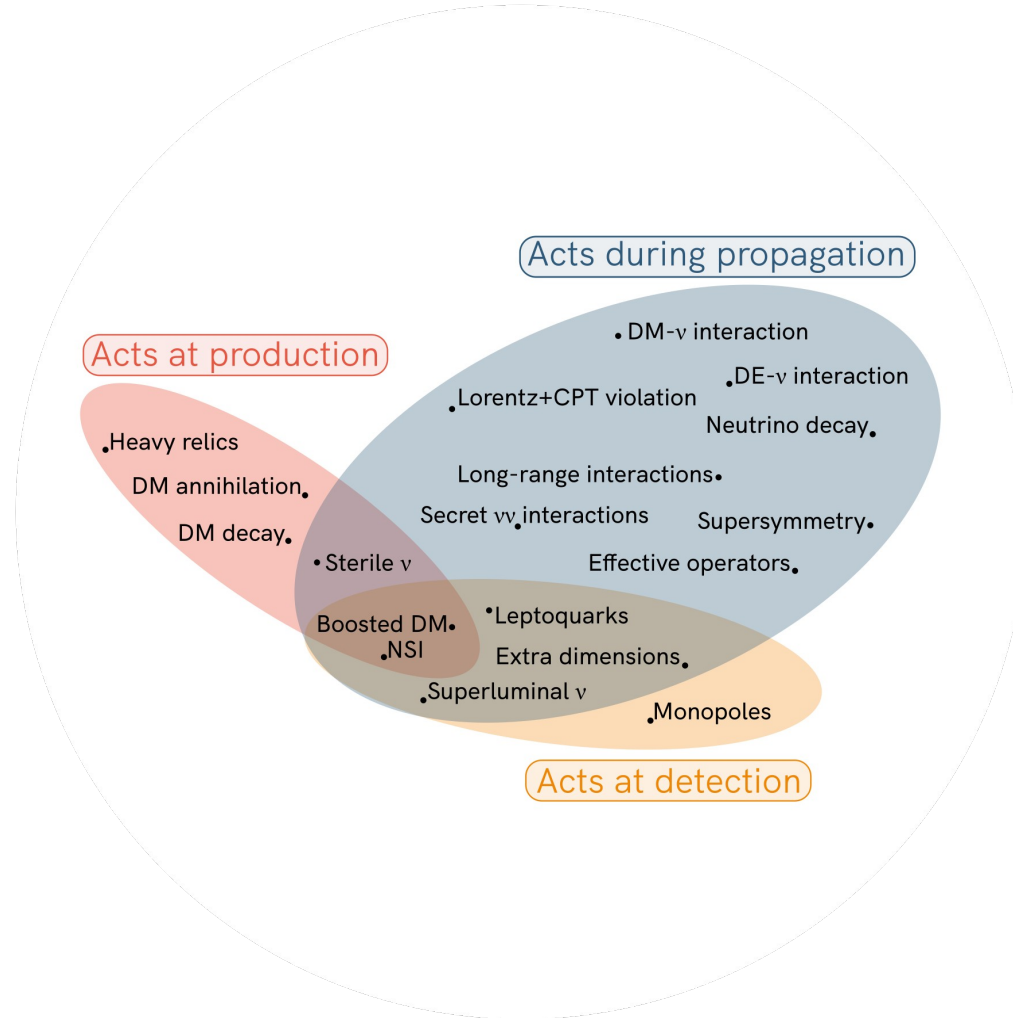
Note: Not an exhaustive list



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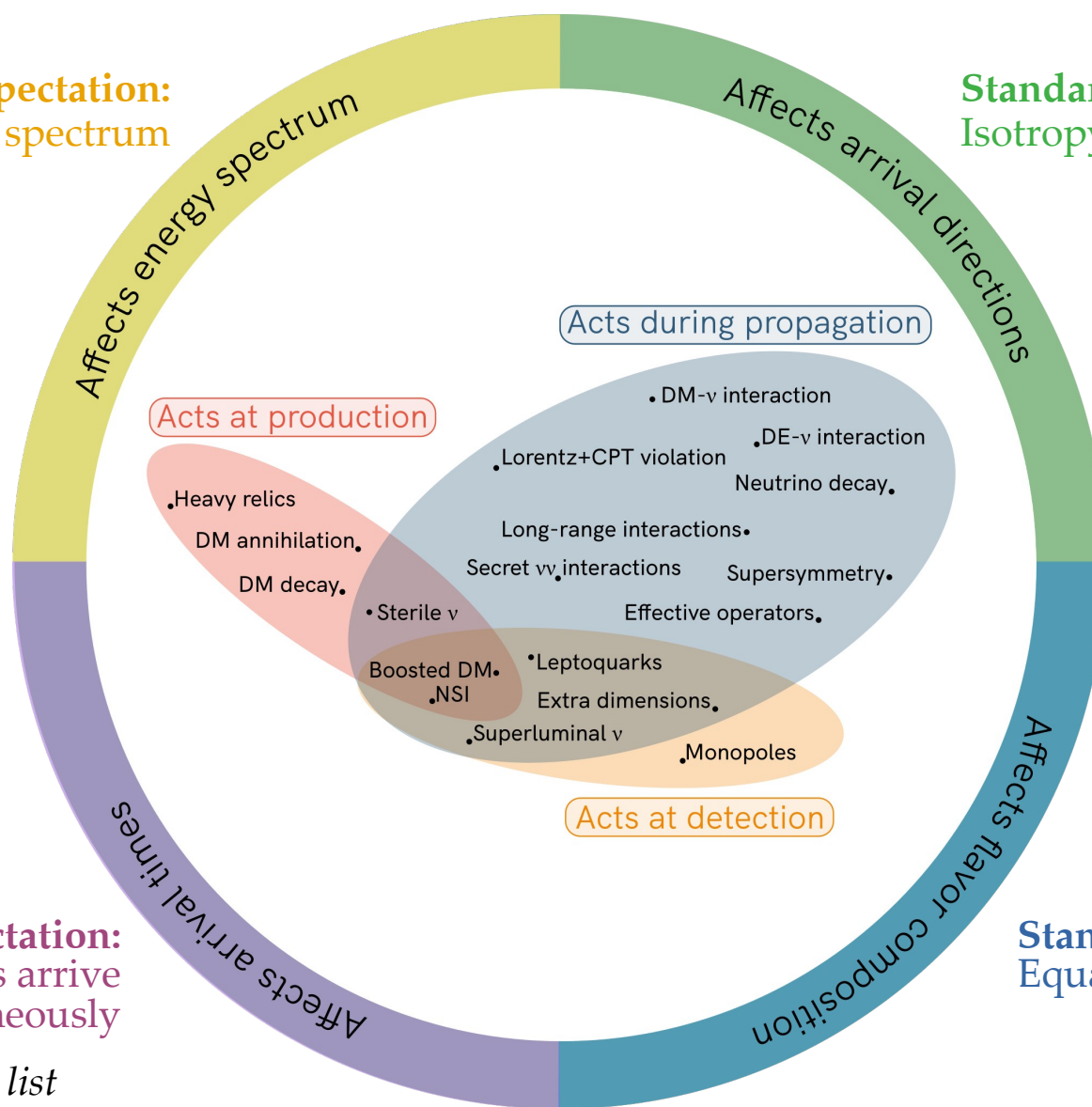
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Standard expectation:
Power-law energy spectrum

Standard expectation:
Isotropy (for diffuse flux)



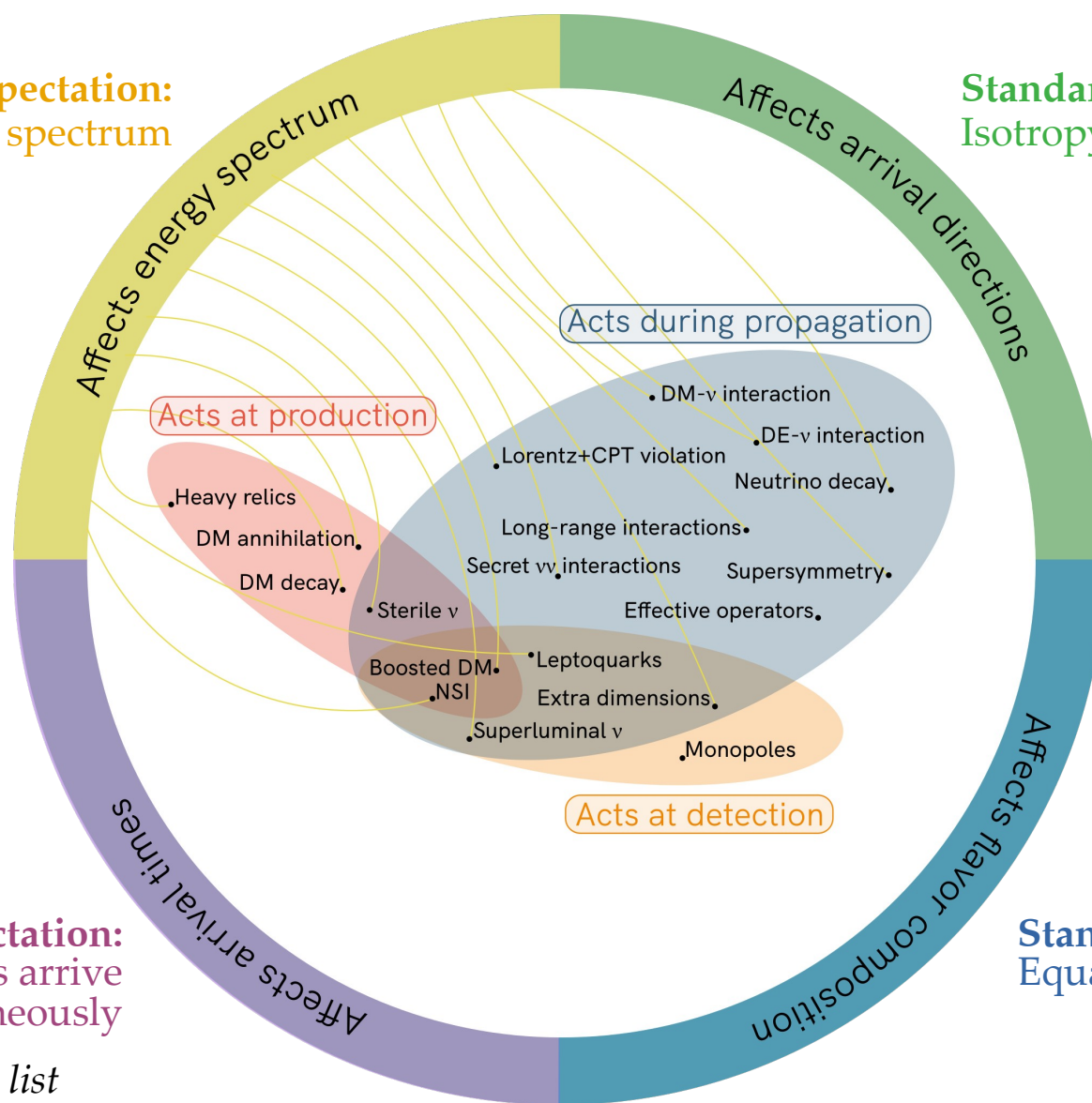
Standard expectation:
Equal number of ν_e, ν_μ, ν_τ

Standard expectation:
 ν and γ from transients arrive
simultaneously

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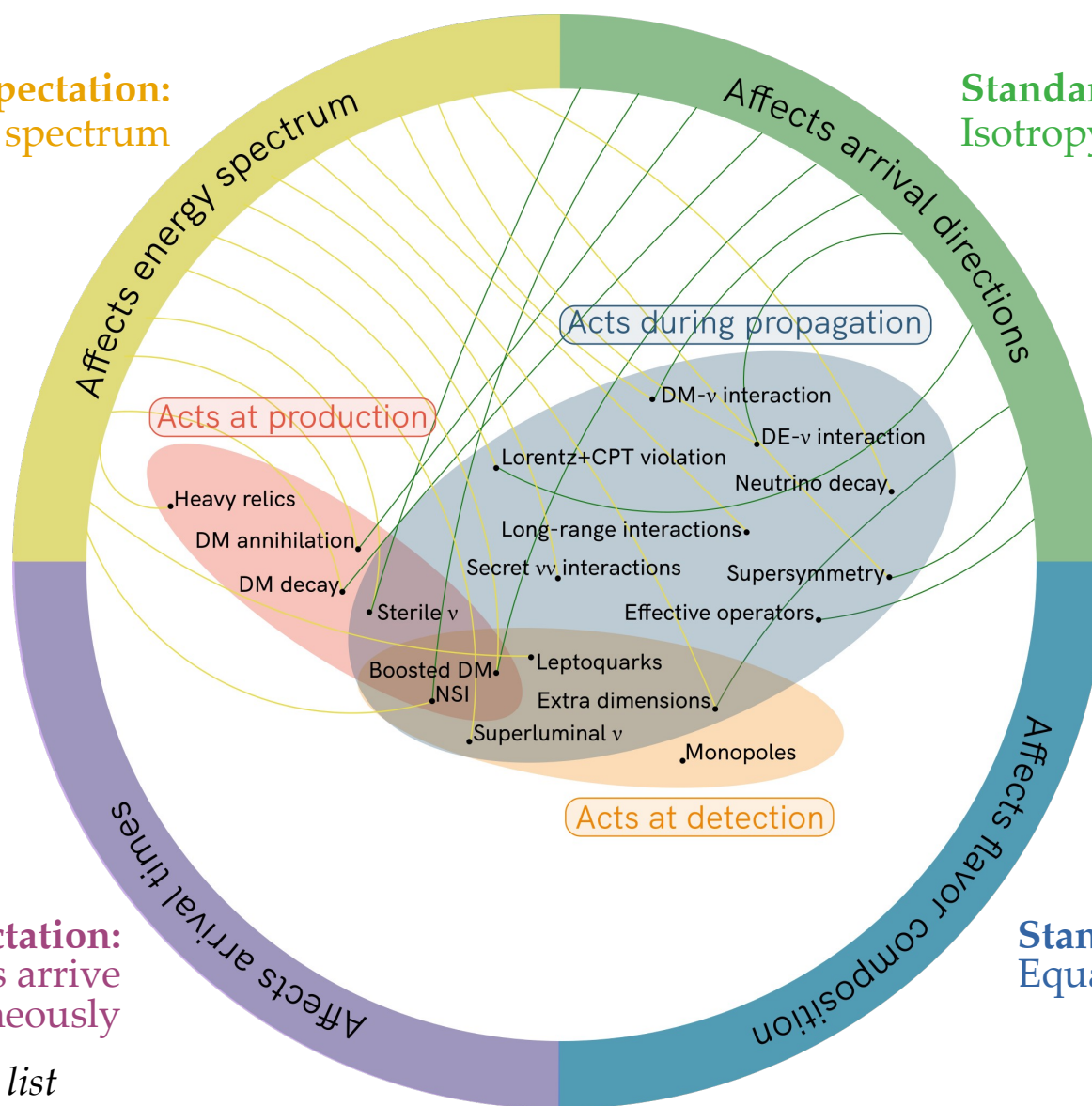
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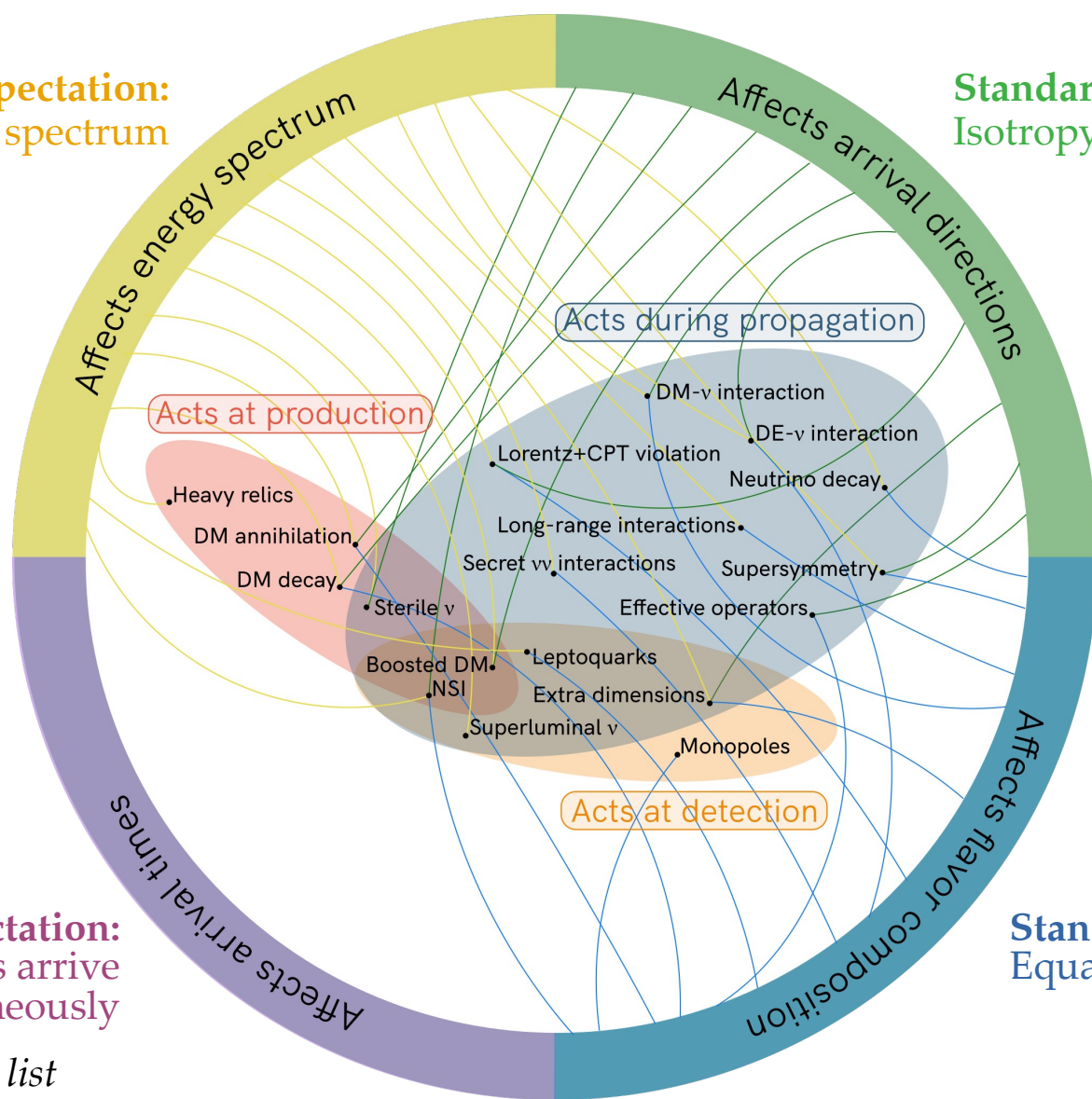
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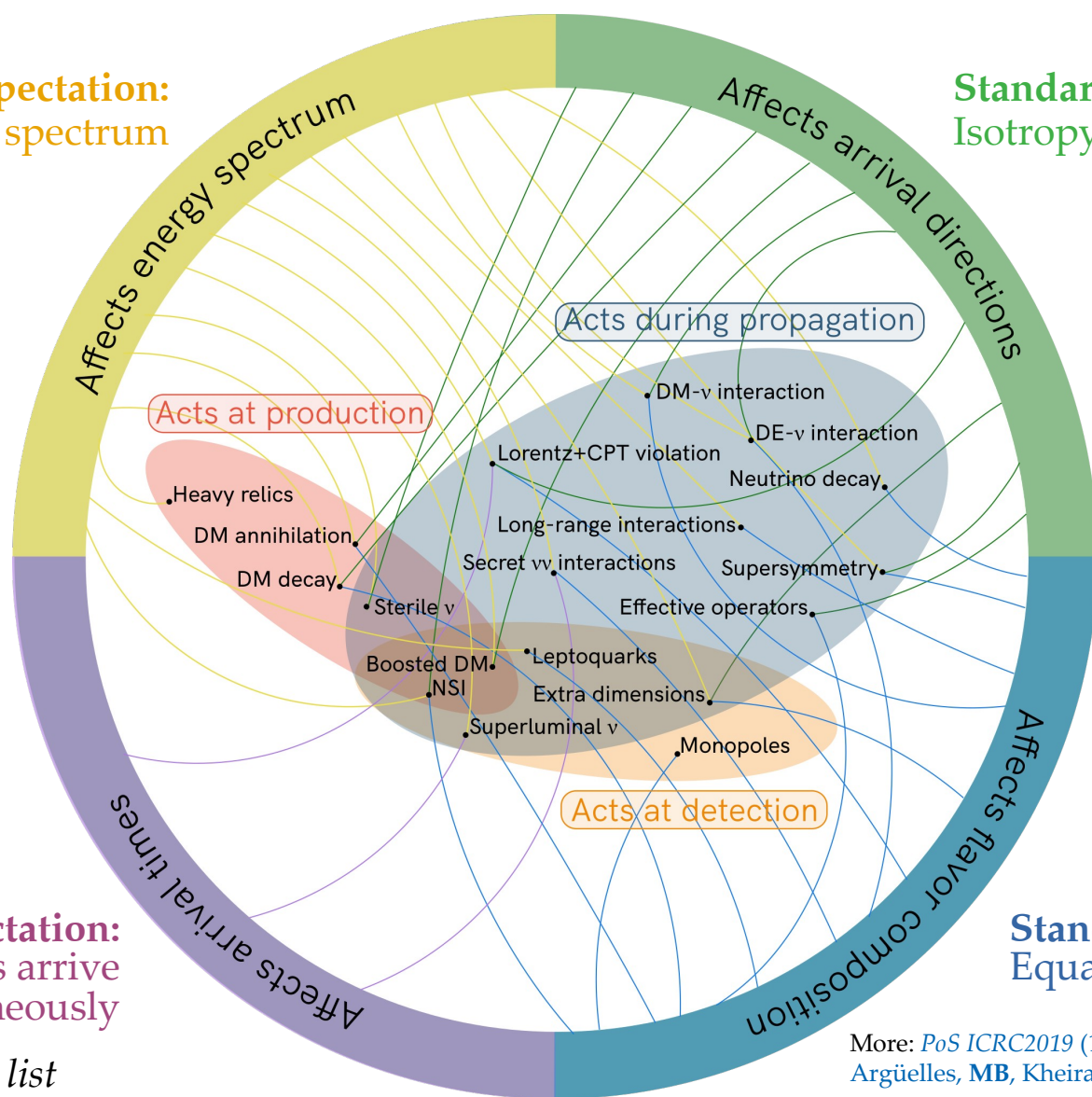
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More: *PoS ICRC2019 (1907.08690)*
Argüelles, MB, Kheirandish, Palomares-Ruiz, Salvadó, Vincent

Standard expectation:
Power-law energy spectrum

Standard expectation:
Isotropy (for diffuse flux)

Affects energy spectrum

Affects arrival directions

Acts during propagation

Acts at production

Reviews:

Ahlers, Helbing, De los Heros, *EPJC* 2018

Argüelles, MB, Kheirandish, Palomares-Ruiz, Salvadó, Vincent, *ICRC* 2019 [1907.08690]

Ackermann, Ahlers, Anchordoqui, MB, et al., *Astro2020 Decadal Survey* [1903.04333]

Affects arrival times

Affects flavor composition

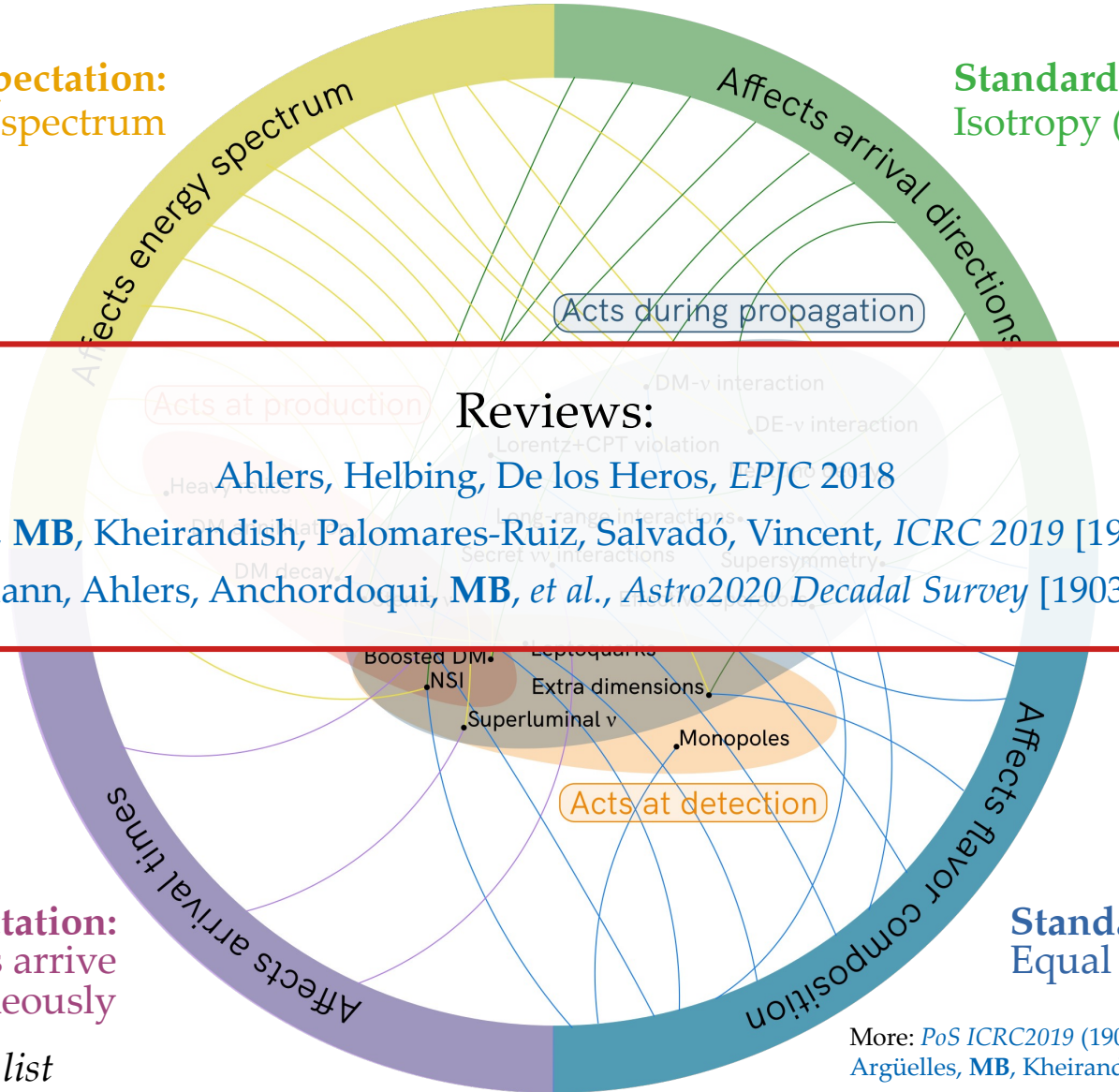
Acts at detection

Standard expectation:
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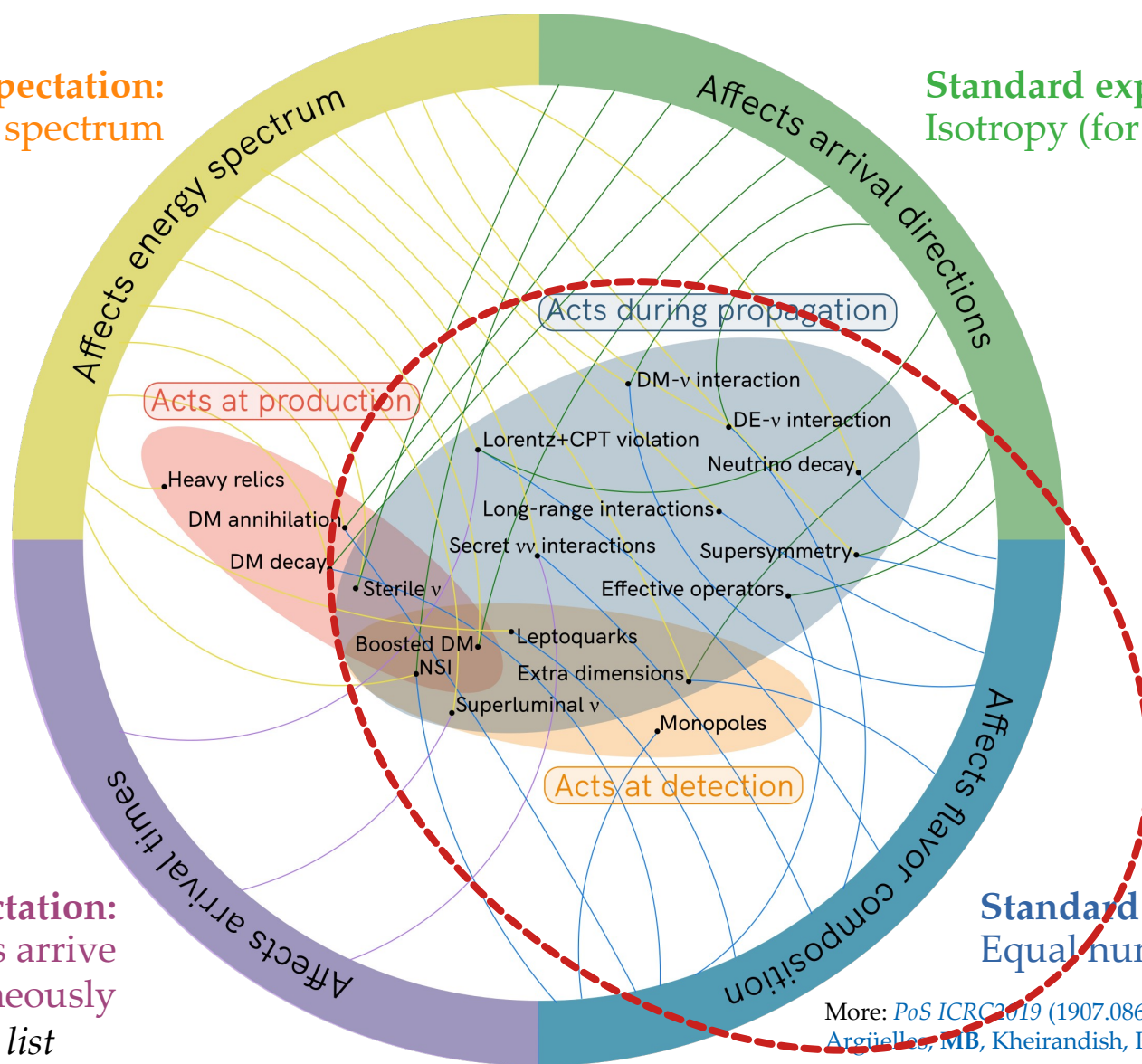
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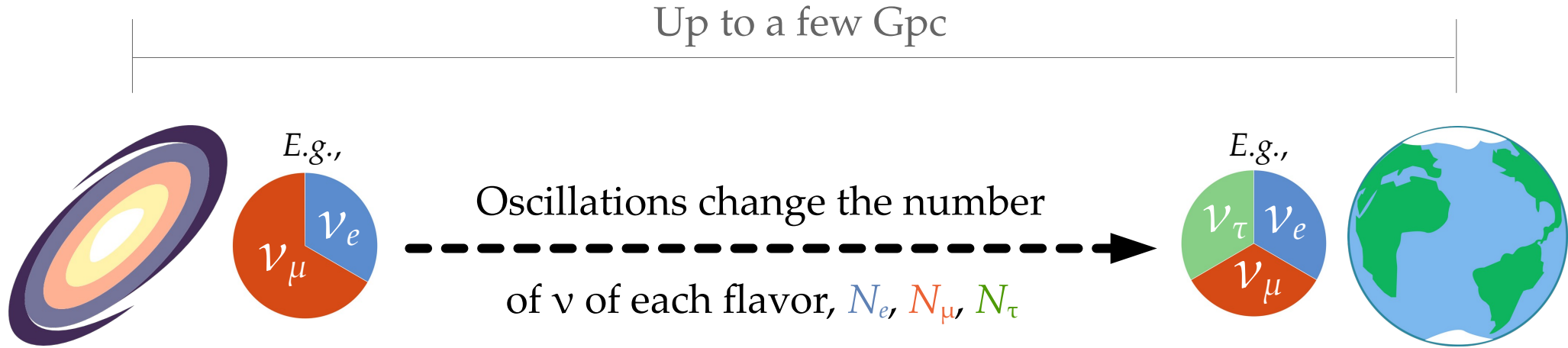
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Predicting the flavor composition at Earth

Astrophysical sources

Earth



Different production mechanisms yield different flavor ratios:

$$(f_{e,S}, f_{\mu,S}, f_{\tau,S}) \equiv (N_{e,S}, N_{\mu,S}, N_{\tau,S})/N_{\text{tot}}$$

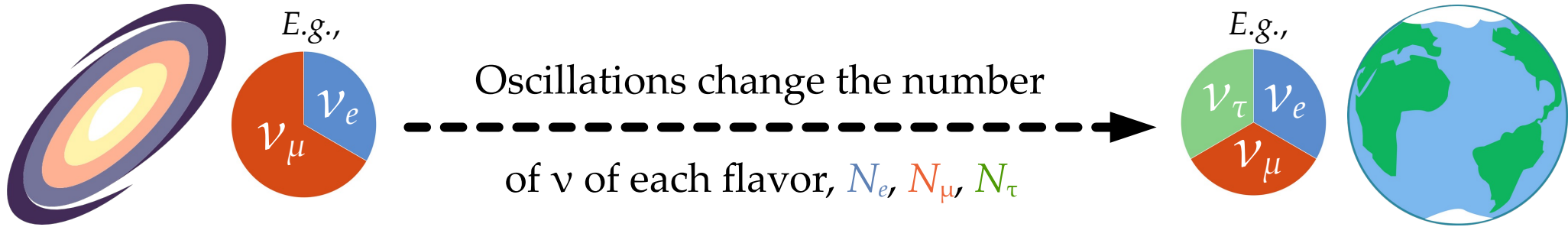
Flavor ratios at Earth ($\alpha = e, \mu, \tau$):

$$f_{\alpha,\oplus} = \sum_{\beta=e,\mu,\tau} P_{\nu_\beta \rightarrow \nu_\alpha} f_{\beta,S}$$

Astrophysical sources

Earth

Up to a few Gpc



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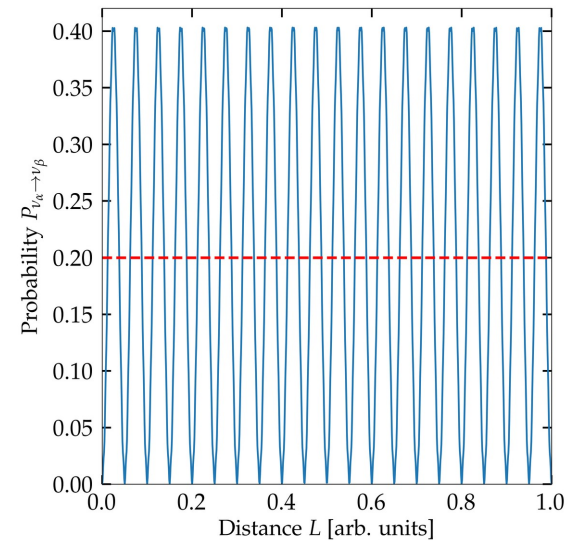
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Standard oscillations
or
new physics

Full 3ν oscillation probability:

$$P_{\alpha\beta}(E, L) = \delta_{\alpha\beta} - 4 \sum_{i>j} \operatorname{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + 2 \sum_{i>j} \operatorname{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right)$$



Oscillation length for 1-TeV ν: $2\pi \times 2E/\Delta m^2 \sim 0.1$ pc

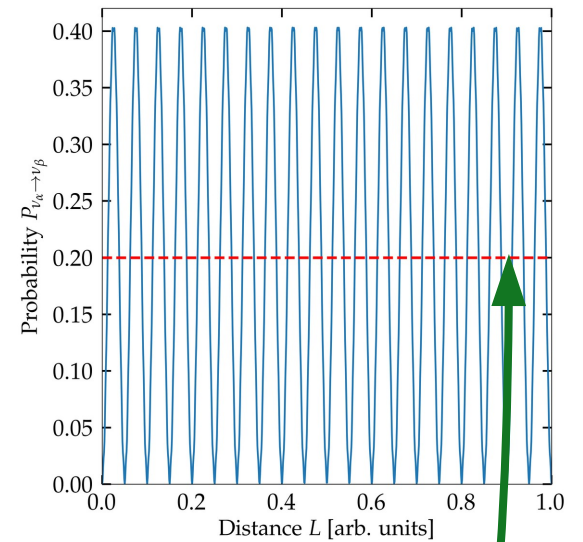
- ~ 8% of the way to Proxima Centauri
- ≪ Distance to Galactic Center (8 kpc)
- ≪ Distance to Andromeda (1 Mpc)
- ≪ Cosmological distances (few Gpc)

We cannot resolve oscillations, so we use instead the average probability:

$$P_{\alpha\beta} = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2$$

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Flavor composition at the Earth:

$$\begin{pmatrix} f_{e,\oplus} \\ f_{\mu,\oplus} \\ f_{\tau,\oplus} \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix} \begin{pmatrix} f_{e,S} \\ f_{\mu,S} \\ f_{\tau,S} \end{pmatrix}$$

Informed by
particle physics



Informed by astrophysics



Flavor composition at the Earth:

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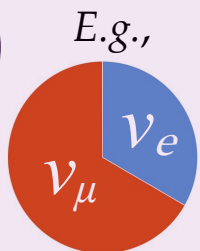
Assume no ν_τ production,
so we only need vary the ν_e fraction

Farzan, *JHEP* 2021

From sources to Earth: we learn what to expect when measuring $f_{\alpha,\oplus}$



Sources



$(f_{e,S}, f_{\mu,S}, f_{\tau,S})$

Oscillations



$(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{CP})$

Earth



$(f_{e,\oplus}, f_{\mu,\oplus}, f_{\tau,\oplus})$

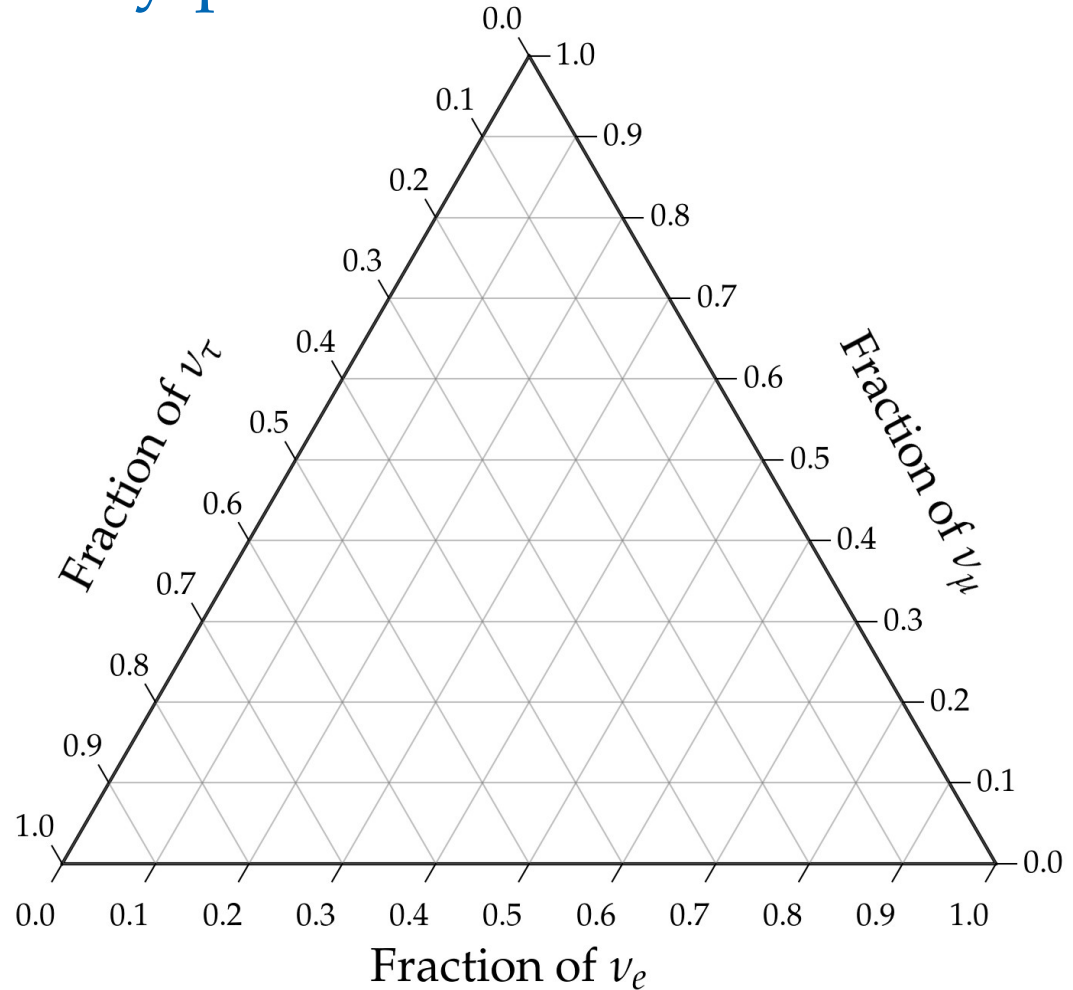
Quick aside: how to read a ternary plot

Assumes underlying unitarity –
sum of projections on each axis is 1

How to read it:

Follow the tilt of the tick marks

Always in this order: (f_e, f_μ, f_τ)



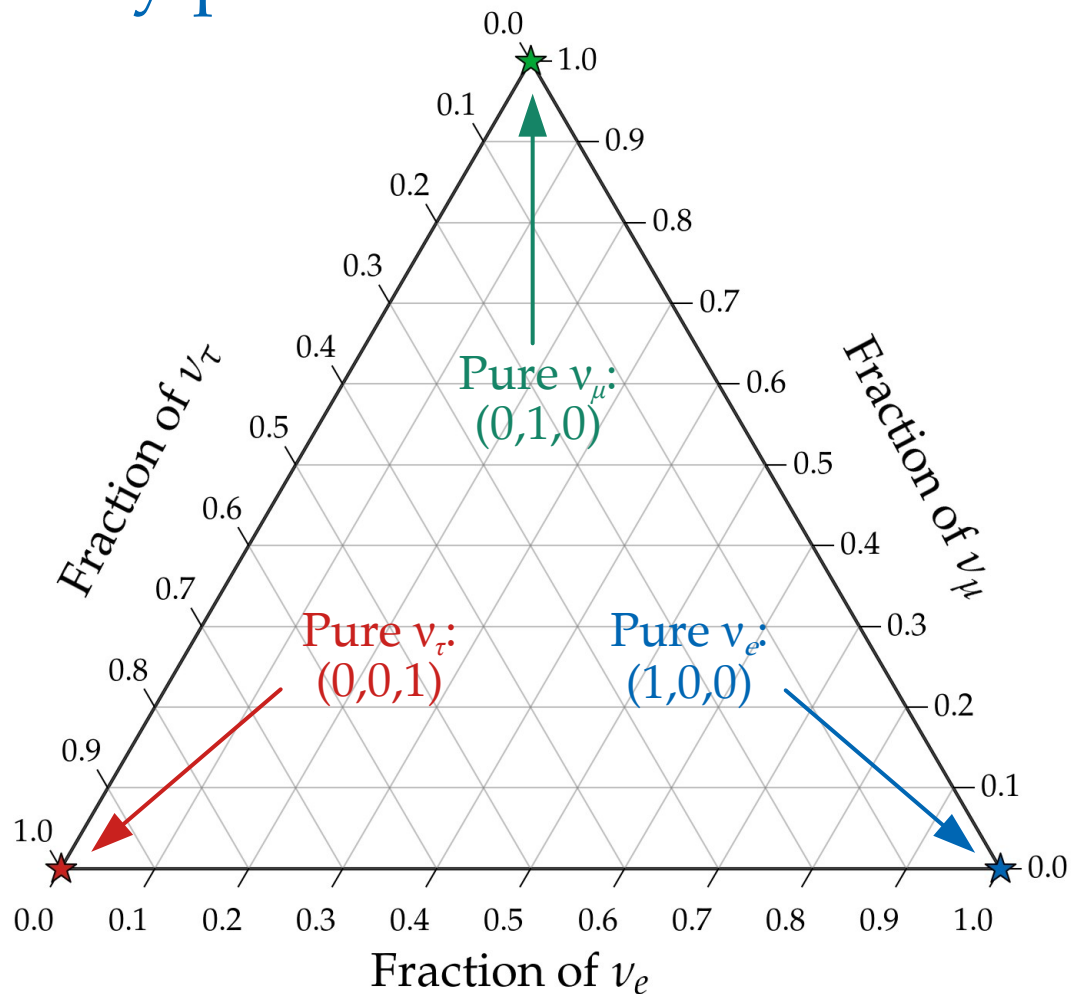
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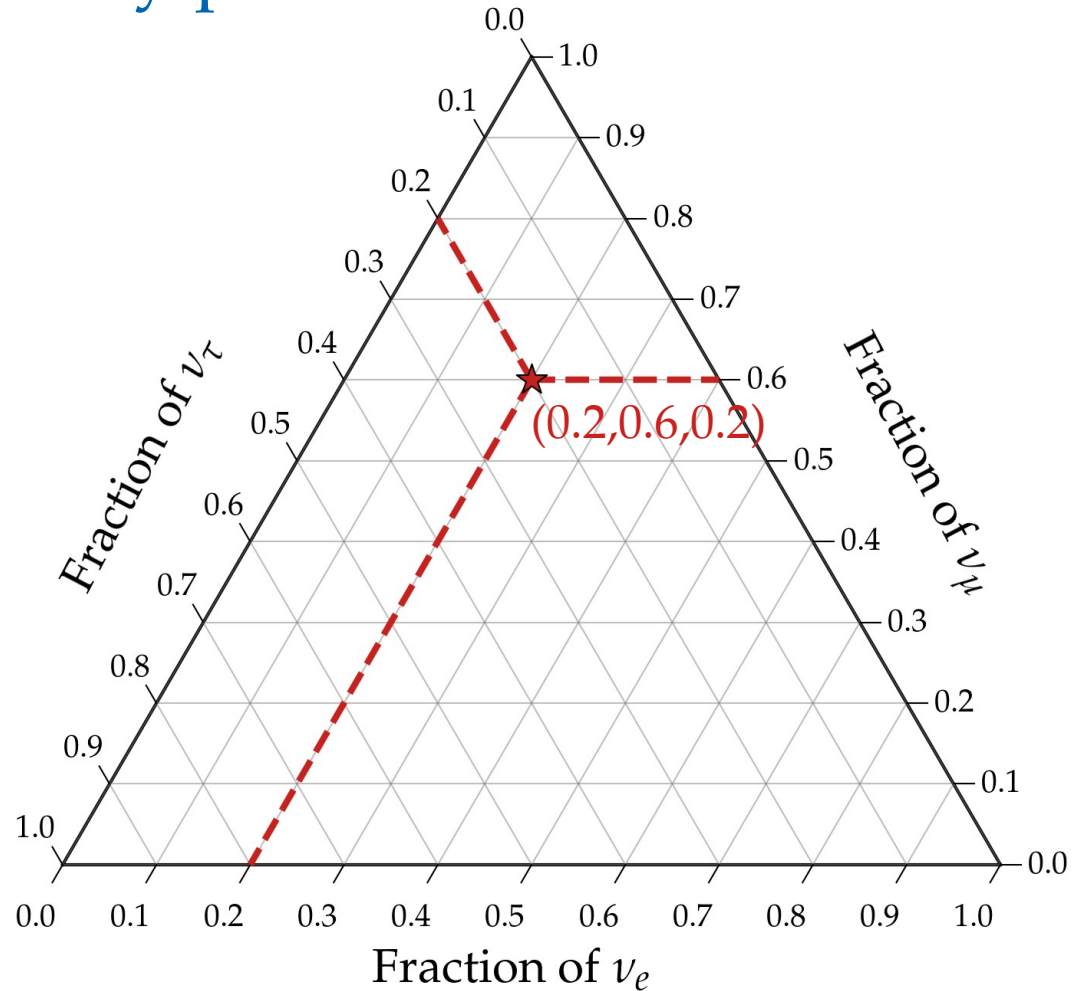
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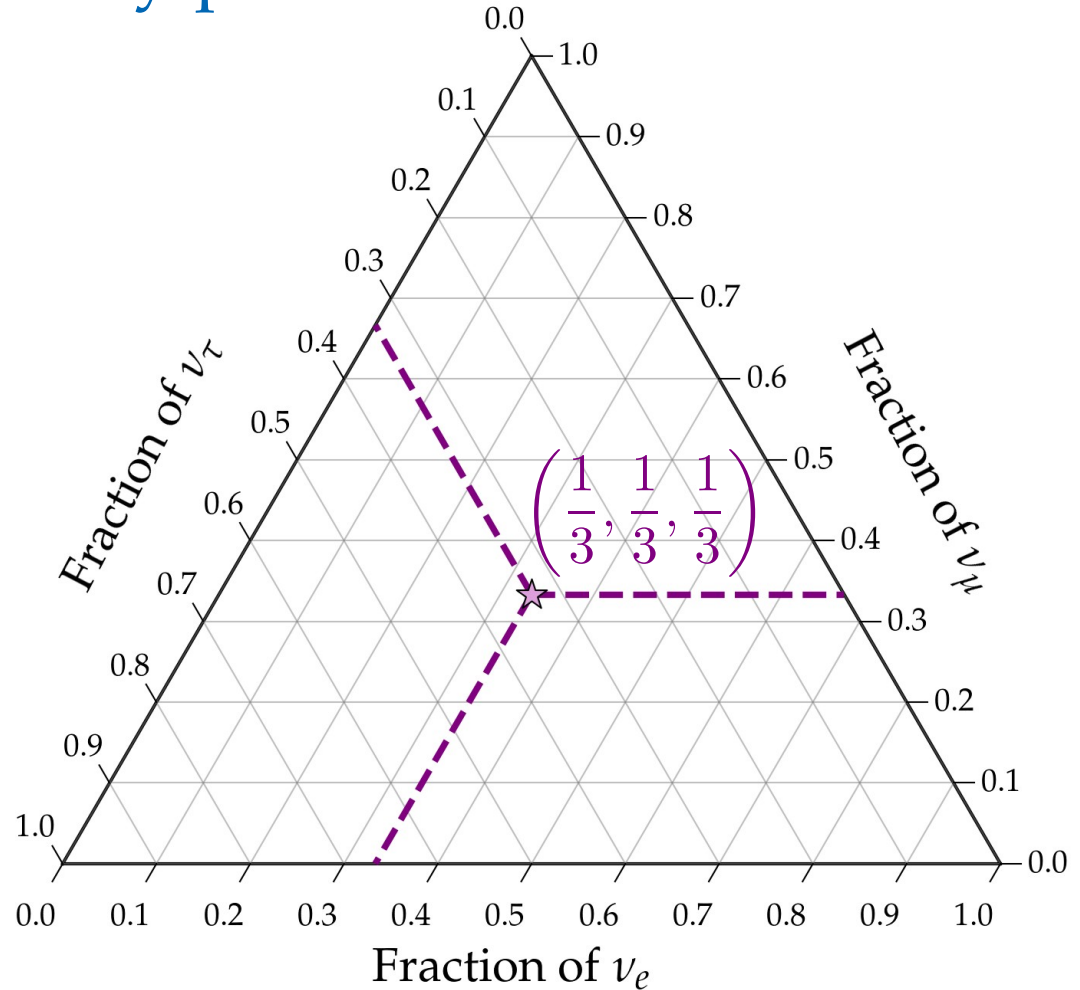
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One likely TeV–PeV ν production scenario:

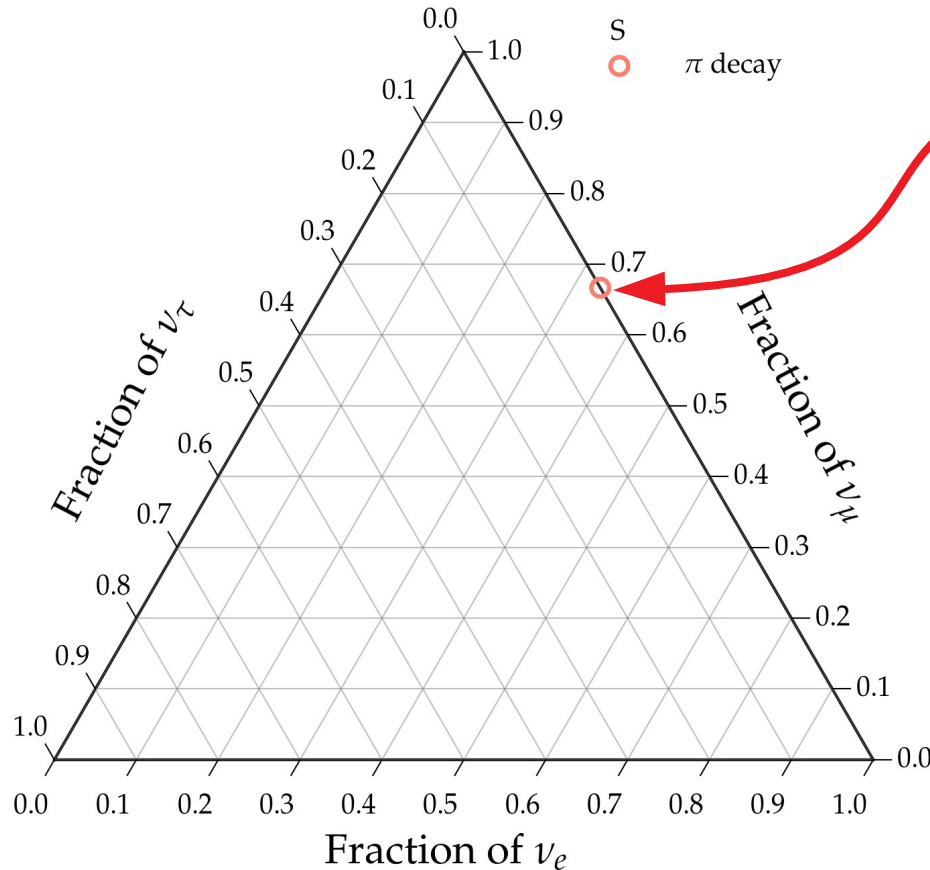
$$p + \gamma \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu \quad \text{followed by} \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

Full π decay chain

$$(1/3:2/3:0)_S$$

Note: ν and $\bar{\nu}$ are (so far) indistinguishable
in neutrino telescopes

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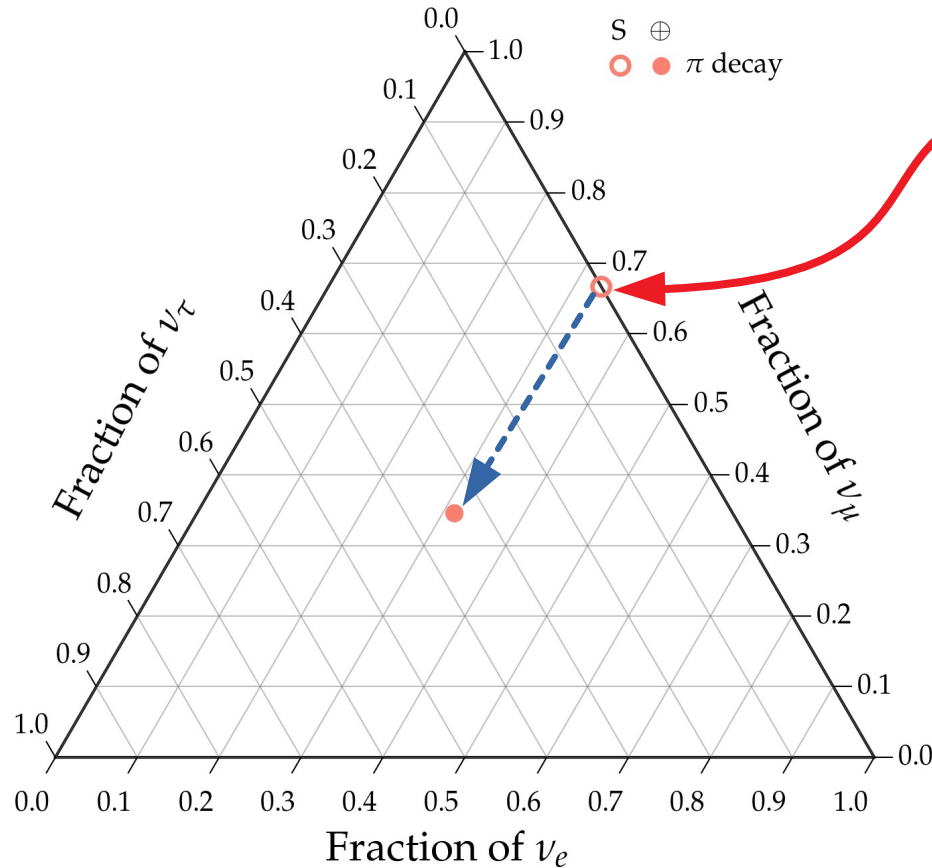


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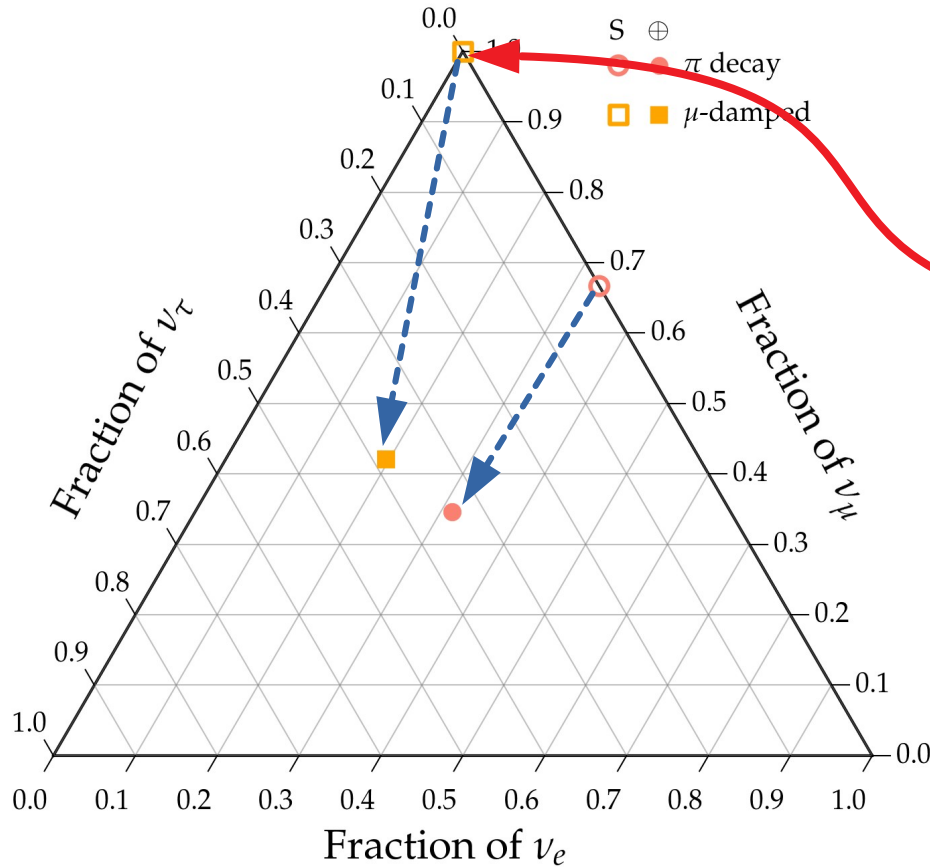


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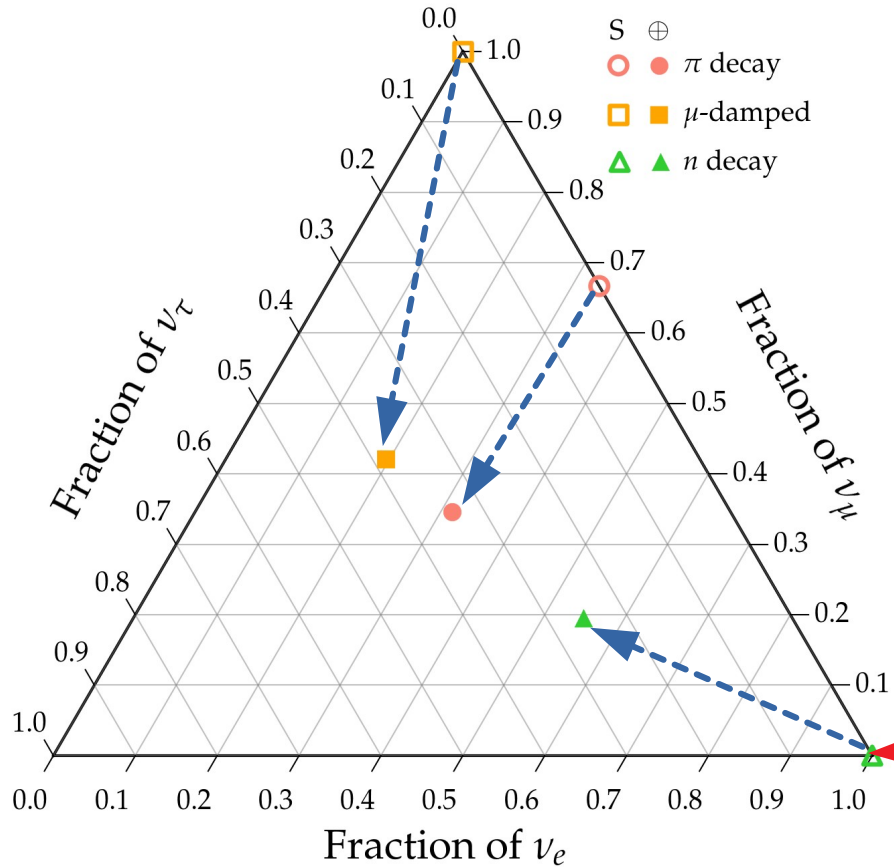
$(1/3:2/3:0)_S$

Muon damped

$(0:1:0)_S$

Note: ν and $\bar{\nu}$ are (so far) indistinguishable in neutrino telescopes

One likely TeV–PeV ν production scenario:



Full π decay chain

$(1/3:2/3:0)_S$

Muon damped

$(0:1:0)_S$

Neutron decay

$(1:0:0)_S$

Note: ν and $\bar{\nu}$ are (so far) indistinguishable in neutrino telescopes

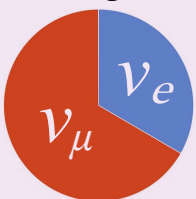
From sources to Earth: we learn what to expect when measuring $f_{\alpha,\oplus}$



Sources



E.g.,



$(f_{e,S}, f_{\mu,S}, f_{\tau,S})$

Oscillations



$(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{CP})$

Earth

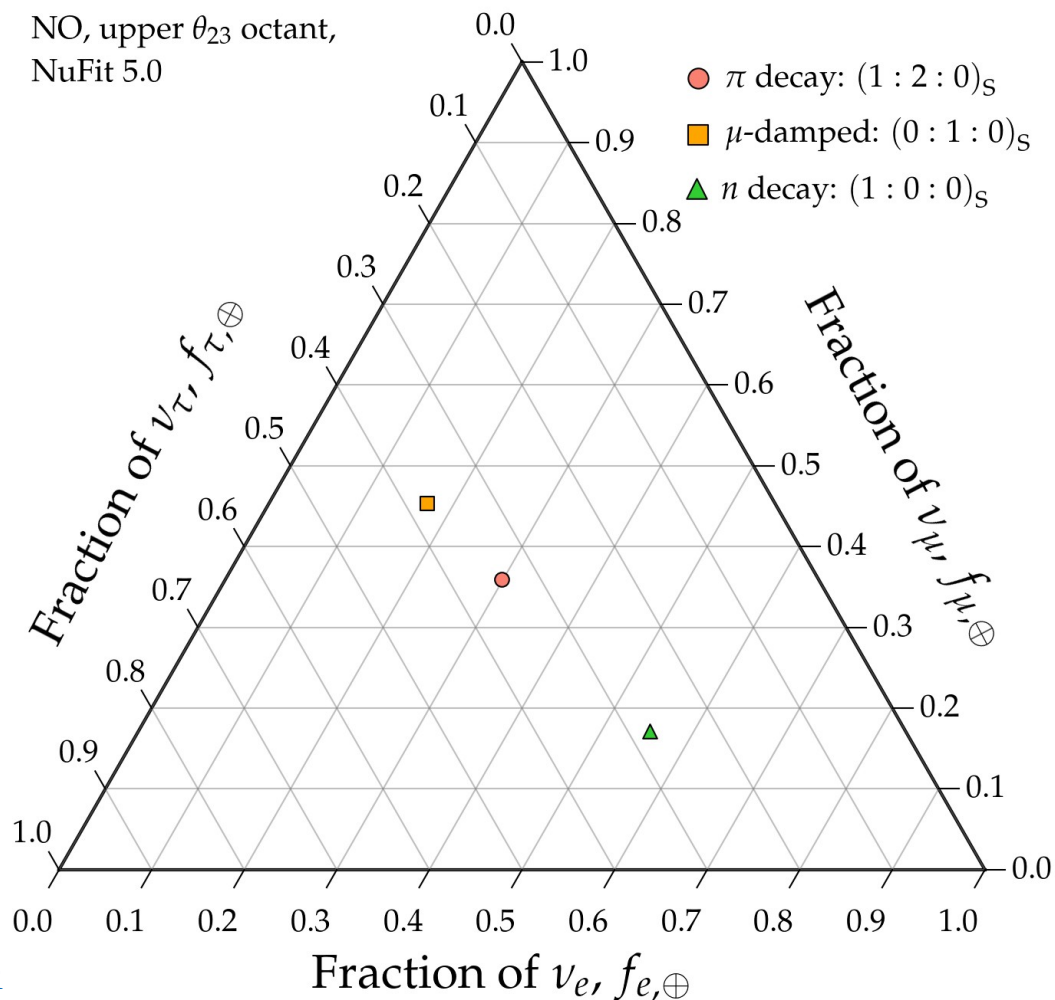


$(f_{e,\oplus}, f_{\mu,\oplus}, f_{\tau,\oplus})$

Known from oscillation experiments, to different levels of precision

Theoretically palatable regions: today

NO, upper θ_{23} octant,
NuFit 5.0



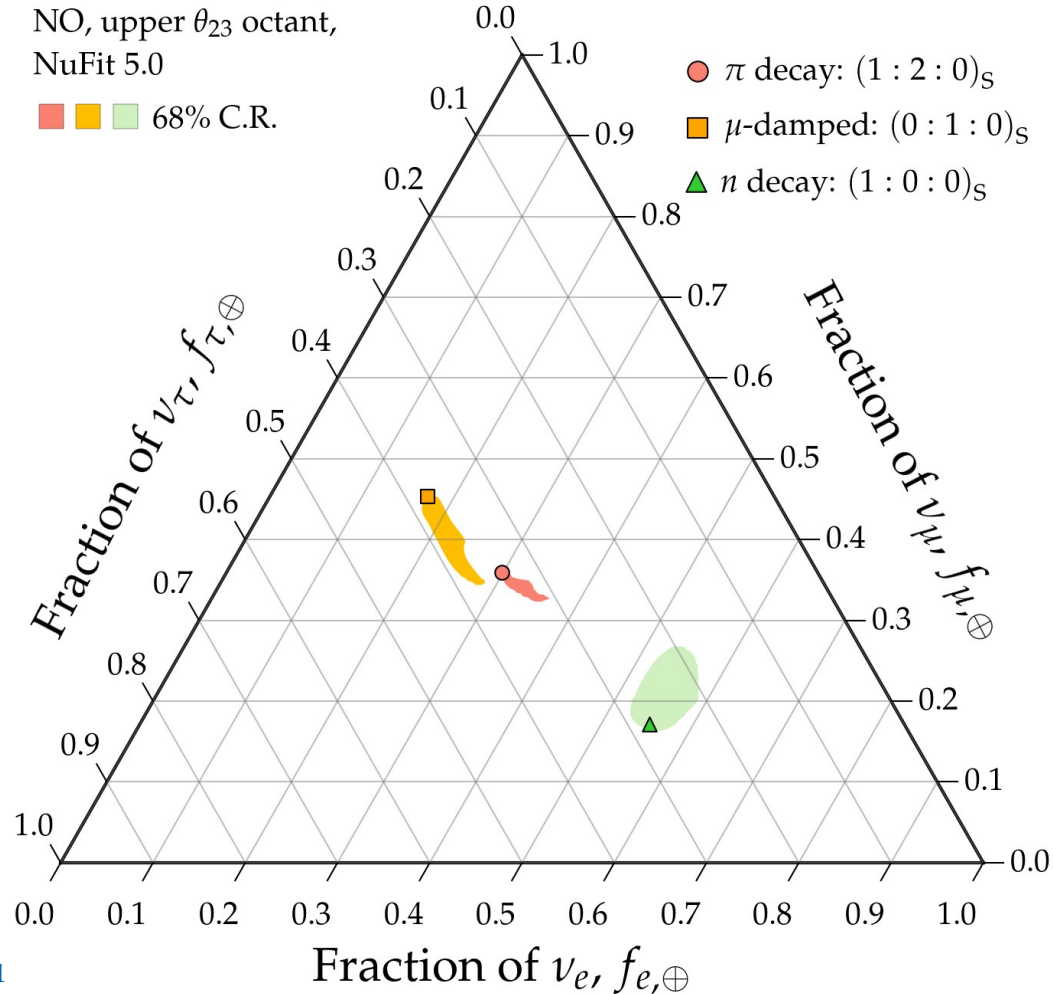
Note:

All plots shown are for normal neutrino mass ordering (NO); inverted ordering looks similar

Song, Li, Argüelles, MB, Vincent, JCAP 2021

See also: MB, Beacom, Winter, PRL 2015

Theoretically palatable regions: today



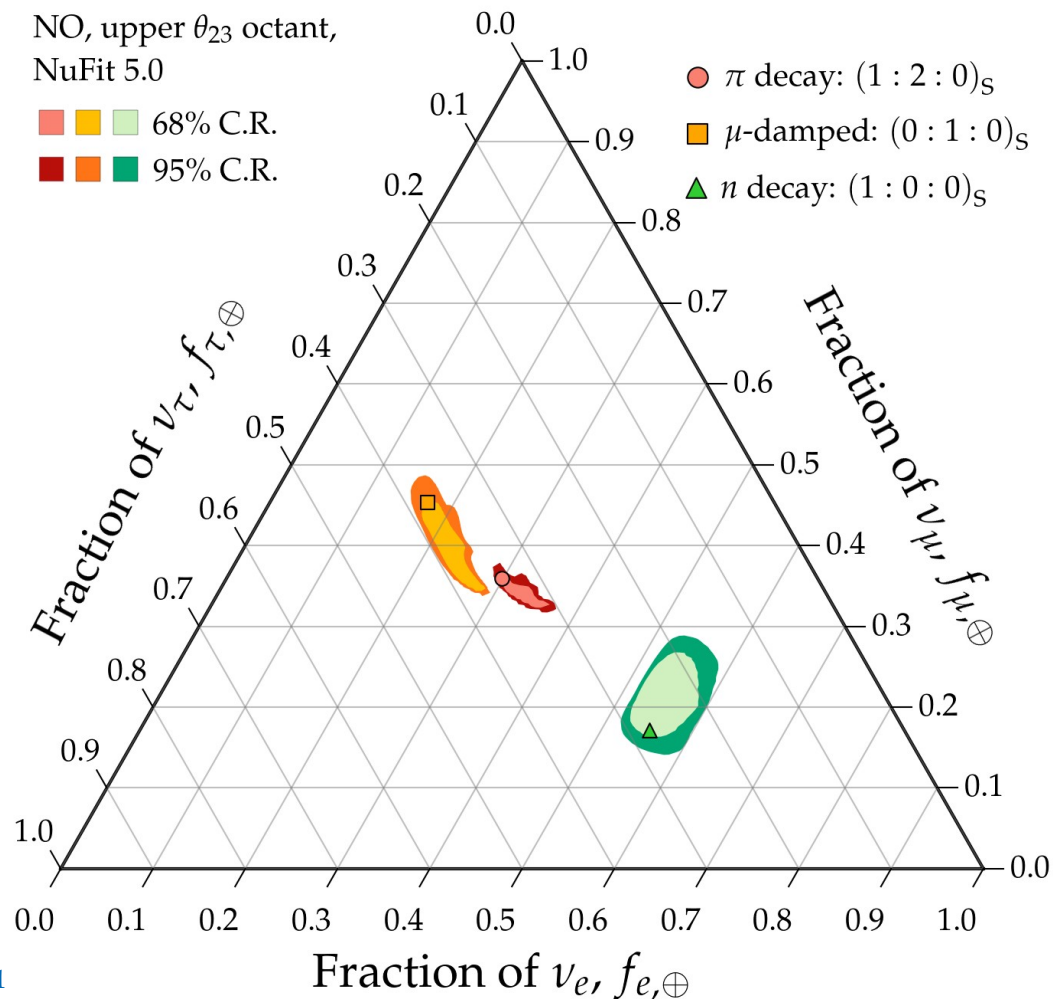
Note:

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Song, Li, Argüelles, MB, Vincent, JCAP 2021

See also: MB, Beacom, Winter, PRL 2015

Theoretically palatable regions: today



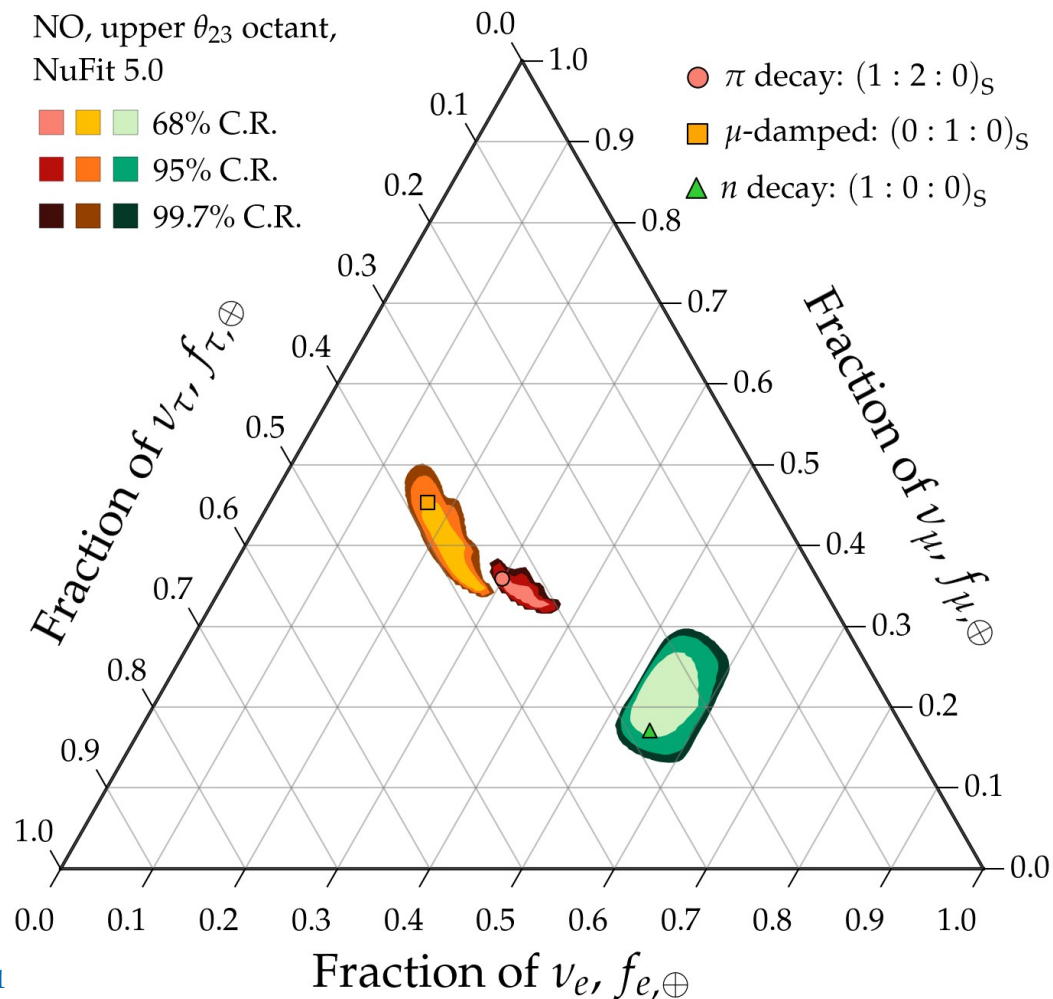
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Theoretically palatable regions: today



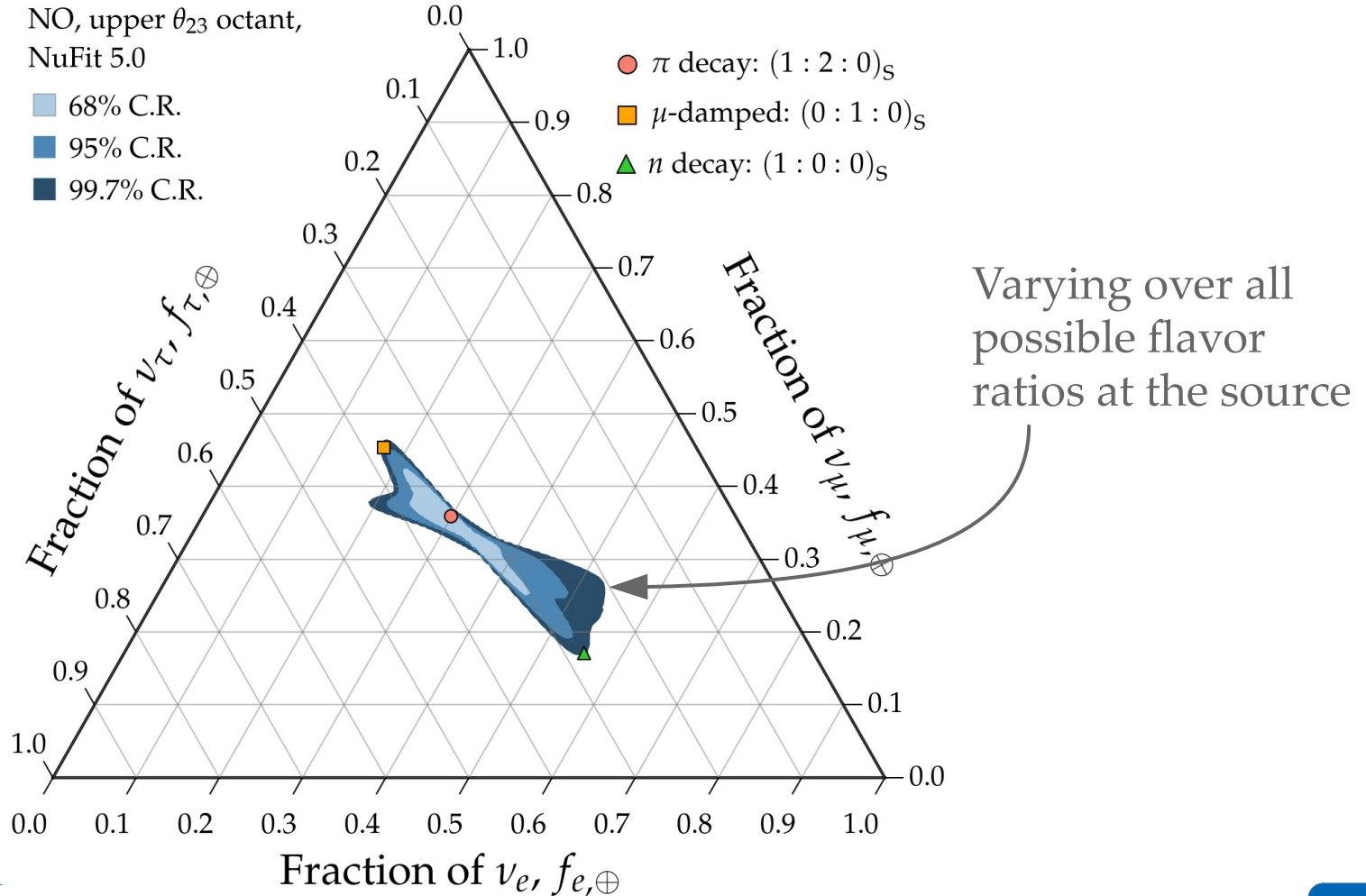
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Theoretically palatable regions: today

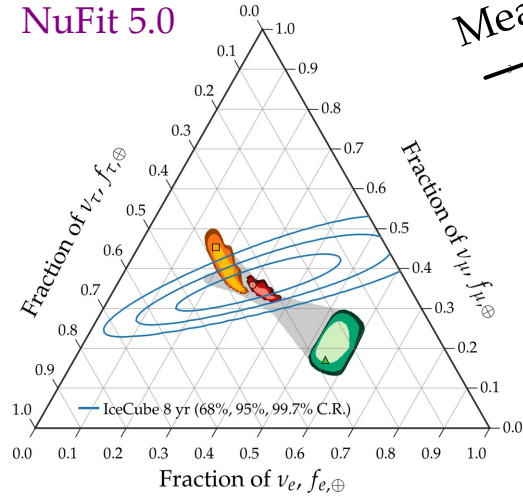


Note:
All plots shown are for normal neutrino mass ordering (NO); inverted ordering looks similar

How knowing the mixing parameters better helps

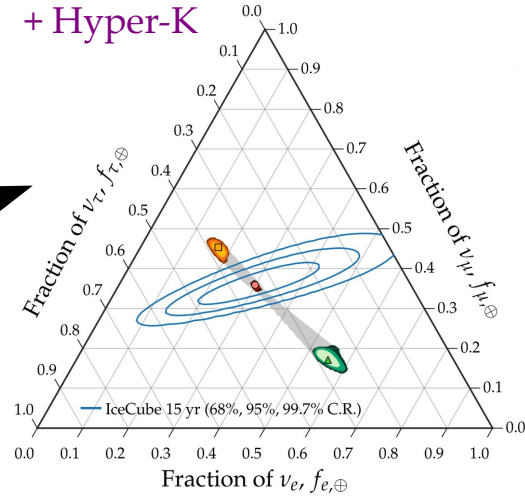
2020

NuFit 5.0

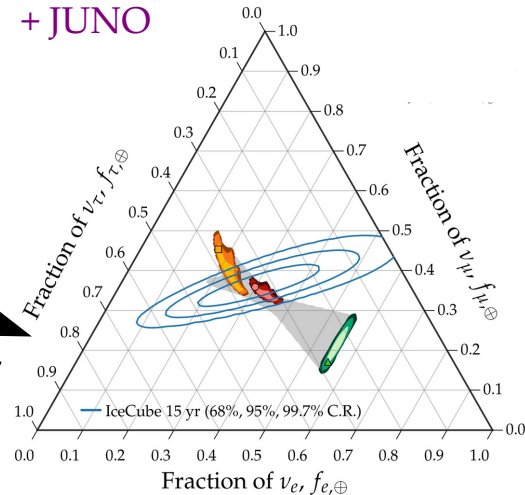


Measure θ_{23} better

+ Hyper-K



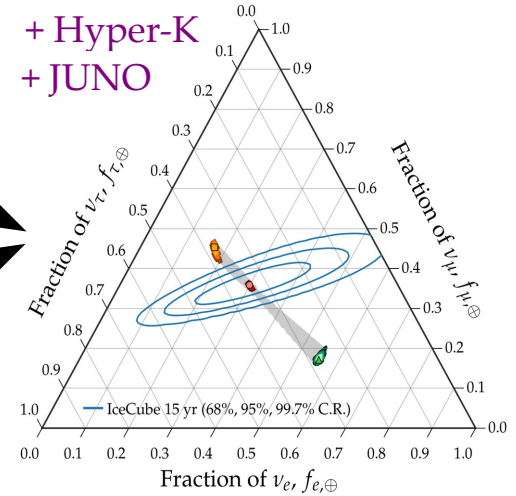
+ JUNO



Measure θ_{12} better

~2030

+ Hyper-K
+ JUNO

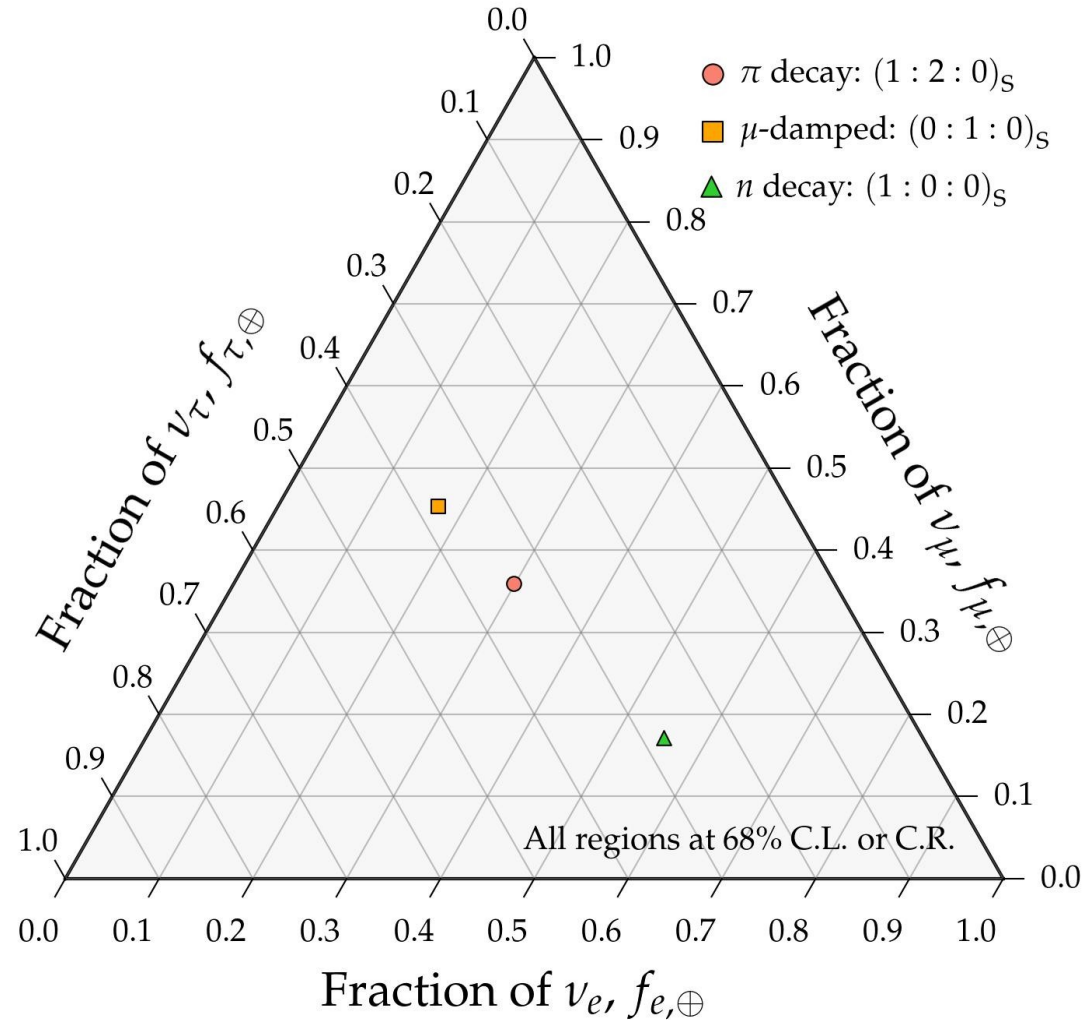


In our results:
JUNO + Hyper-K + DUNE

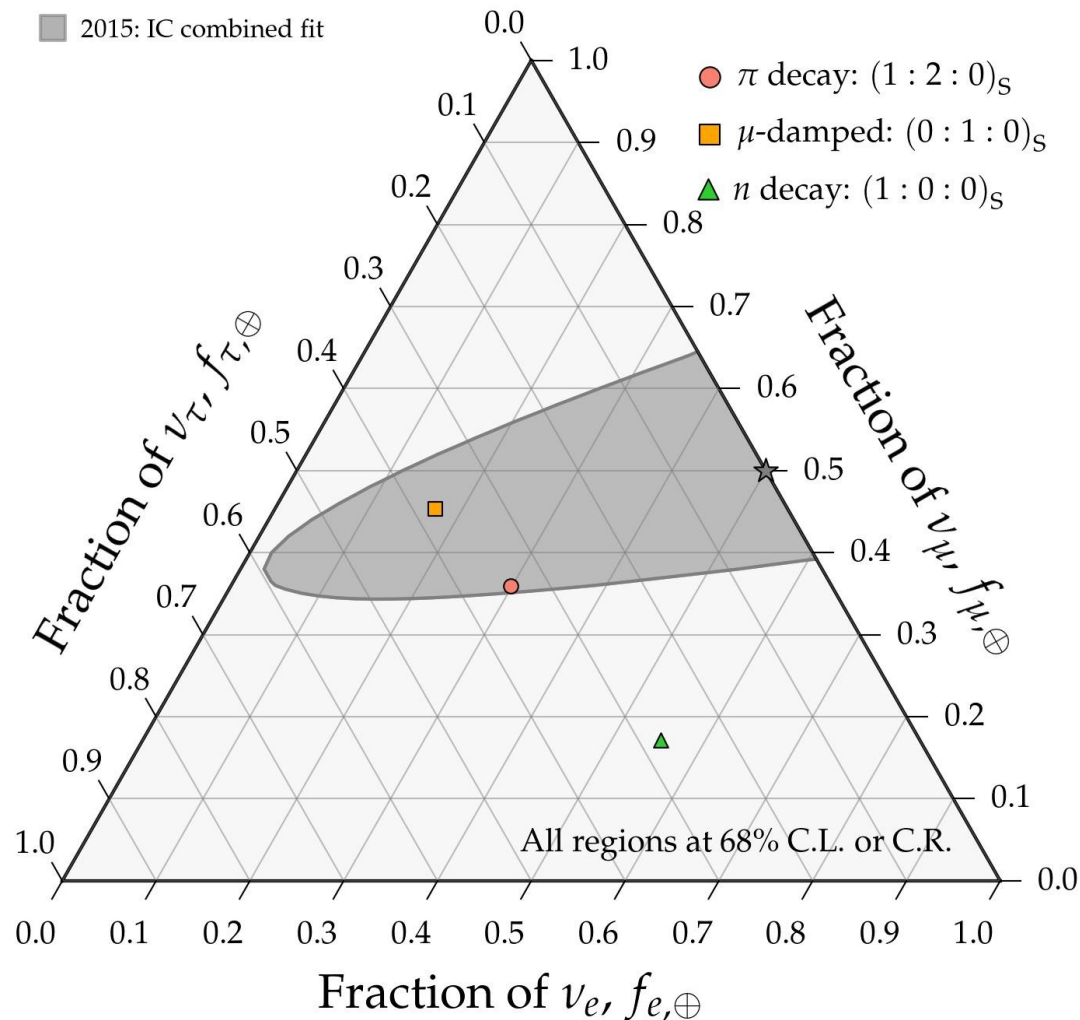
Marginal improvement til 2040

Measuring the flavor composition

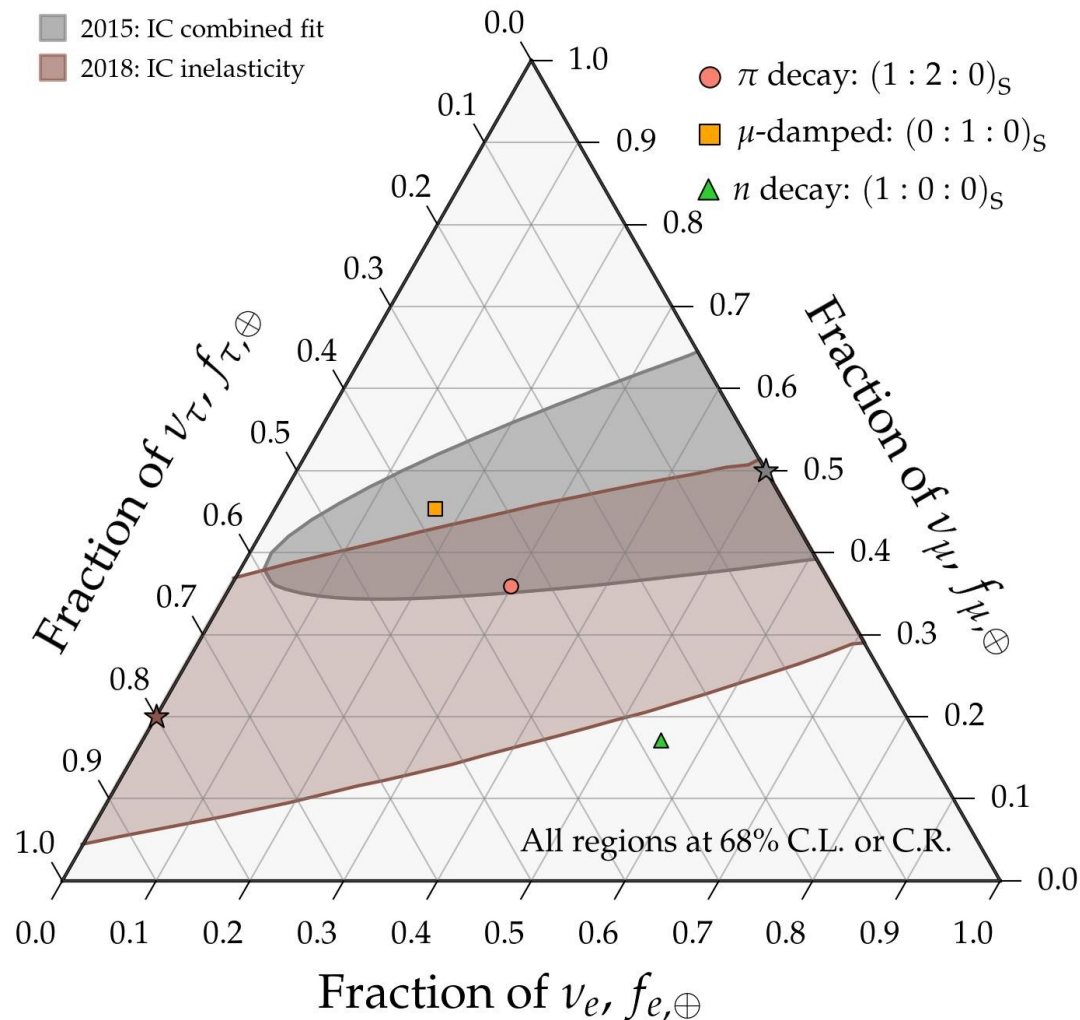
Measuring flavor composition 2015–2025



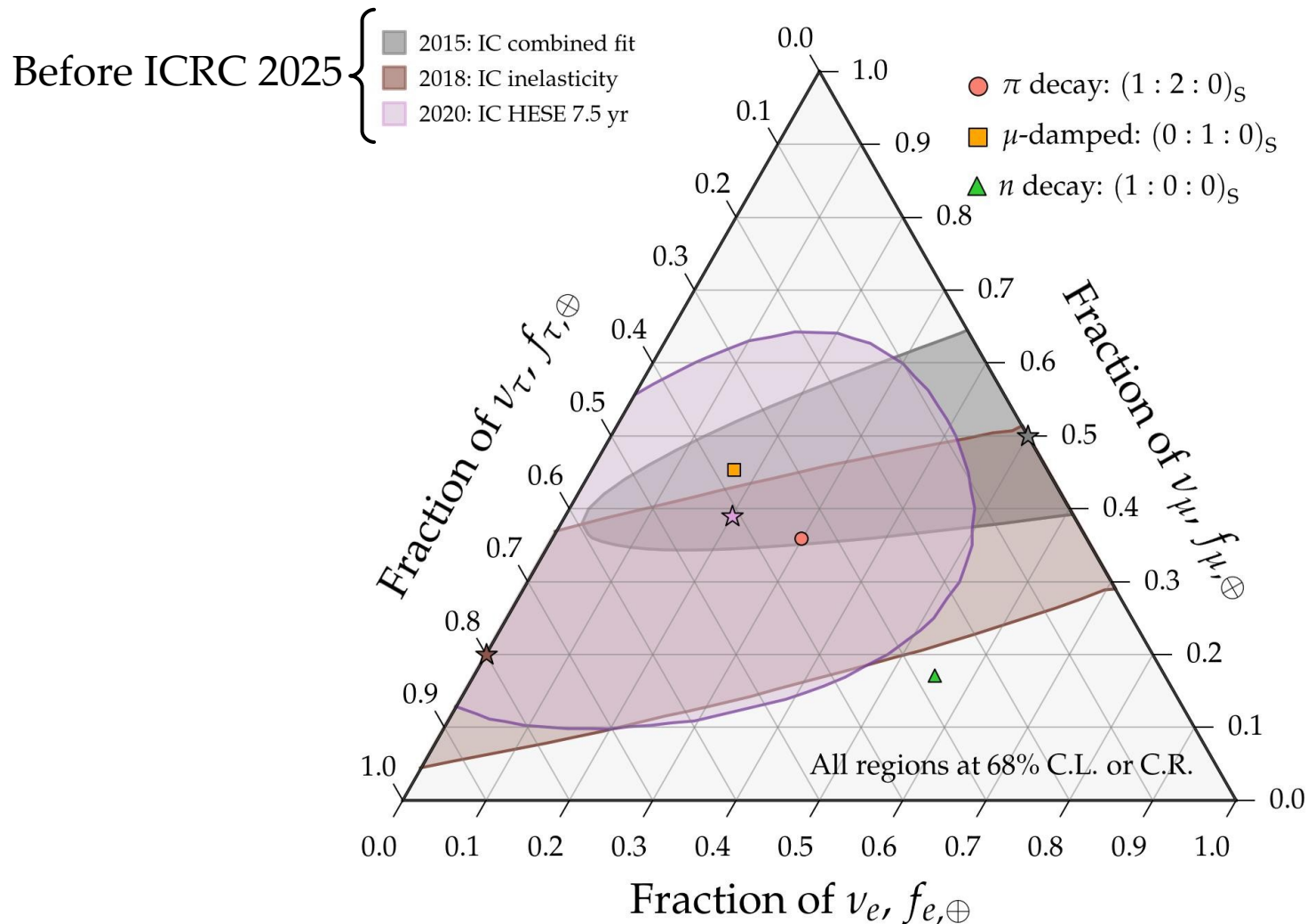
Measuring flavor composition 2015–2025



Measuring flavor composition 2015–2025

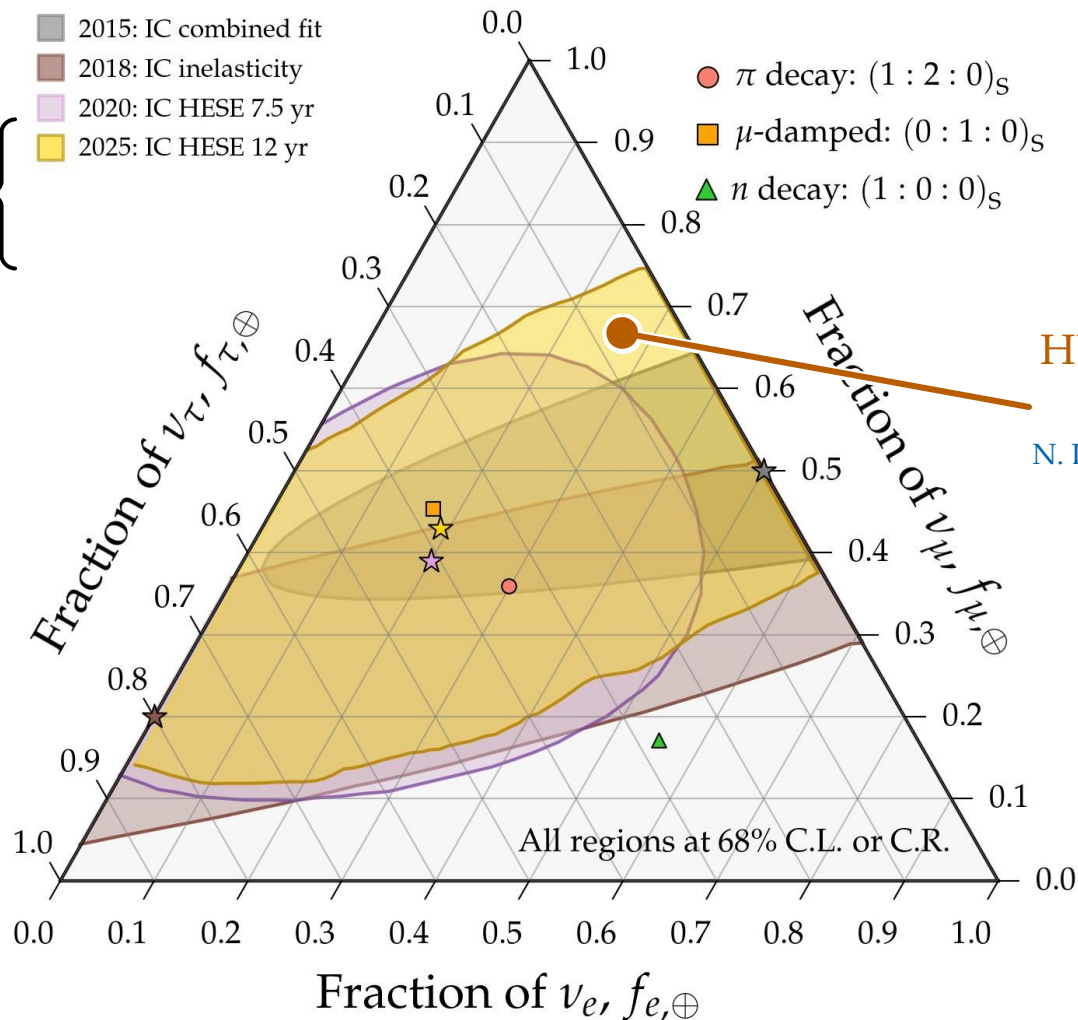


Measuring flavor composition 2015–2025



Measuring flavor composition 2015–2025

Presented at
ICRC 2025

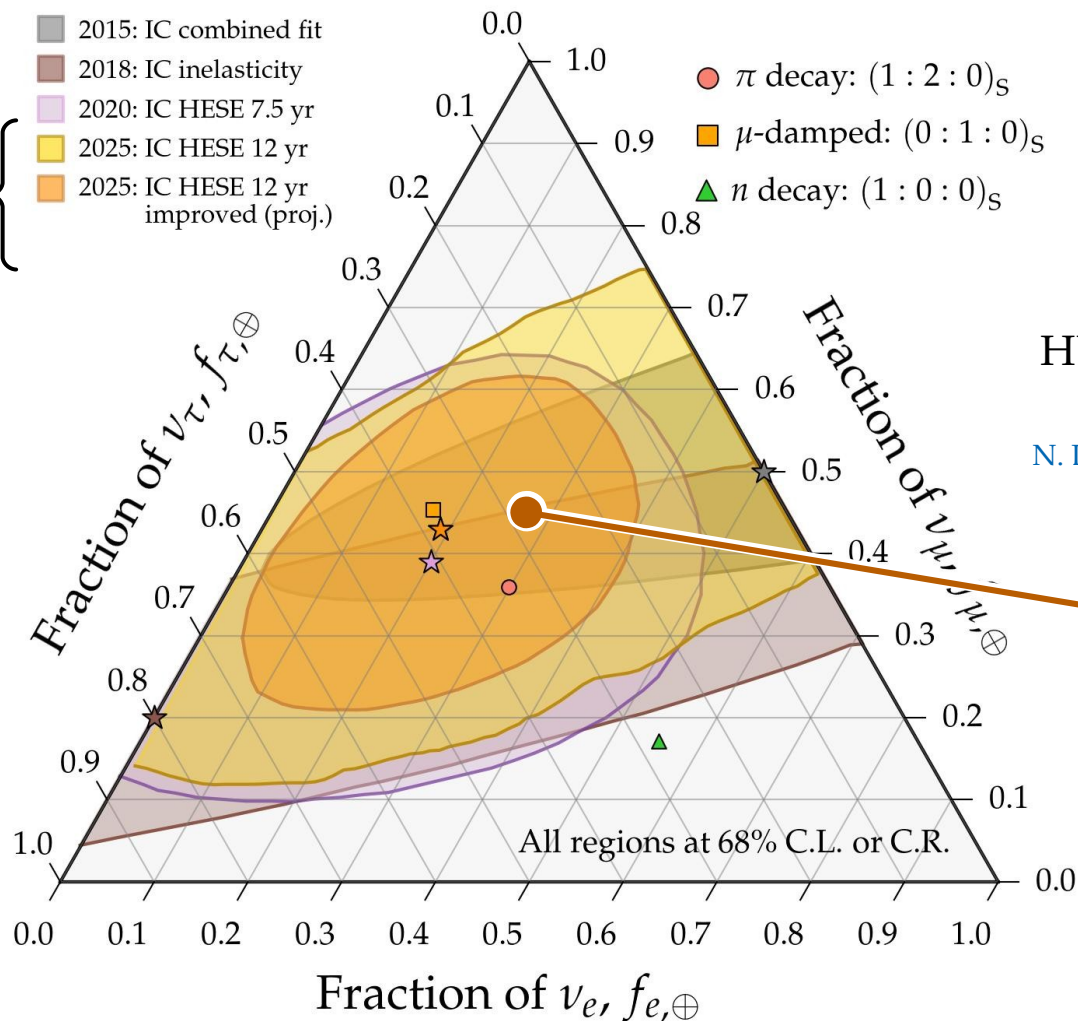


HESE (> 60 TeV) are scarce
(~100 events in 12 yr)

N. Lad, T. J. van Eeden, M. Ackermann
PoS(ICRC2025)1198

Measuring flavor composition 2015–2025

Presented at
ICRC 2025



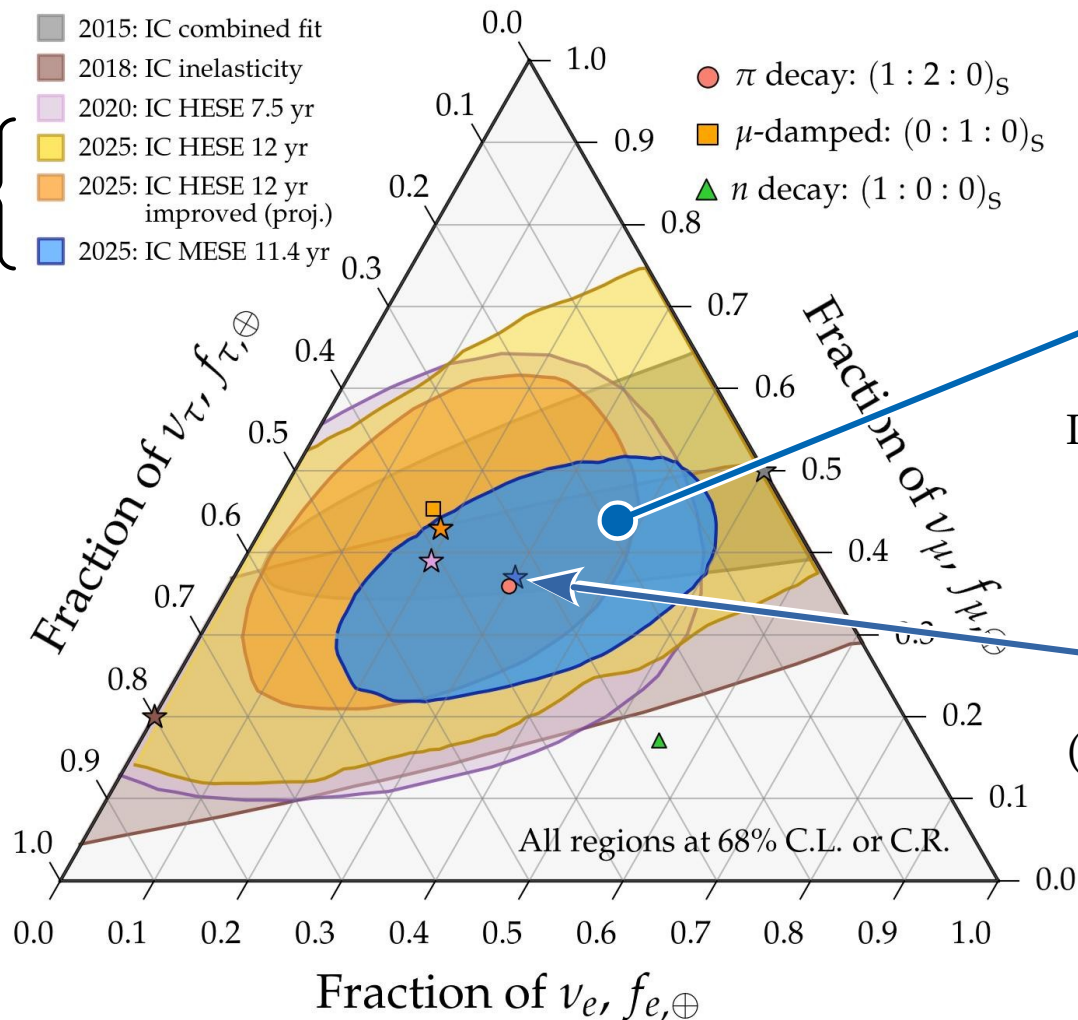
HESE (> 60 TeV) are scarce
(~ 100 events in 12 yr)

N. Lad, T. J. van Eeden, M. Ackermann
PoS(ICRC2025)1198

Improve via a neural
network that uses the
energy asymmetry of
the two bangs
and the direction

Measuring flavor composition 2015–2025

Presented at
ICRC 2025



MESE events (> 1 TeV)
are more abundant

Includes classification of ν_τ

First time all flavors are
nonzero at 68% C.L.

Best fit very close to
nominal expectation of
(1:1:1) from production via
pion decay

A. Balagopal *et al.*
PoS(ICRC2025)983

Can we infer the
mixing parameters?

We use Bayesian inference to infer the values of the mixing parameters:

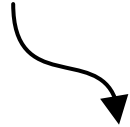
$$\mathcal{P}(\boldsymbol{\theta}) = \int_0^1 df_{e,S} \mathcal{L}(f_{\alpha,\oplus}(\boldsymbol{\theta}, f_{e,S})) \pi(\boldsymbol{\theta}) \pi(f_{e,S})$$

Show 1D marginalized allowed intervals of each mixing parameters \rightarrow

We use Bayesian inference to infer the values of the mixing parameters:

Joint posterior distribution
of mixing parameters

$$\boldsymbol{\theta} \equiv (\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\text{CP}})$$



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Compute flavor
composition at Earth for
each choice $\boldsymbol{\theta}$ and $f_{e,S}$

$$\mathcal{P}(\boldsymbol{\theta}) = \int_0^1 df_{e,S} \underbrace{\mathcal{L}(f_{\alpha,\oplus}(\boldsymbol{\theta}, f_{e,S}))}_{\text{Compute flavor composition at Earth for each choice } \boldsymbol{\theta} \text{ and } f_{e,S}} \pi(\boldsymbol{\theta}) \pi(f_{e,S})$$

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$$\mathcal{P}(\boldsymbol{\theta}) = \int_0^1 df_{e,S} \underbrace{\mathcal{L}(f_{\alpha,\oplus}(\boldsymbol{\theta}, f_{e,S}))}_{\text{Likelihood function of flavor-composition measurements (from IceCube, etc.)}} \pi(\boldsymbol{\theta}) \pi(f_{e,S})$$

Compute flavor
composition at Earth for
each choice $\boldsymbol{\theta}$ and $f_{e,S}$

Likelihood function of
flavor-composition
measurements
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Compute flavor
composition at Earth for
each choice $\boldsymbol{\theta}$ and $f_{e,S}$

Prior on $\boldsymbol{\theta}$
(flat or Haar
measure on U_{PMNS})



Likelihood function of
flavor-composition
measurements
(from IceCube, etc.)

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Compute flavor
composition at Earth for
each choice $\boldsymbol{\theta}$ and $f_{e,S}$

Prior on $\boldsymbol{\theta}$
(flat or Haar
measure on U_{PMNS})

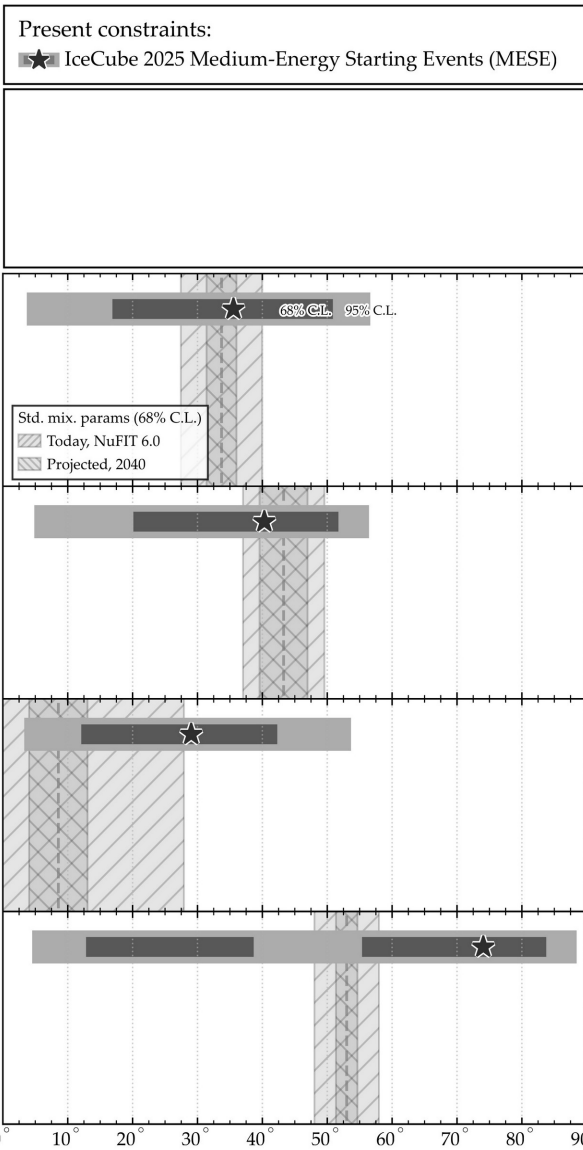
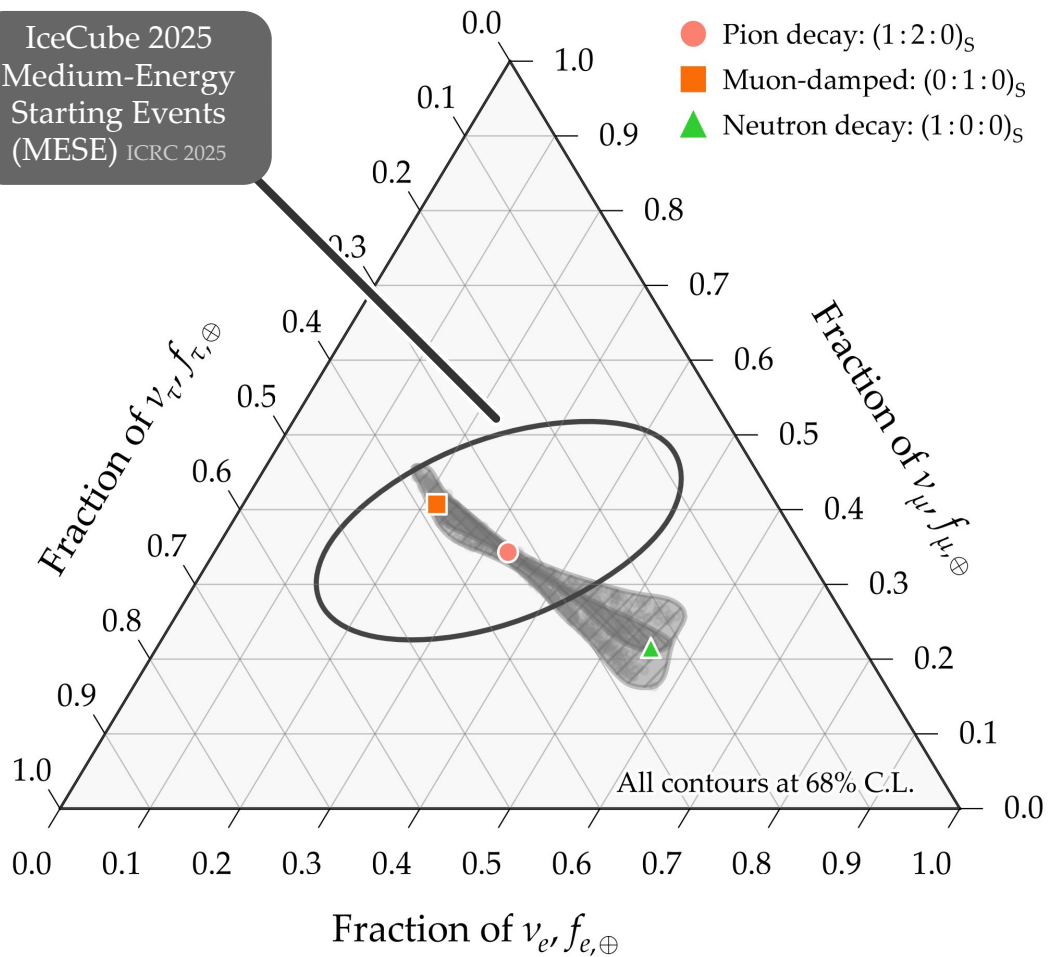
Likelihood function of
flavor-composition
measurements
(from IceCube, etc.)

Prior on $f_{e,S}$
(flat in $[0,1]$)

Show 1D marginalized allowed intervals of each mixing parameters \rightarrow

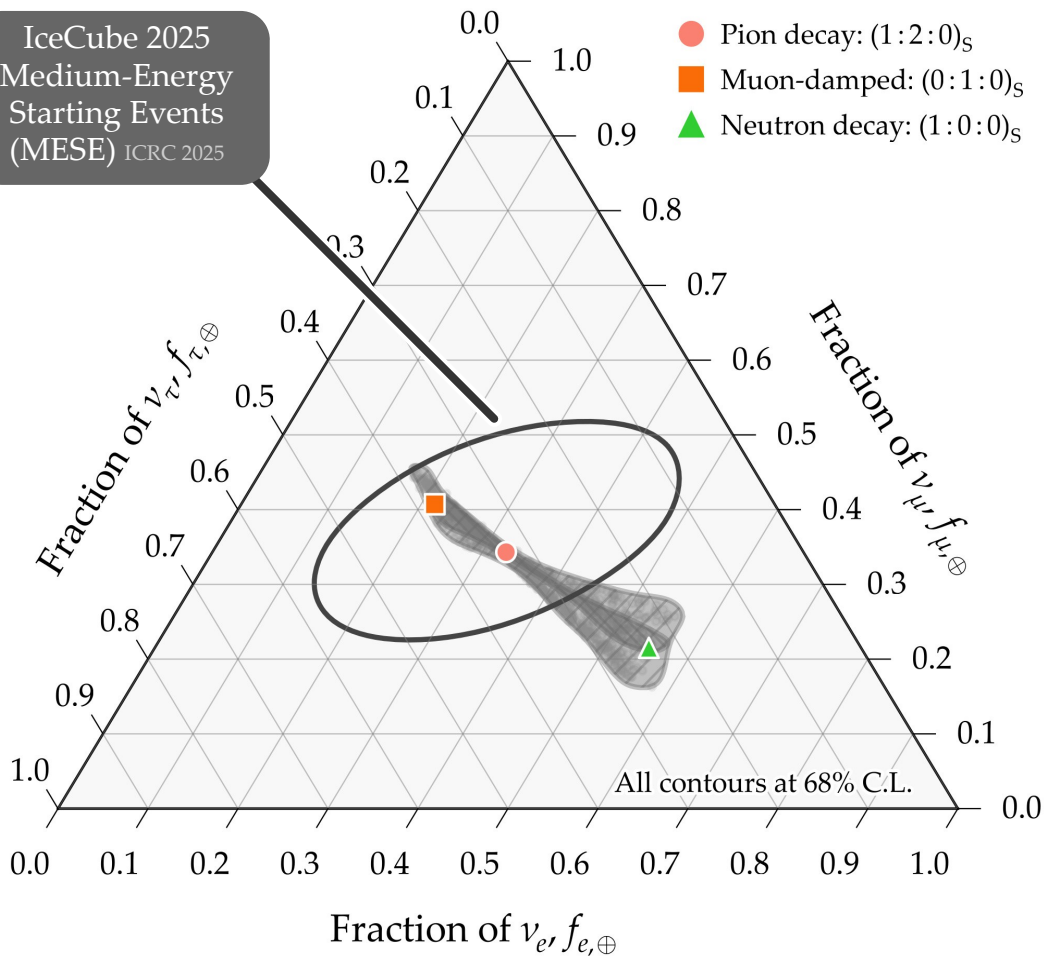
Measurement power *today* —

IceCube 2025
Medium-Energy
Starting Events
(MESE) ICRC 2025



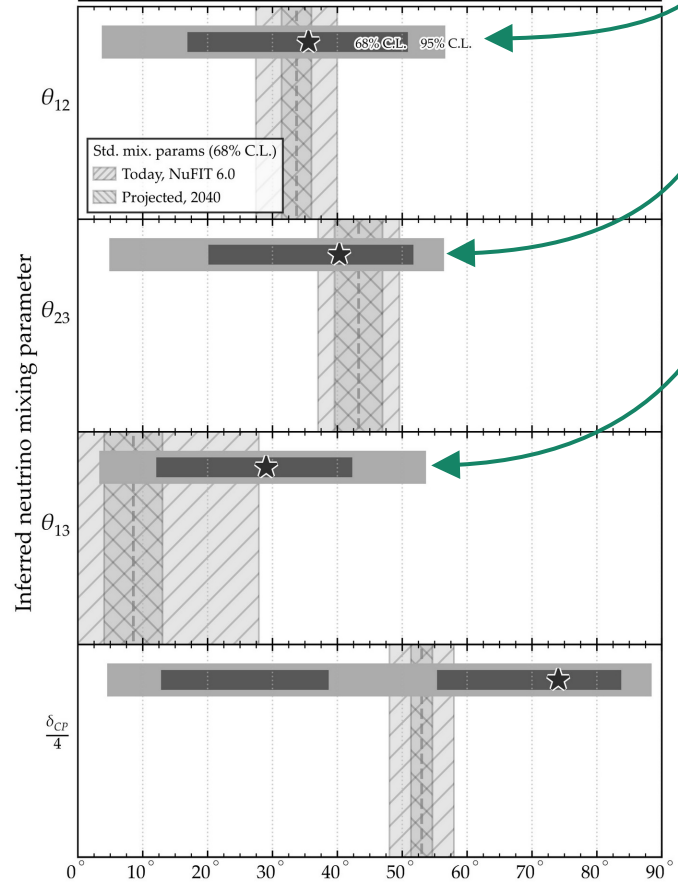
Measurement power *today* —

IceCube 2025
Medium-Energy
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(MESE) ICRC 2025

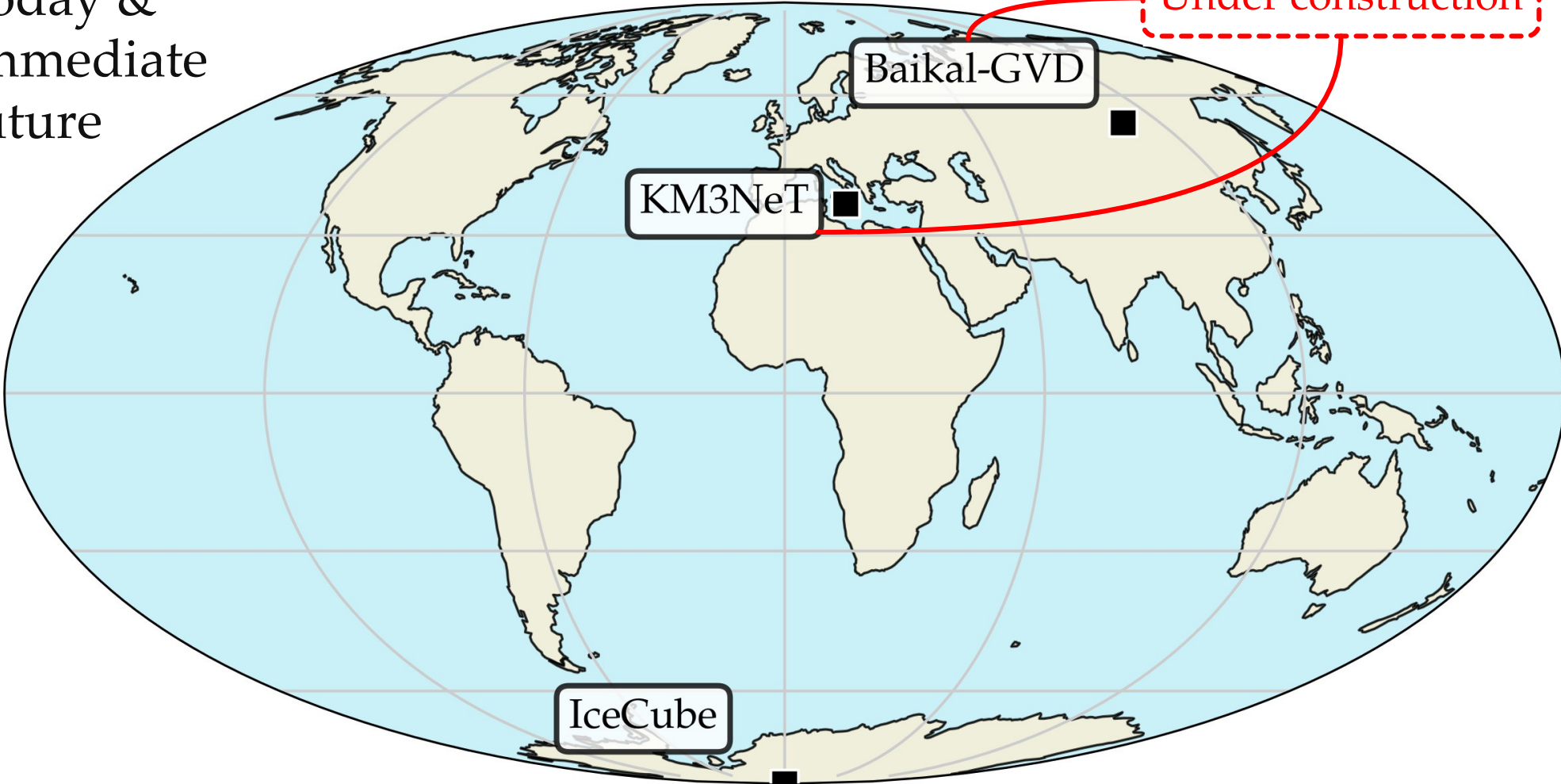


Present constraints:
 ★ IceCube 2025 Medium-Energy Starting Events (MESE)

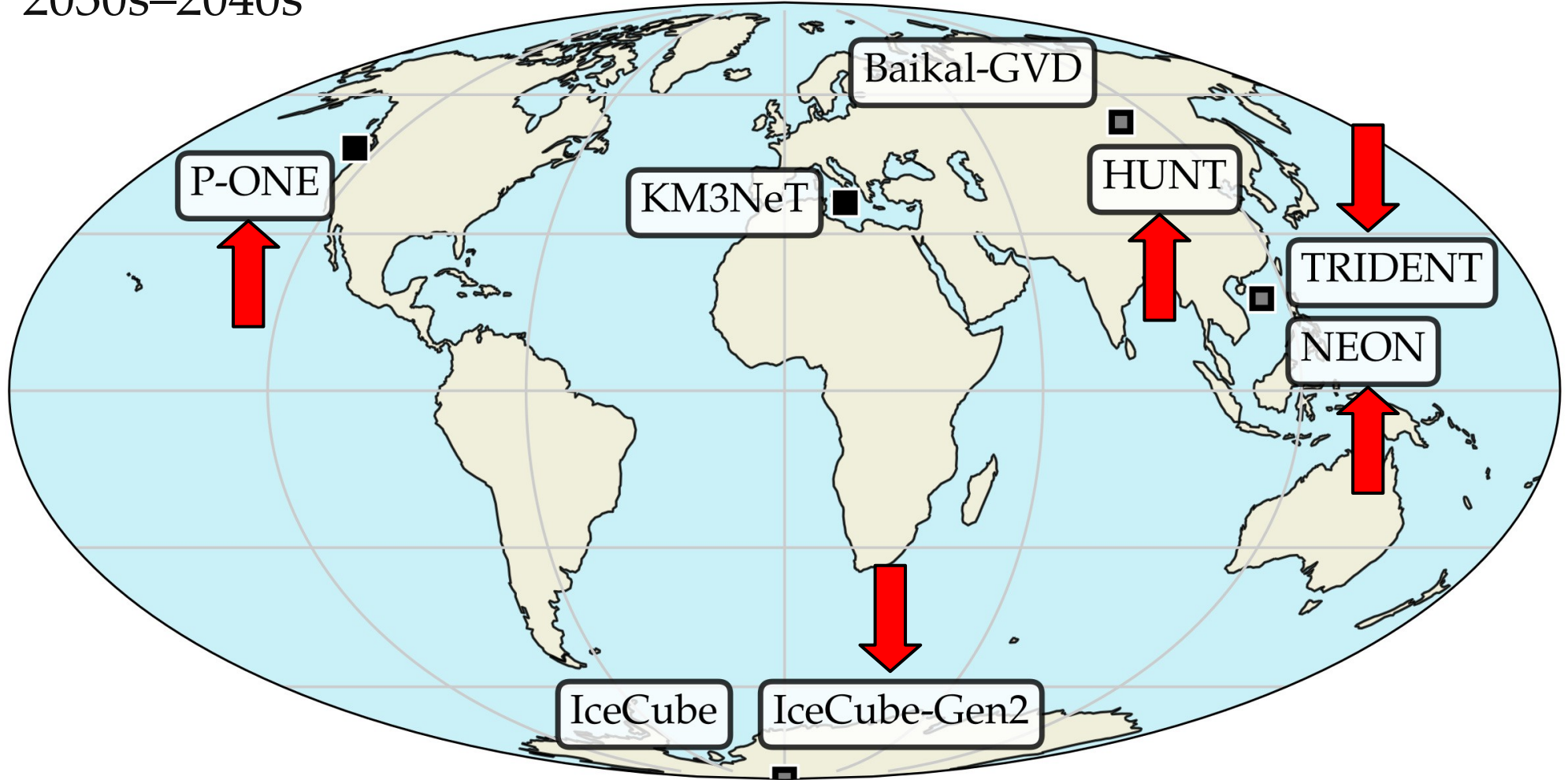
From high-energy neutrinos *alone*
 we can infer the mixing angles,
 though with limited precision



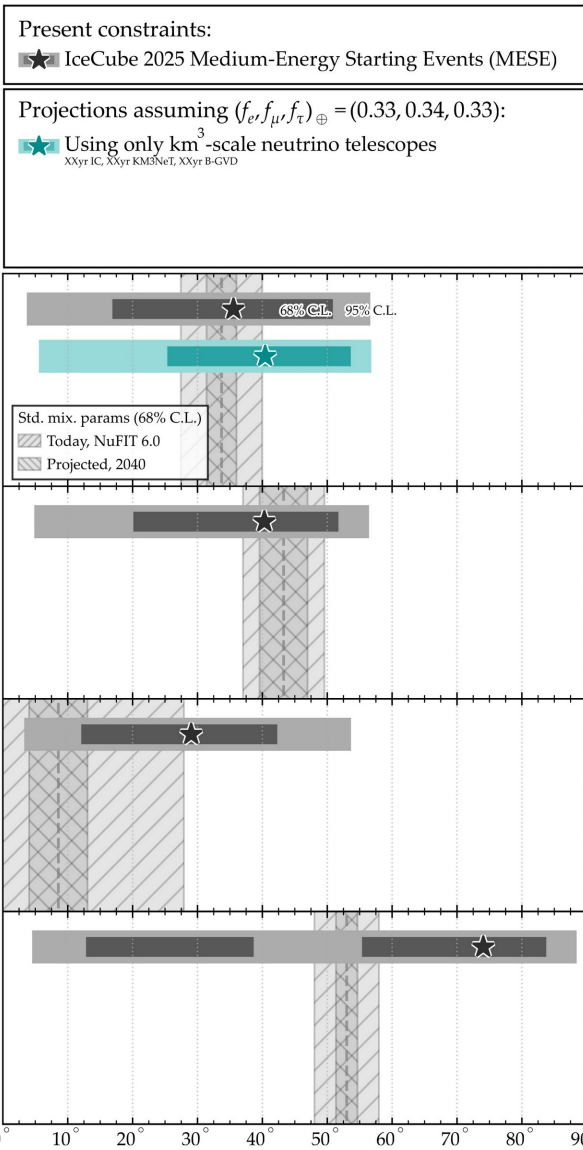
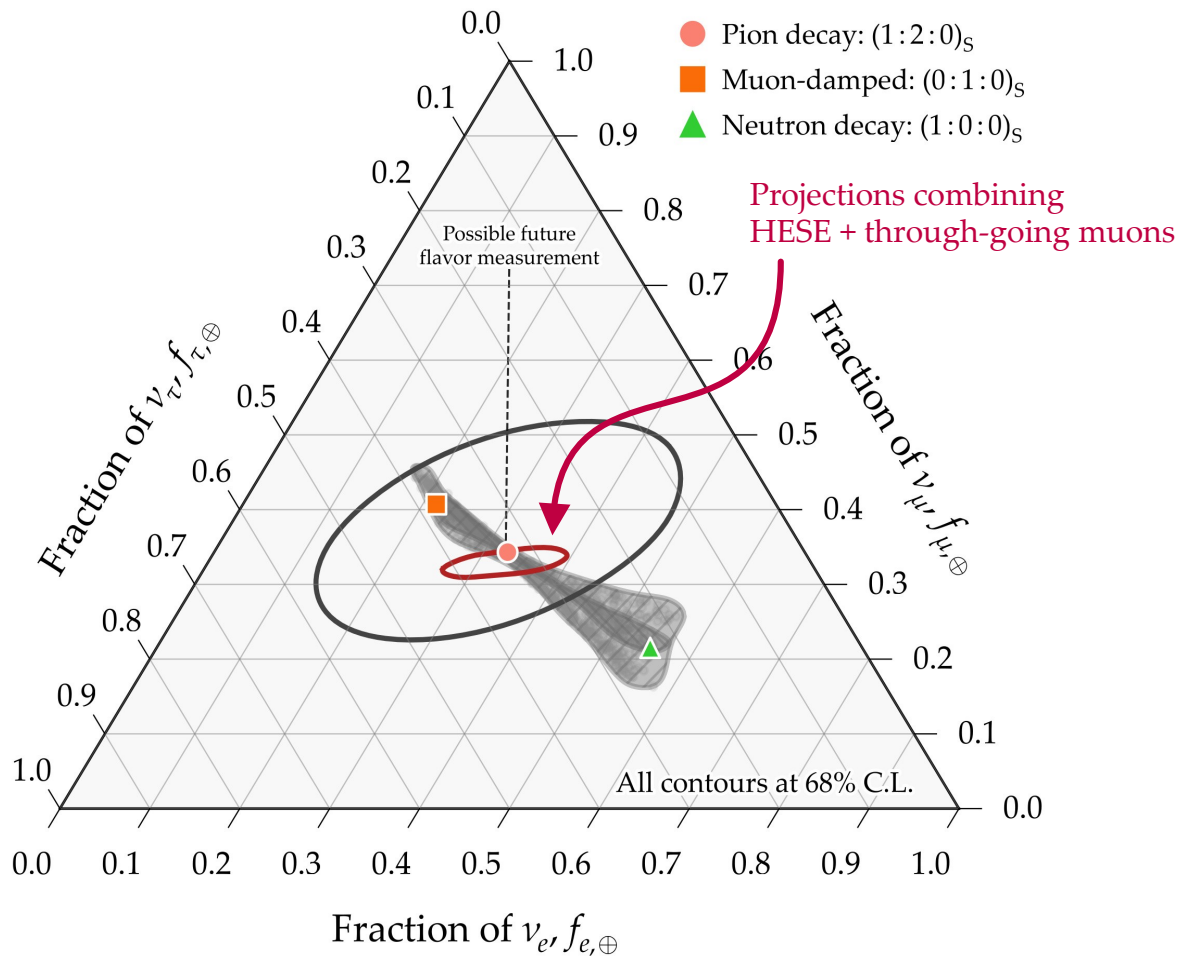
Today &
immediate
future



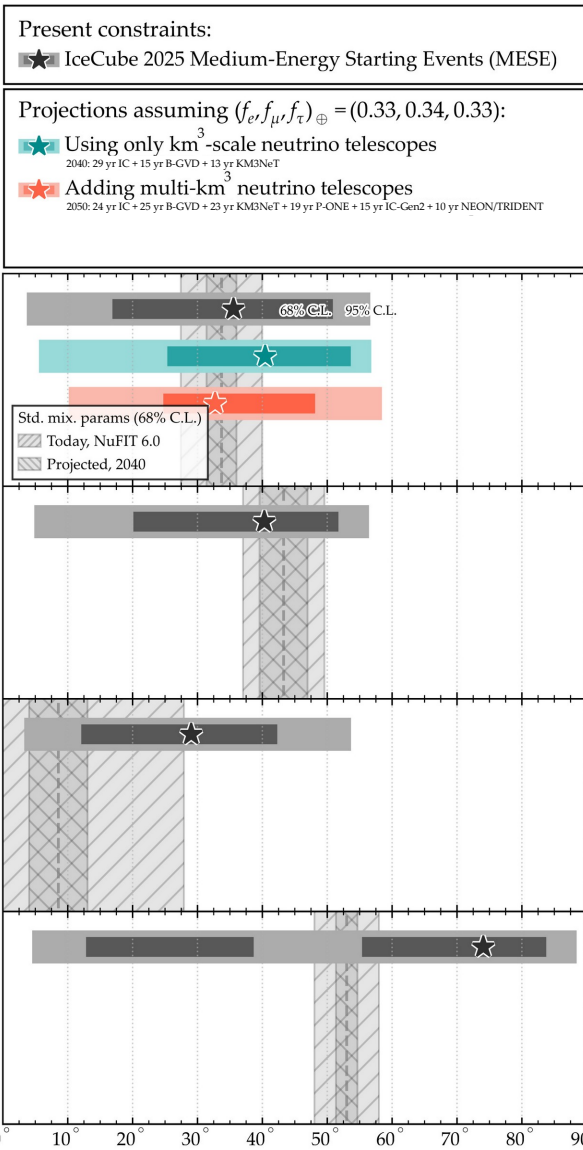
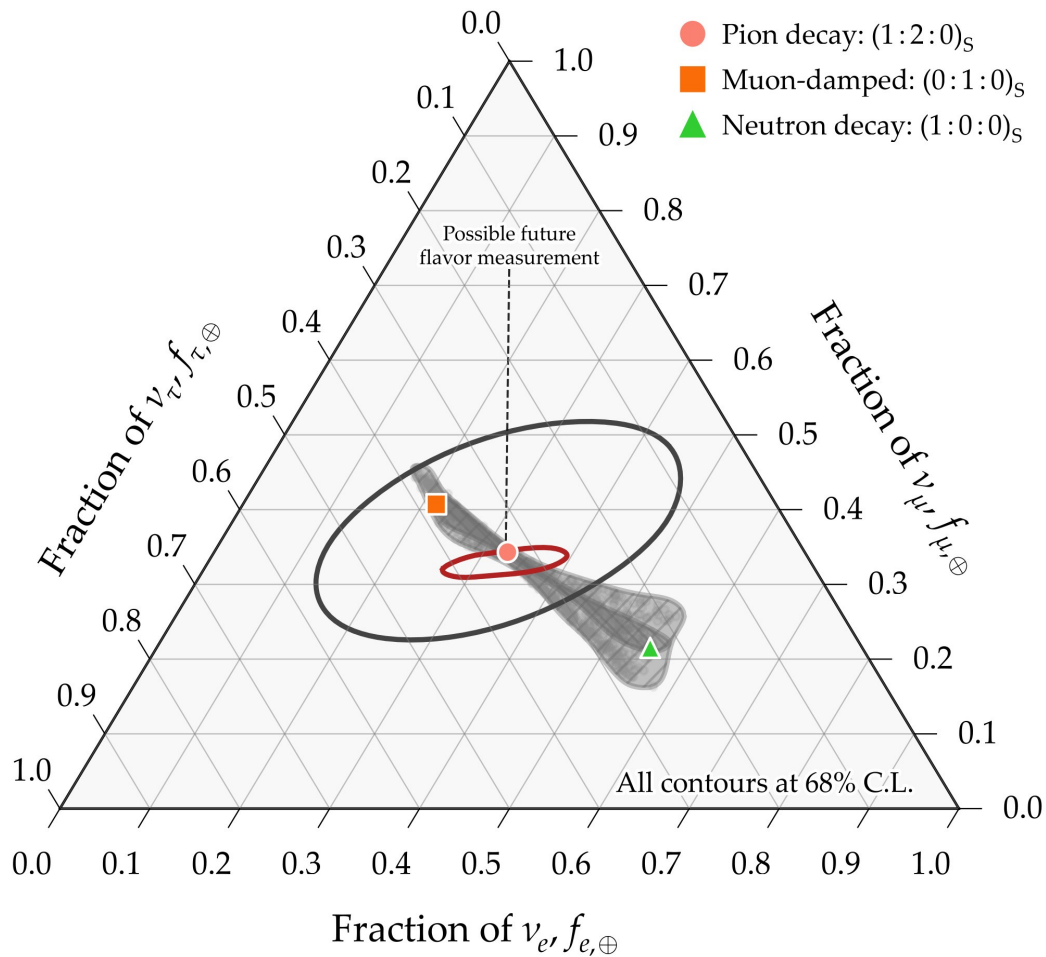
2030s–2040s



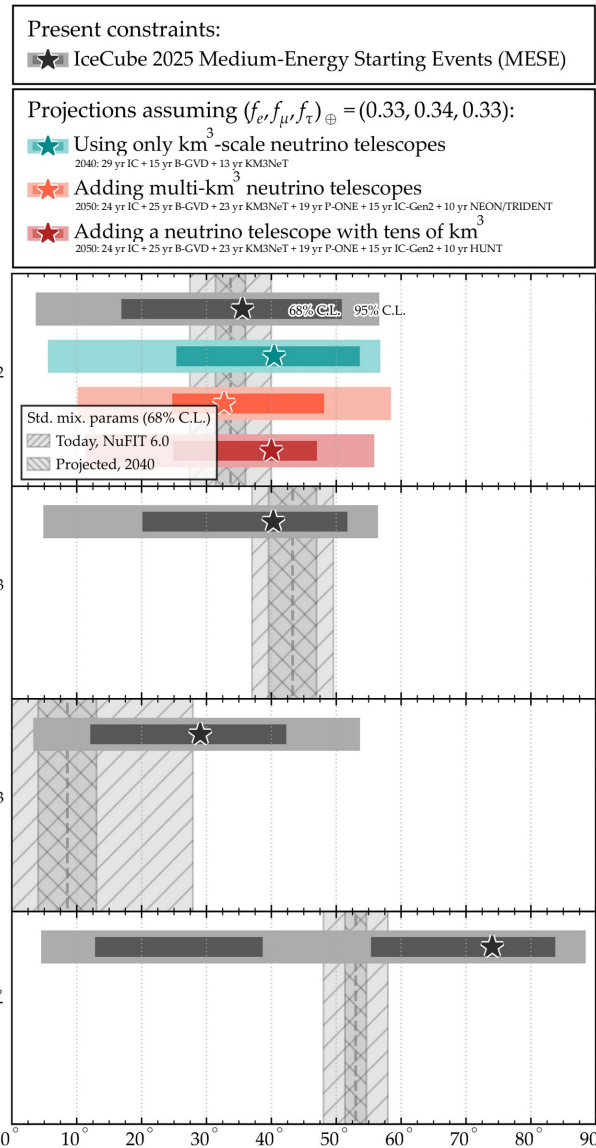
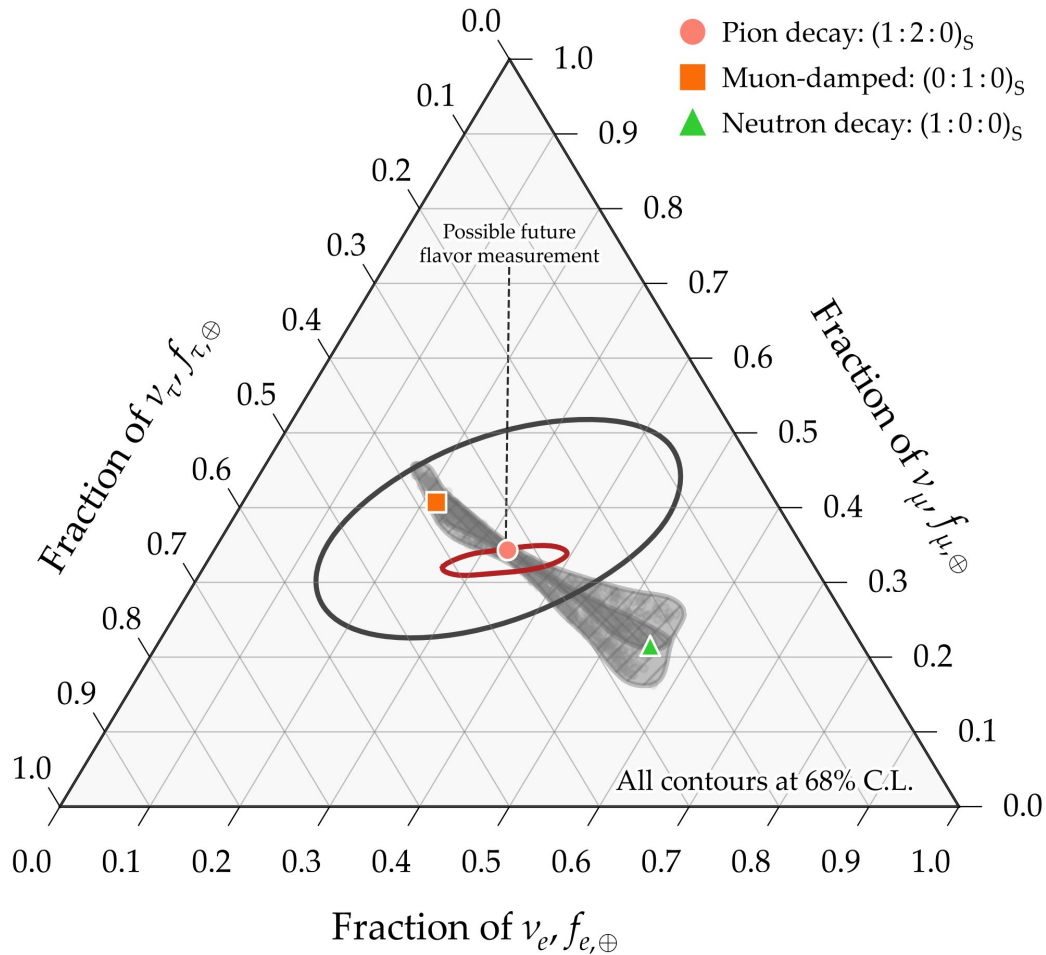
Measurement power *in the future* —



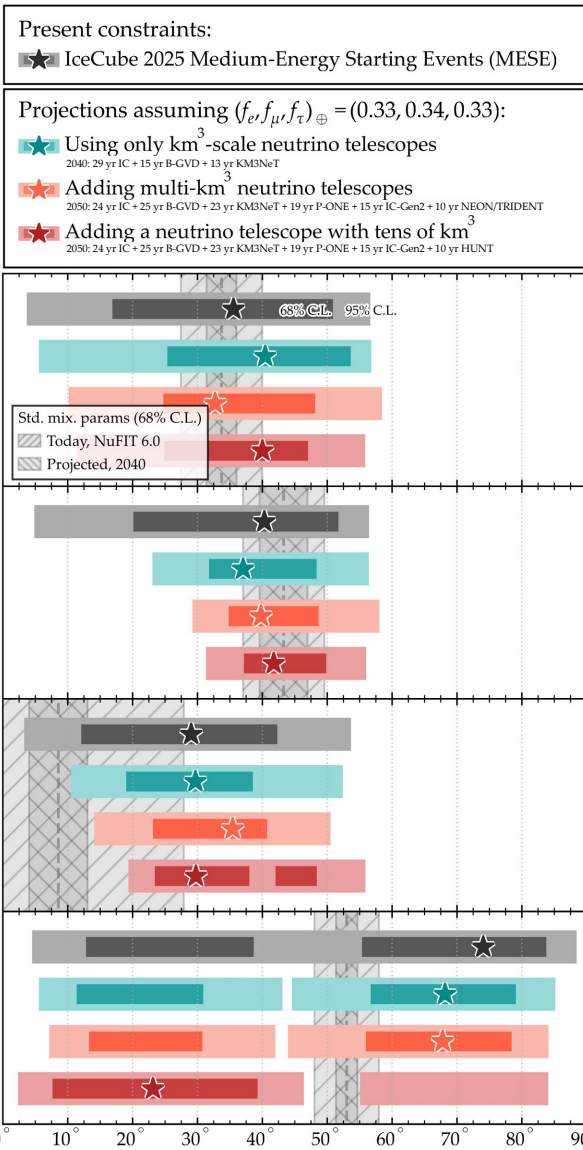
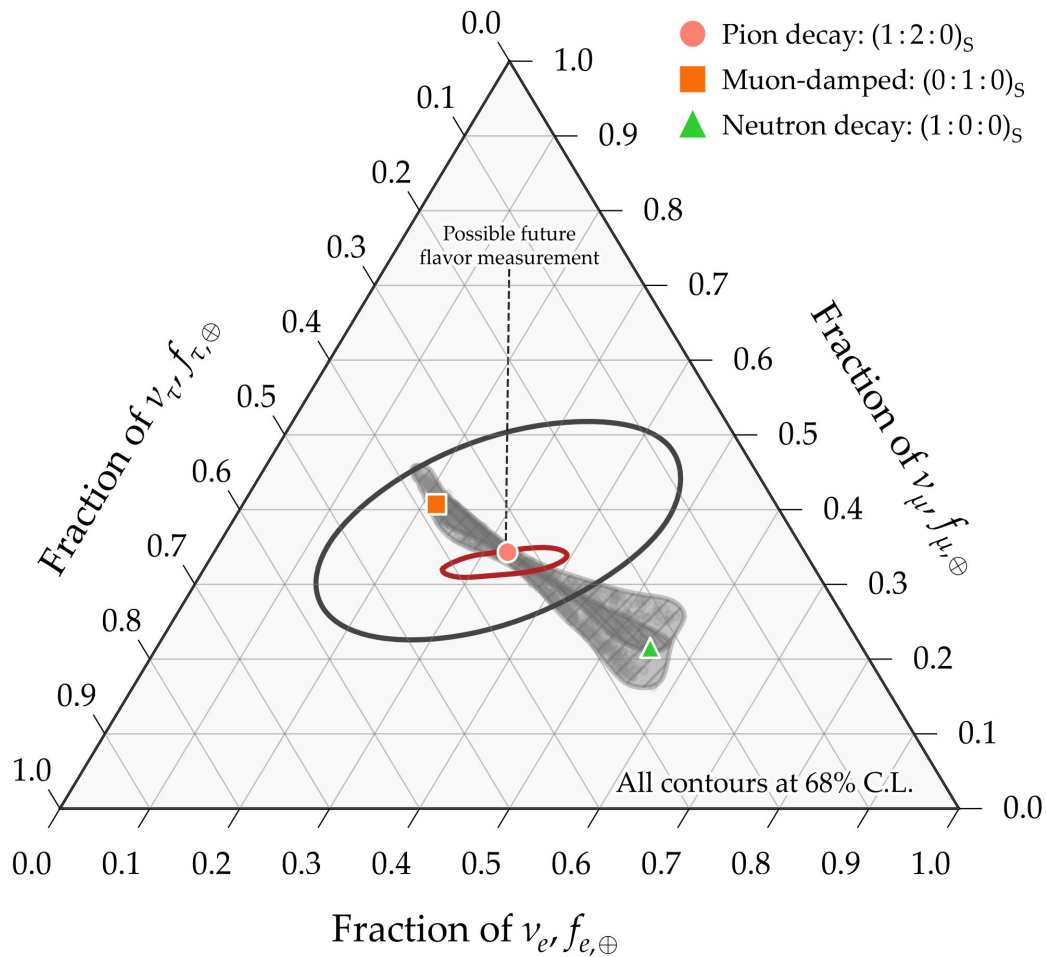
Measurement power *in the future* —



Measurement power *in the future* —



Measurement power *in the future* —



What can we expect
from unitarity

Standard oscillations — driven by neutrino masses:

$$H_{\text{std}} = \frac{1}{2E} U_{\text{PMNS}}^\dagger \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) U_{\text{PMNS}}$$

Remember: U_{PMNS} is unitary



Average flavor-transition probability:

$$P_{\alpha\beta} = \sum_{i=1}^3 |(U_{\text{PMNS}})_{\alpha i}|^2 |(U_{\text{PMNS}})_{\beta i}|^2 = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix}$$

Standard oscillations — driven by neutrino masses:

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9 independent entries



6 independent entries

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Average flavor-transition probability:

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$$\begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix}$$

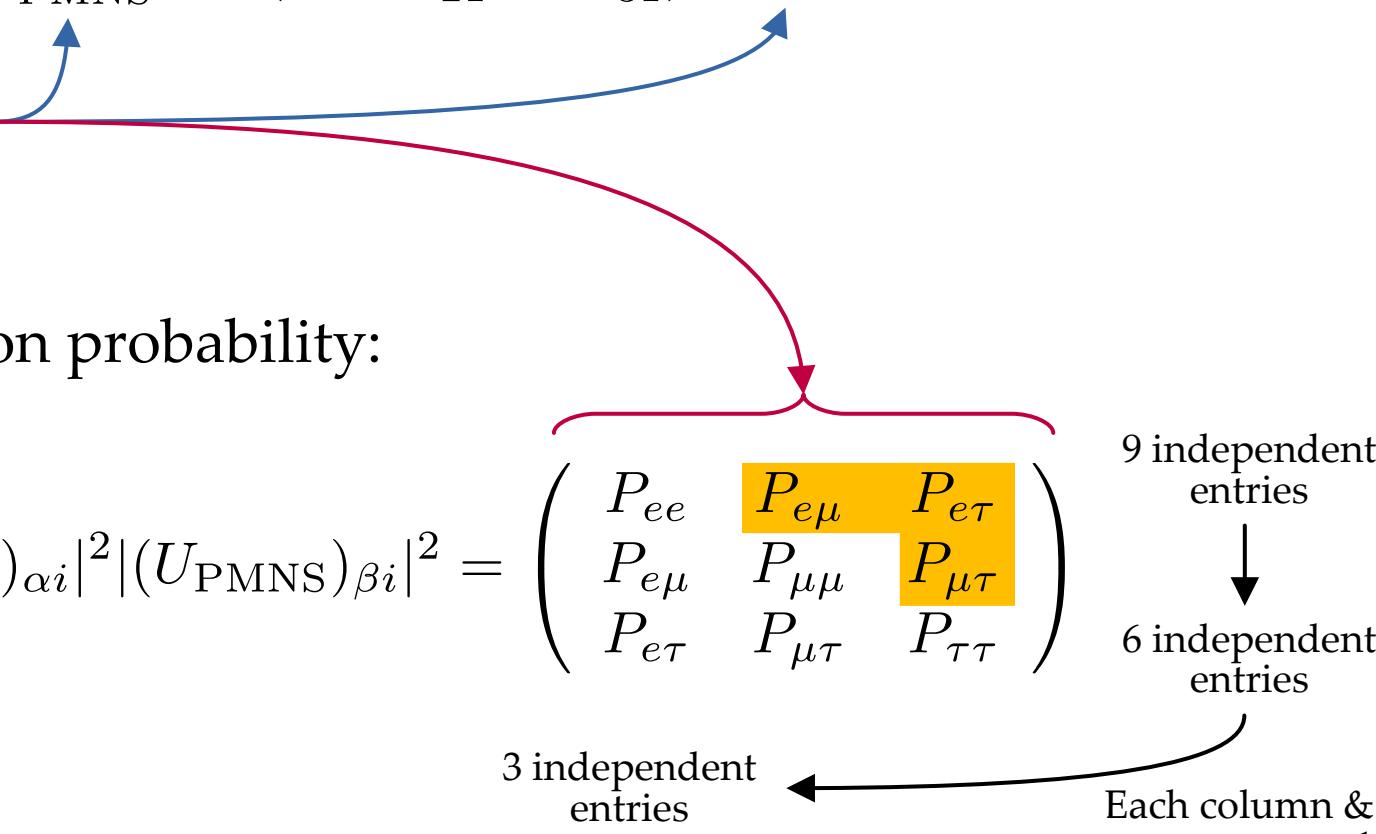
9 independent entries



6 independent entries

3 independent entries

Each column & row must sum to 1



Let's add a new interaction **that conserves the number of ν** :

$$H_{\text{new}} = \left(\frac{E}{\Lambda} \right)^n U_{\text{new}}^\dagger \text{diag}(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3) U_{\text{new}}$$

U_{new} is unitary
It rotates from the eigenbasis of H_{new} to the flavor basis



The total Hamiltonian,

$$H_{\text{tot}} = H_{\text{std}} + H_{\text{new}} ,$$

is diagonalized by a new unitary rotation matrix U_{tot} , that is a function of all the standard and new parameters (and the energy).

Since U_{tot} is unitary, the probability matrix has the same shape as before:

$$P_{\alpha\beta} = \sum_{i=1}^3 |(U_{\text{tot}})_{\alpha i}|^2 |(U_{\text{tot}})_{\beta i}|^2 = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix}$$

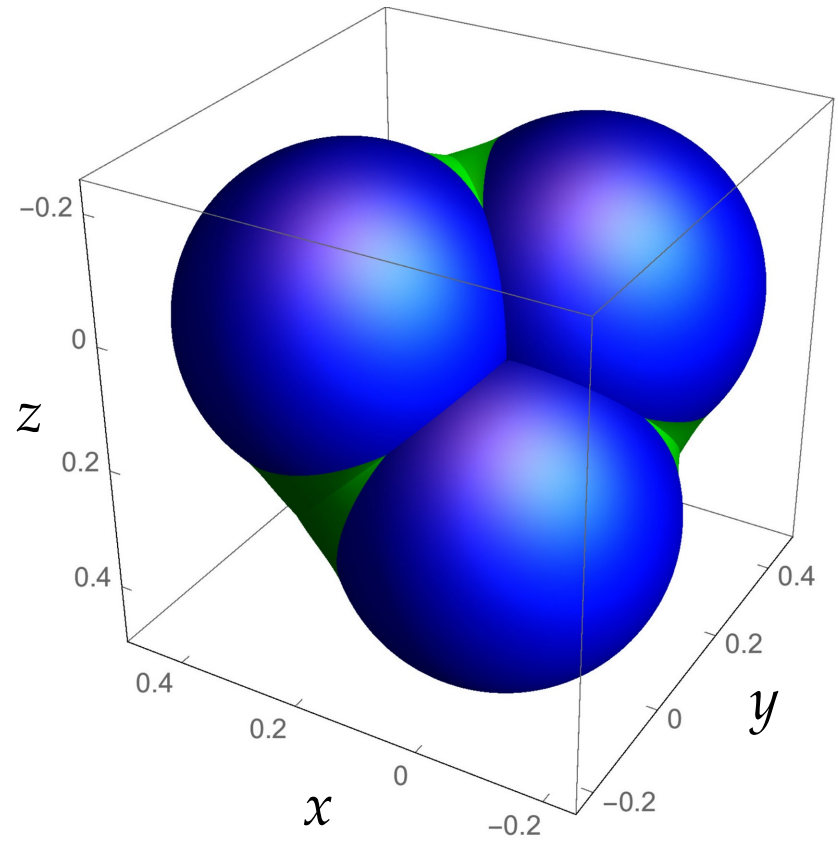
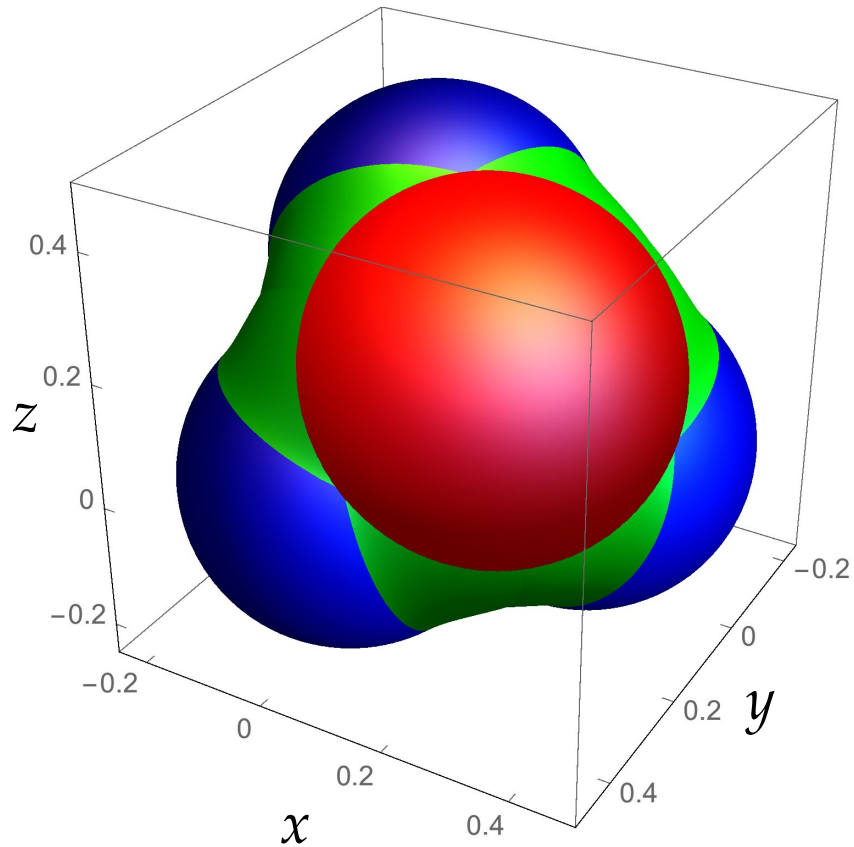
The unitarity of U_{tot} bounds the space of linear combinations

$$xP_{\mu\tau} + yP_{e\tau} + zP_{e\mu} \leq B(x, y, z)$$

Arbitrary coefficients

Complex function that traces over all possible values of the components of U_{tot}

(If you were curious, this is how the boundary function B looks:



Flavor composition at the Earth:

$$\begin{pmatrix} f_{e,\oplus} \\ f_{\mu,\oplus} \\ f_{\tau,\oplus} \end{pmatrix} = \underbrace{\begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix}}_{\text{Because the probabilities are bound by unitarity...}} \begin{pmatrix} f_{e,S} \\ 1 - f_{e,S} \\ 0 \end{pmatrix}$$

Because the probabilities
are bound by unitarity...

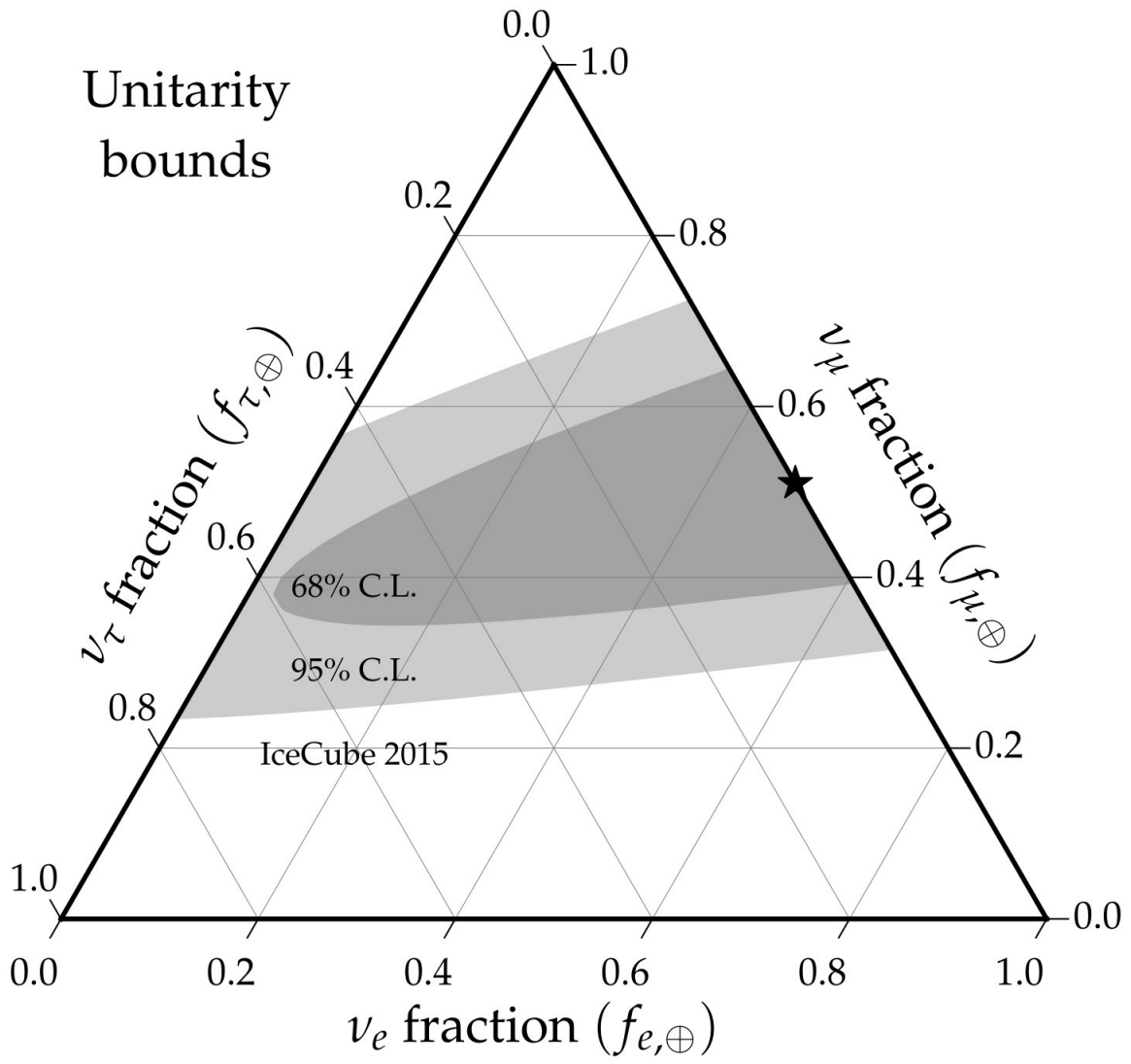
Flavor composition at the Earth:

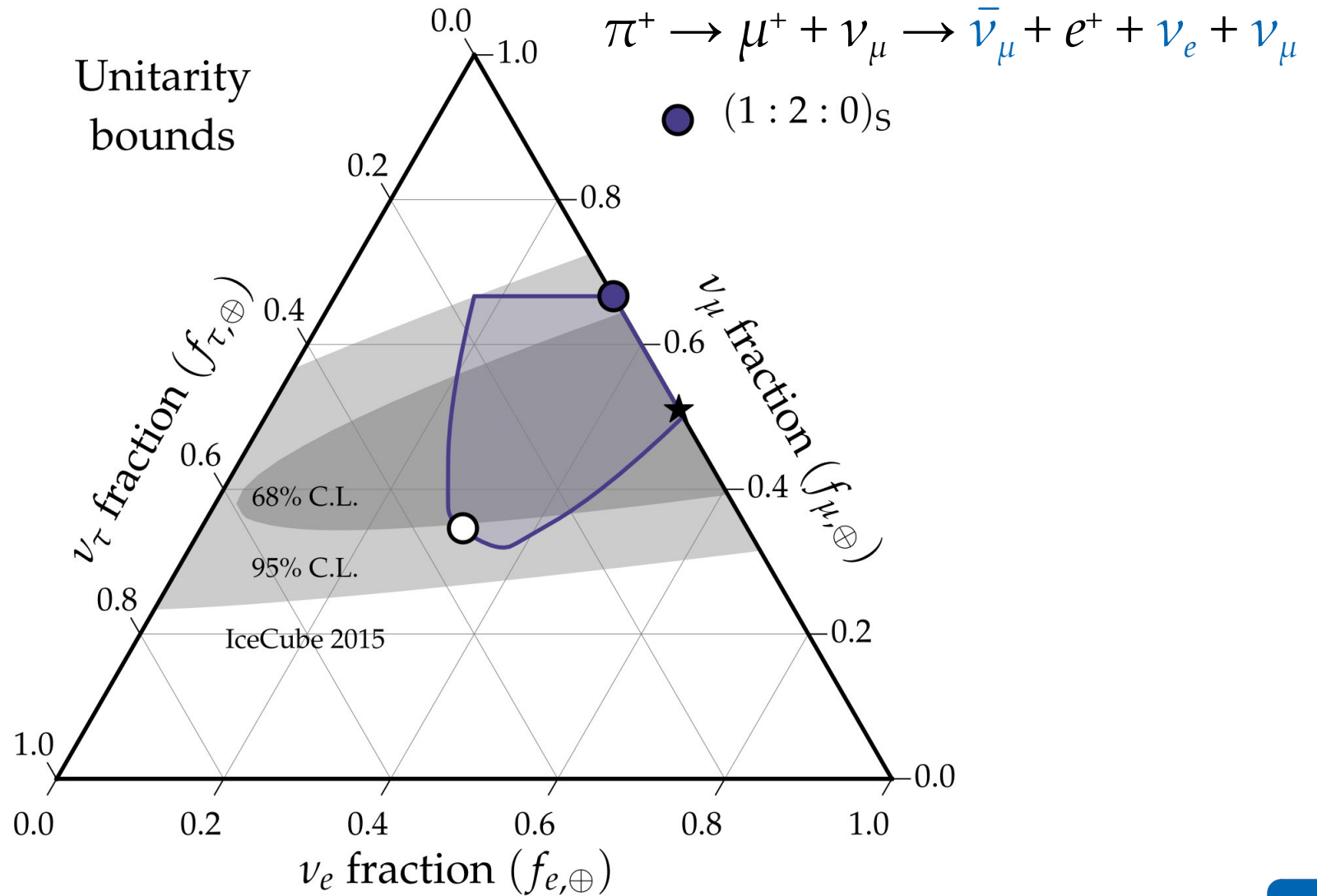
$$\underbrace{\begin{pmatrix} f_{e,\oplus} \\ f_{\mu,\oplus} \\ f_{\tau,\oplus} \end{pmatrix}}_{\text{... so is the predicted flavor composition at the Earth}} = \underbrace{\begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix}}_{\text{Because the probabilities are bound by unitarity...}} \begin{pmatrix} f_{e,S} \\ 1 - f_{e,S} \\ 0 \end{pmatrix}$$

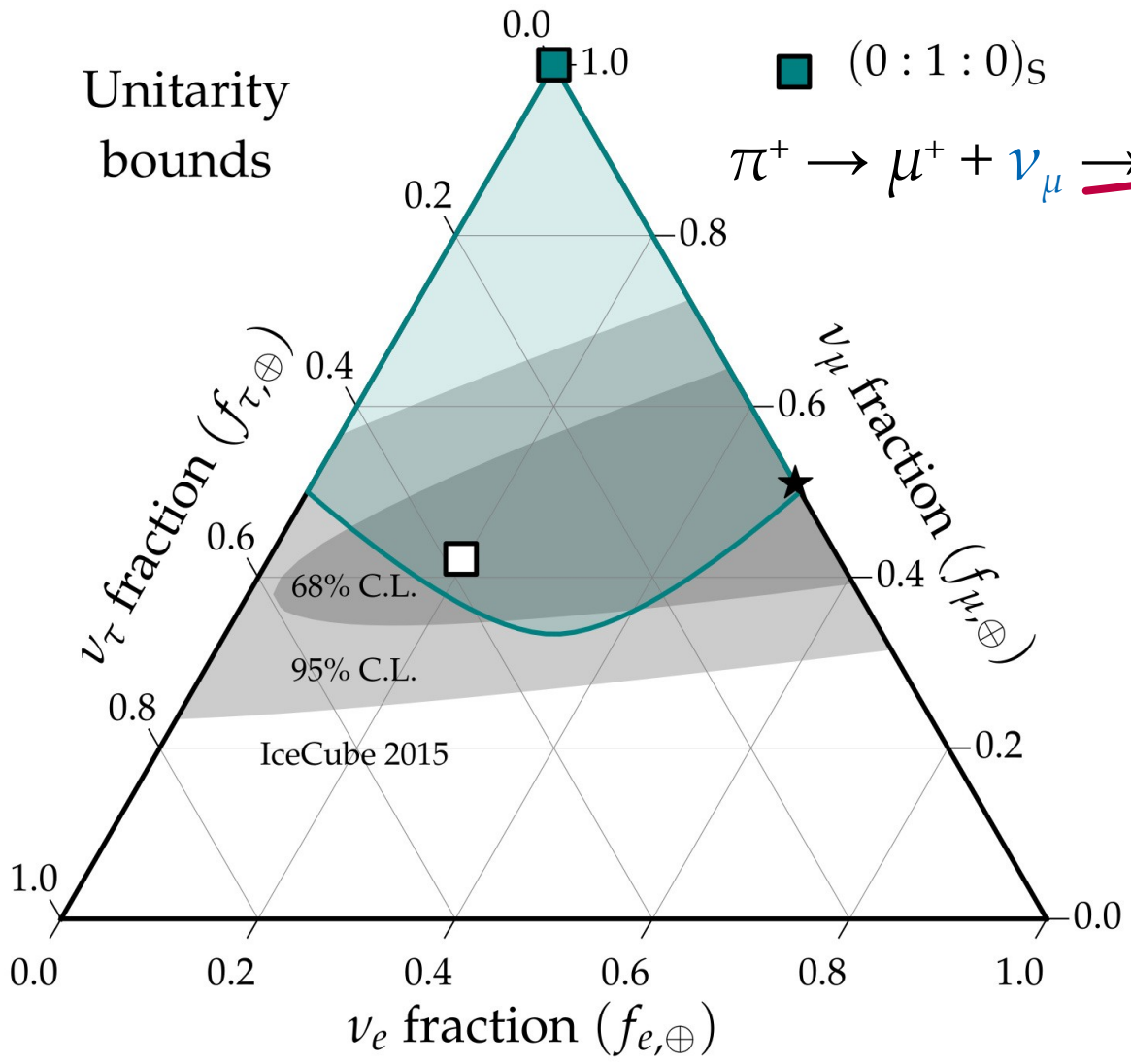
... so is the predicted flavor composition at the Earth

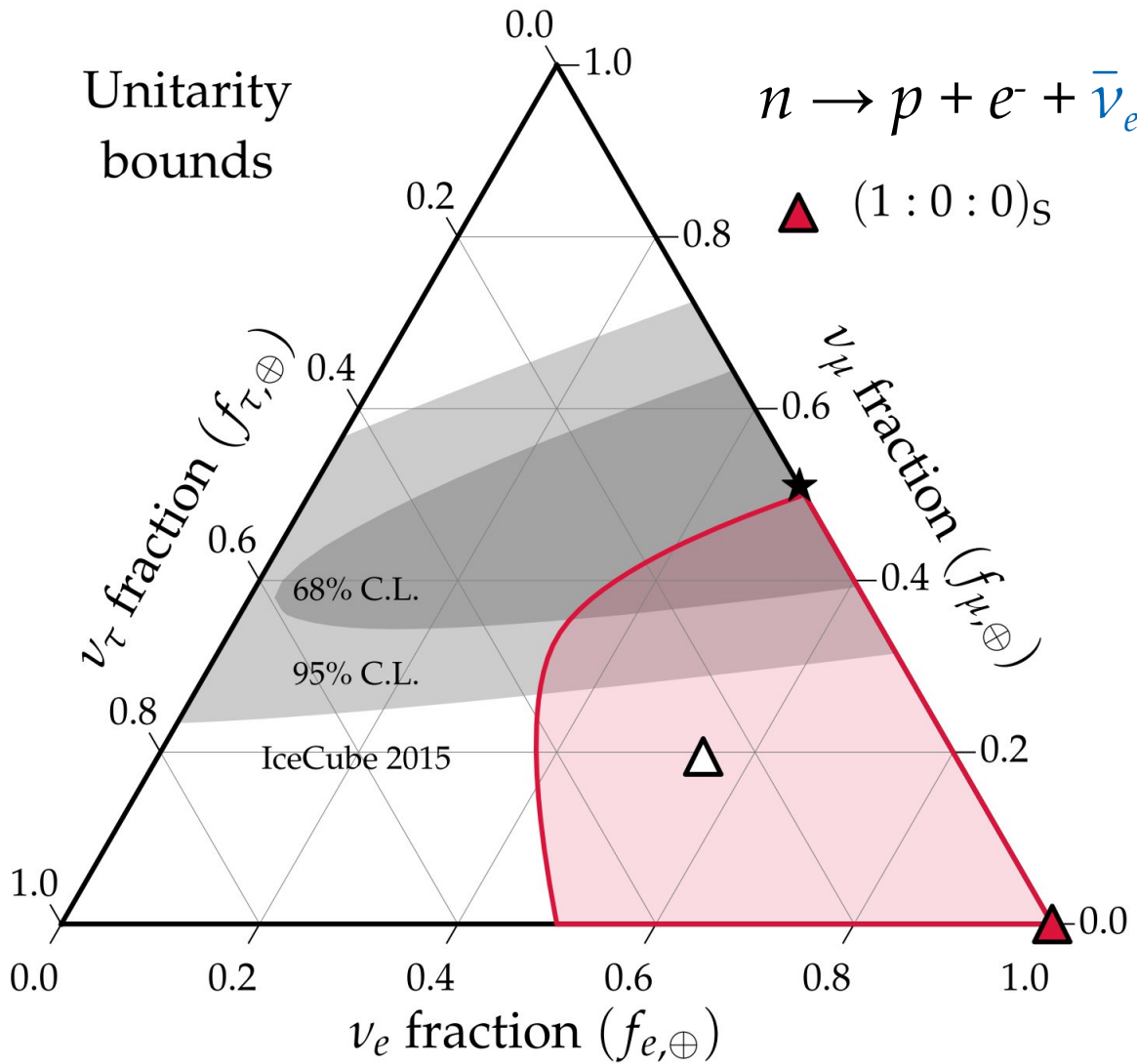
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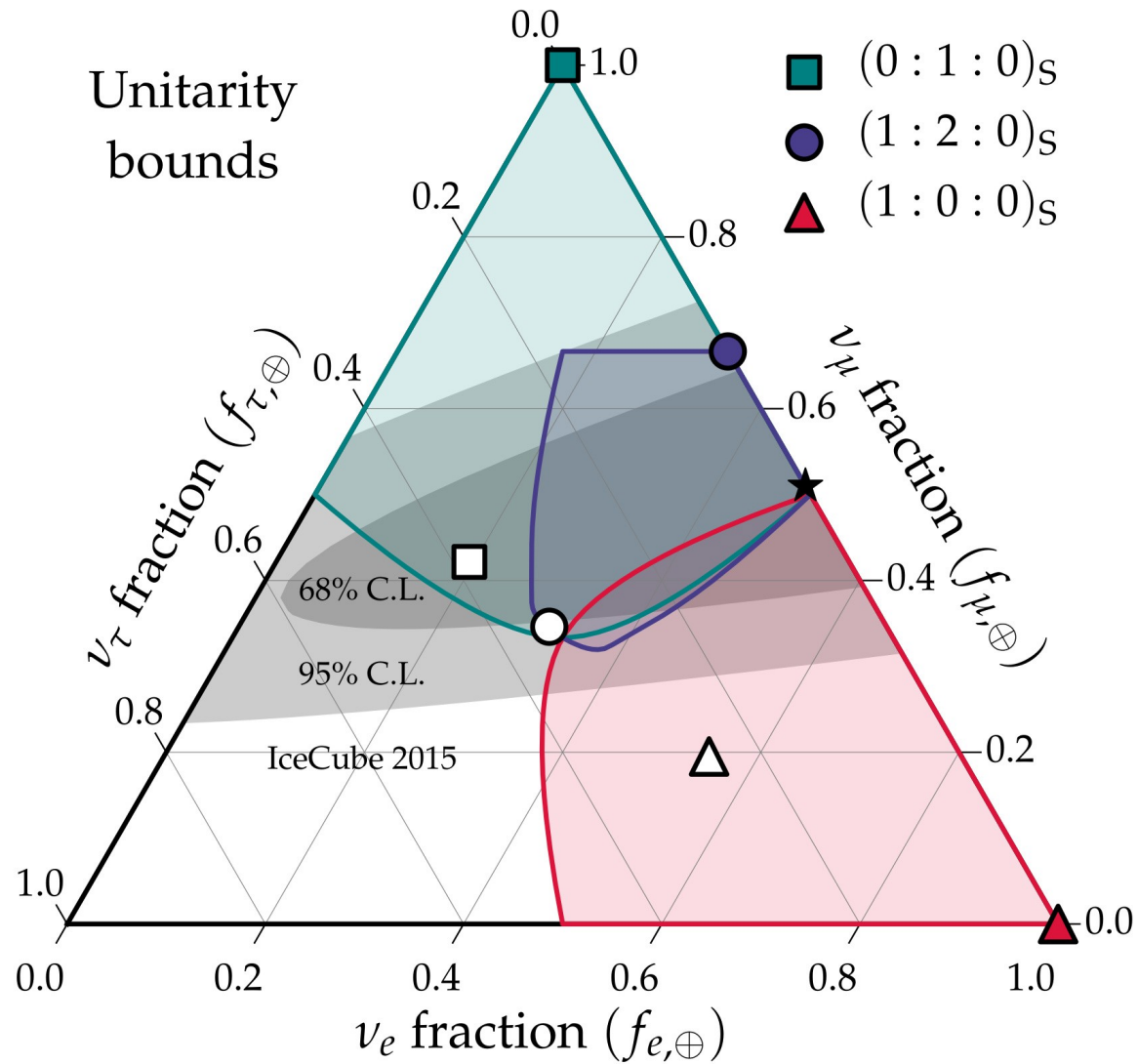






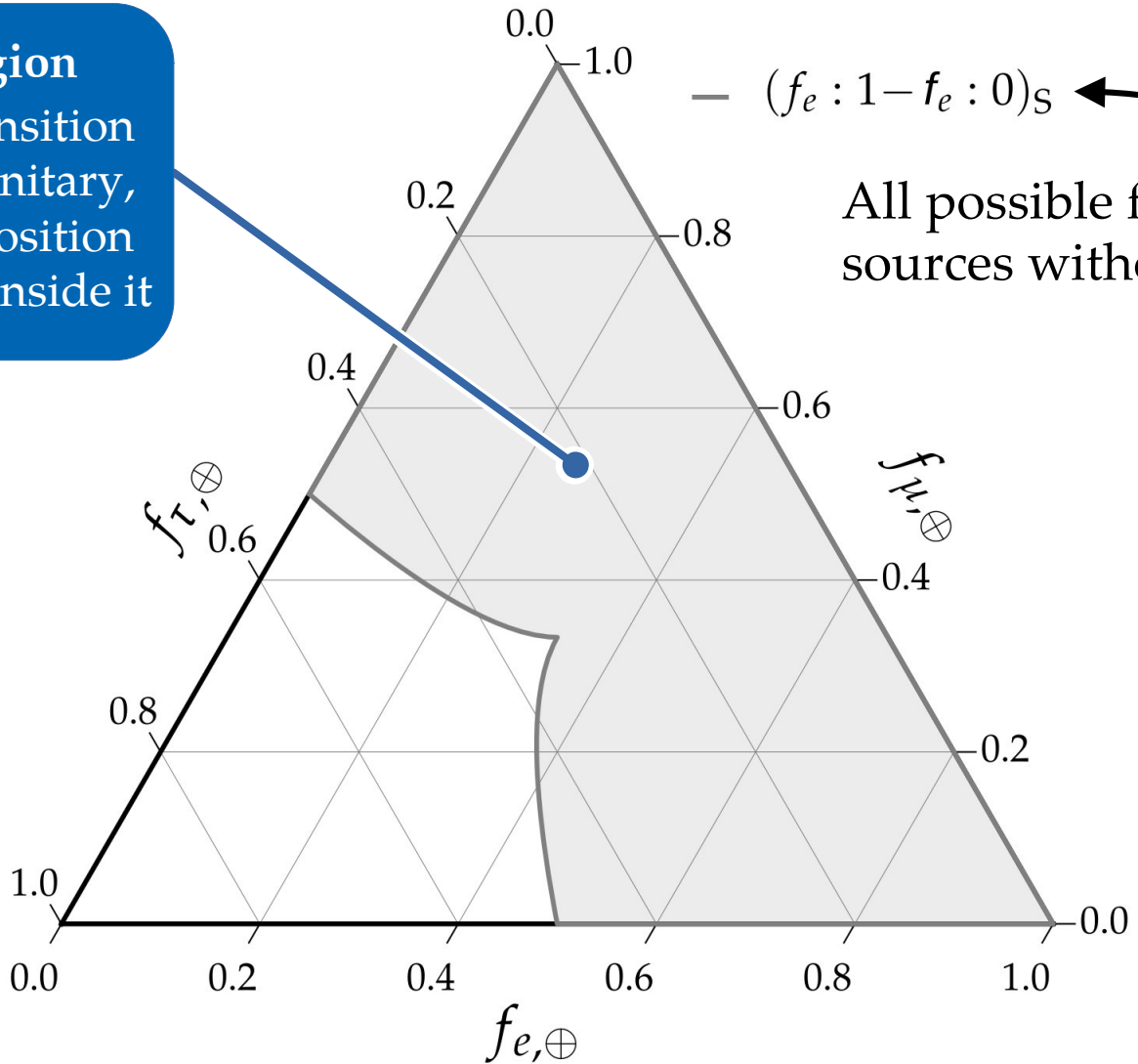


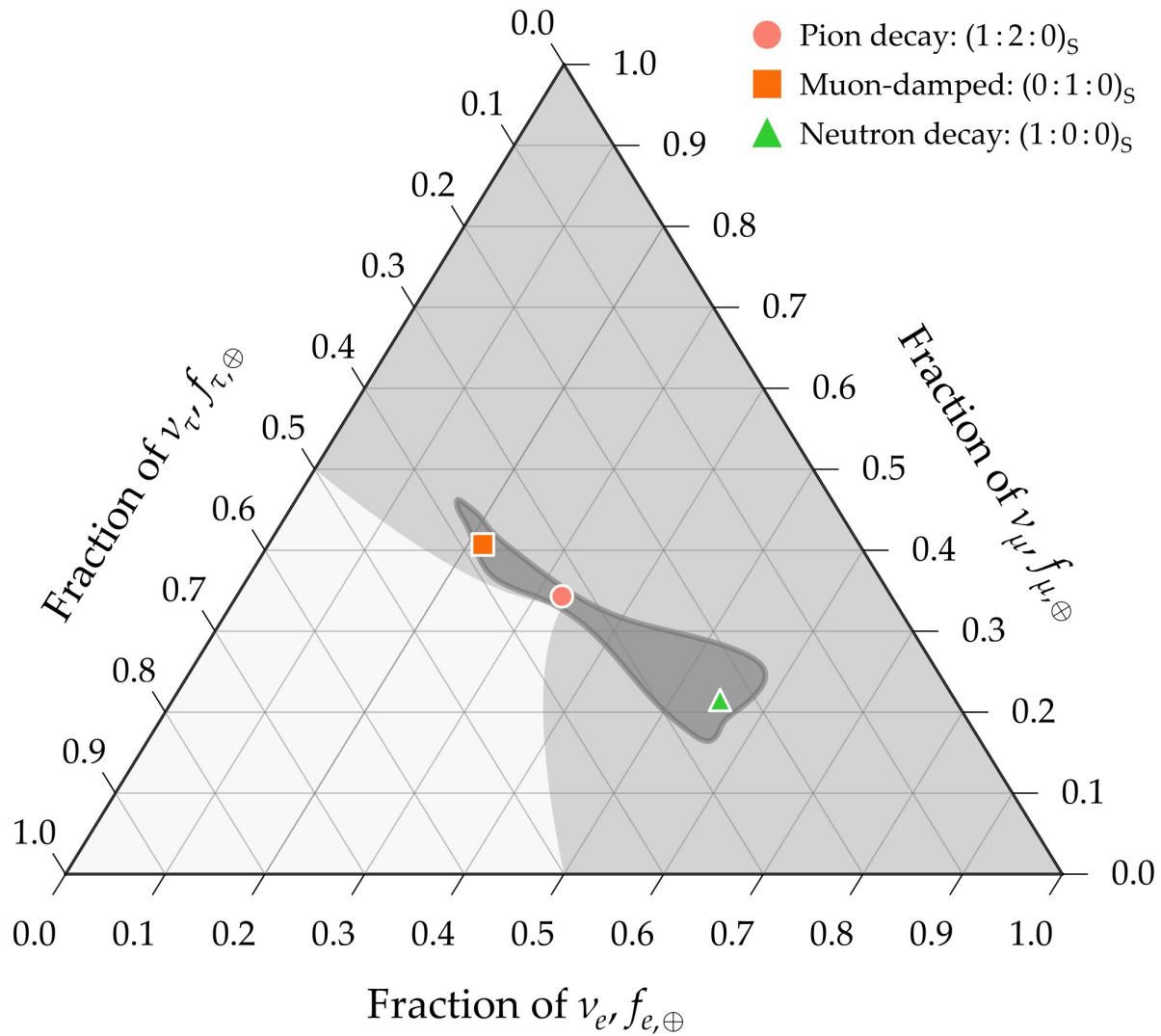




Unitarity region

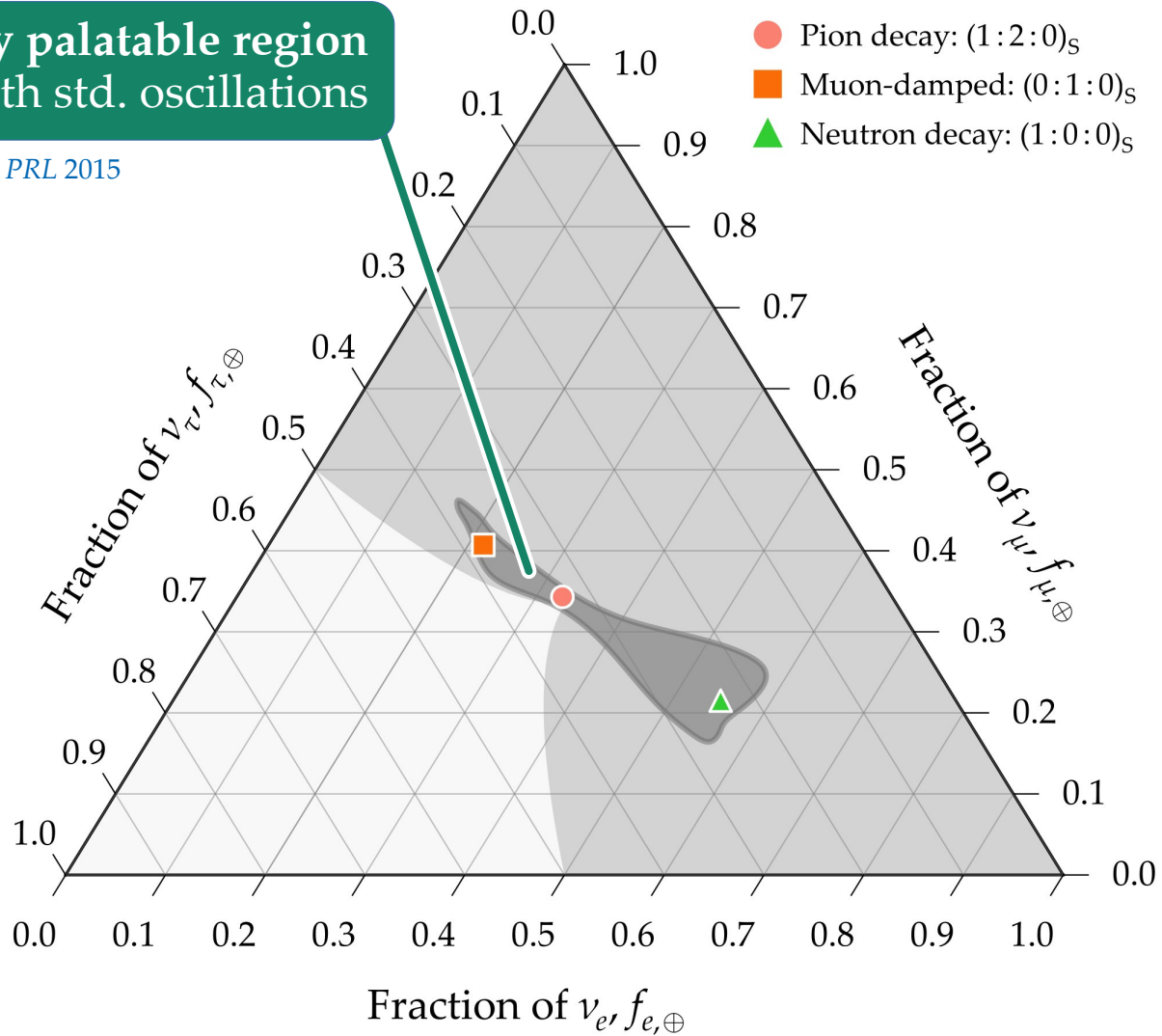
If the flavor-transition mechanism is unitary, the flavor composition at Earth will be inside it





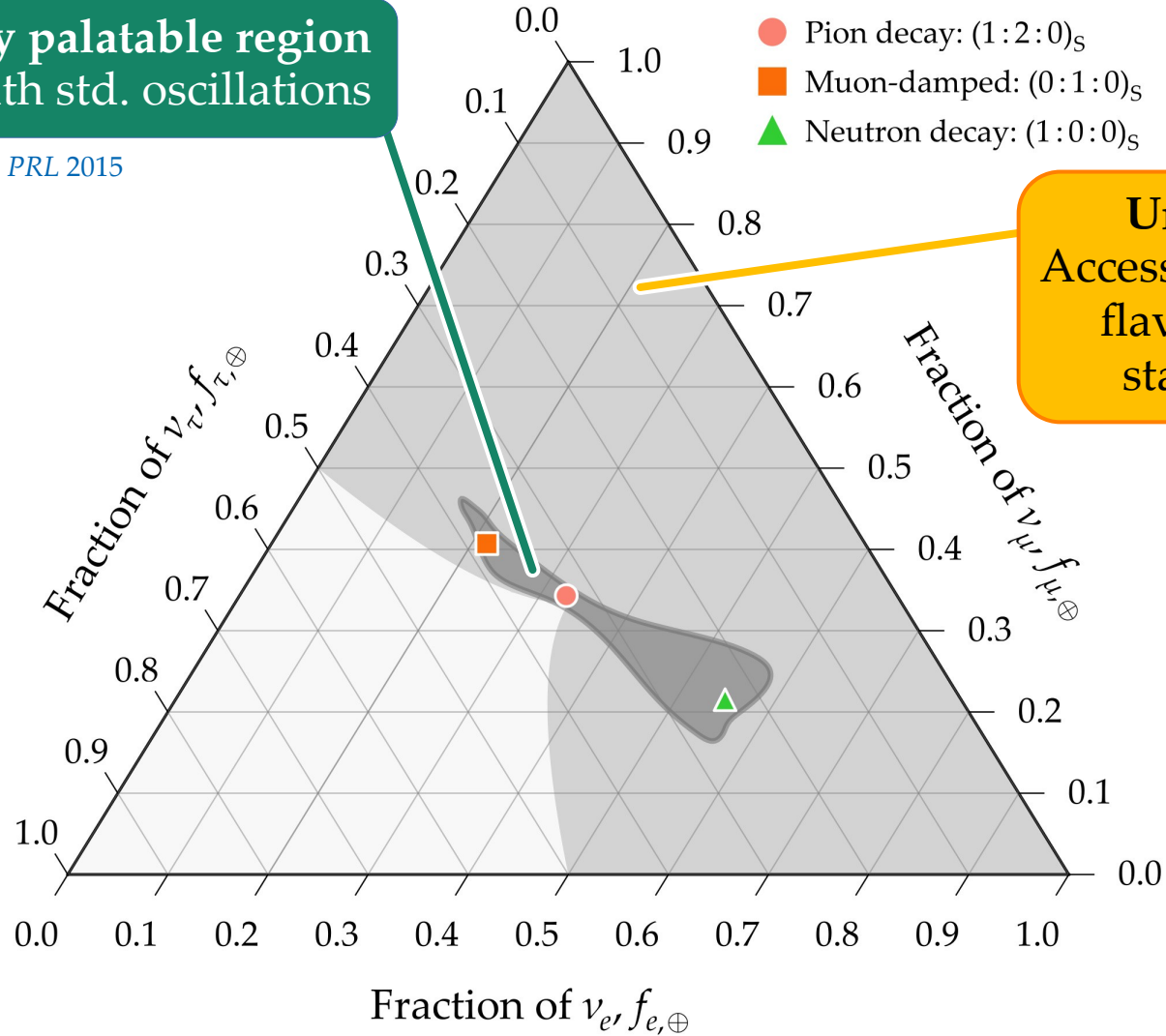
Theoretically palatable region Accessible with std. oscillations

MB, Beacom, Winter, *PRL* 2015



**Theoretically palatable region
Accessible with std. oscillations**

MB, Beacom, Winter, *PRL* 2015



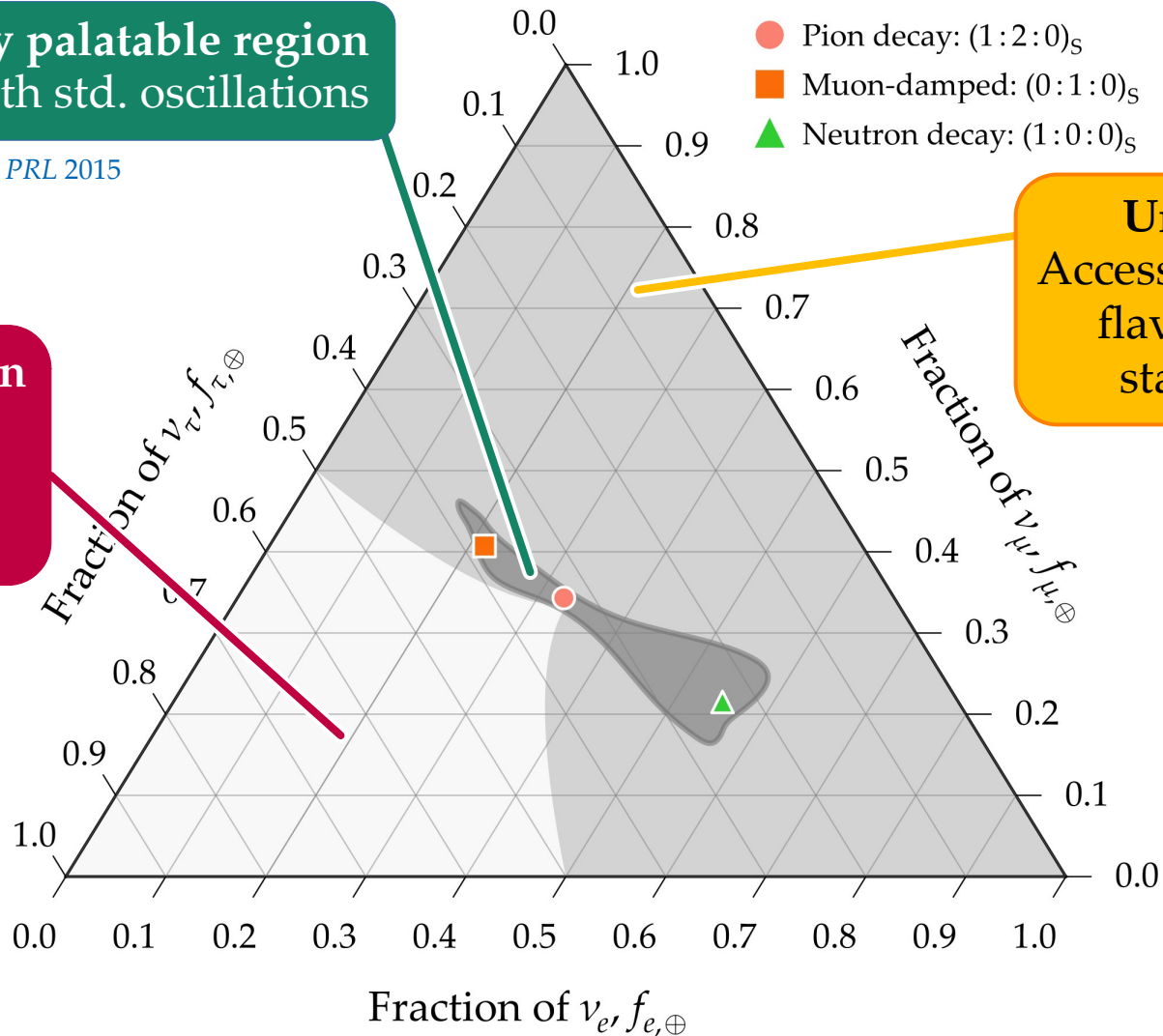
**Unitary region
Accessible with unitary
flavor transitions,
standard or not**

Ahlers, MB, Mu, *PRD* 2018

Theoretically palatable region
Accessible with std. oscillations

MB, Beacom, Winter, *PRL* 2015

Non-unitary region
Accessible only
with non-unitary
flavor transitions



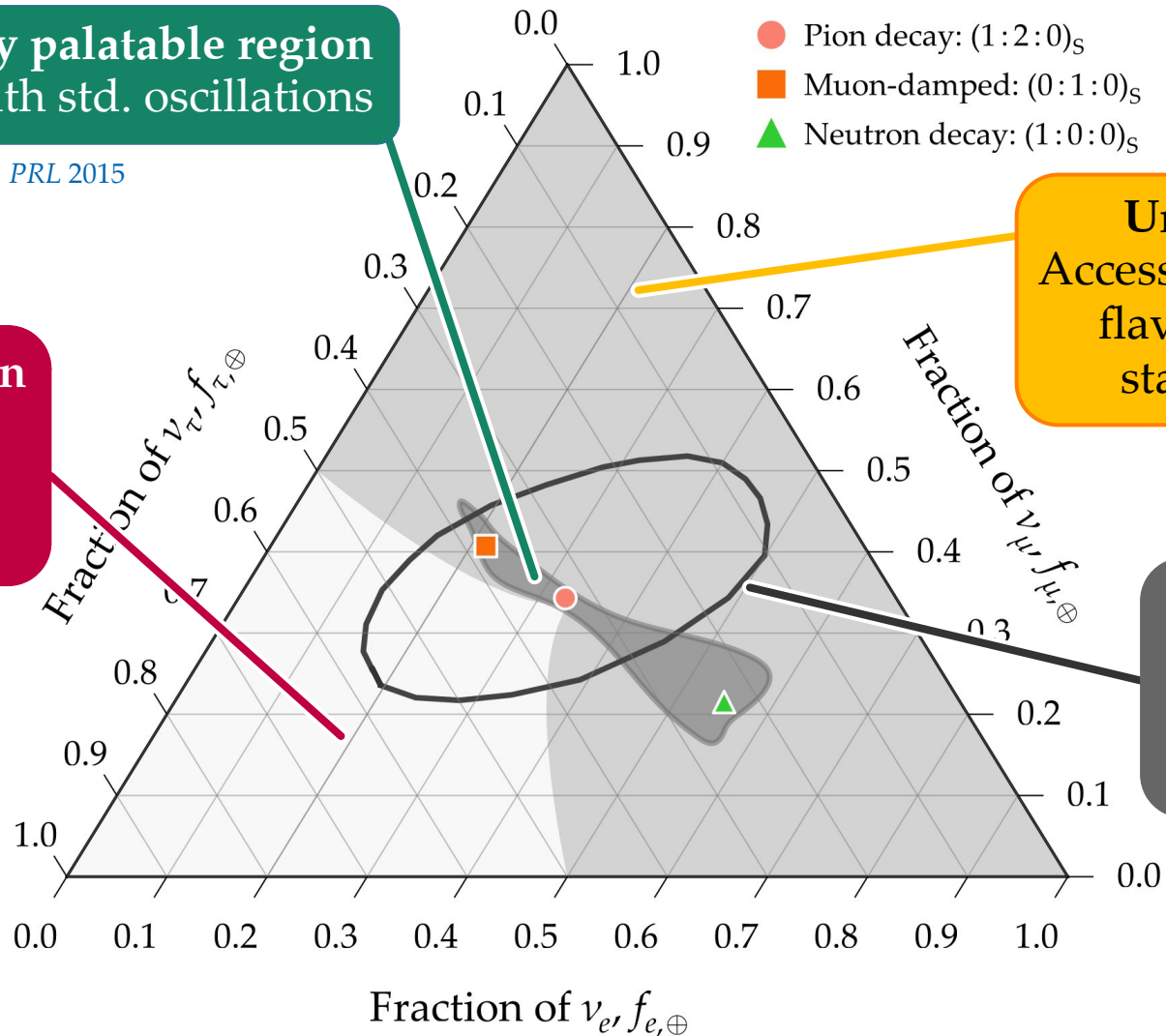
Unitary region
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Ahlers, MB, Mu, *PRD* 2018

Theoretically palatable region
Accessible with std. oscillations

MB, Beacom, Winter, *PRL* 2015

Non-unitary region
Accessible only
with non-unitary
flavor transitions



- Pion decay: $(1:2:0)_S$
- Muon-damped: $(0:1:0)_S$
- ▲ Neutron decay: $(1:0:0)_S$

Unitary region
Accessible with unitary
flavor transitions,
standard or not

Ahlers, MB, Mu, *PRD* 2018

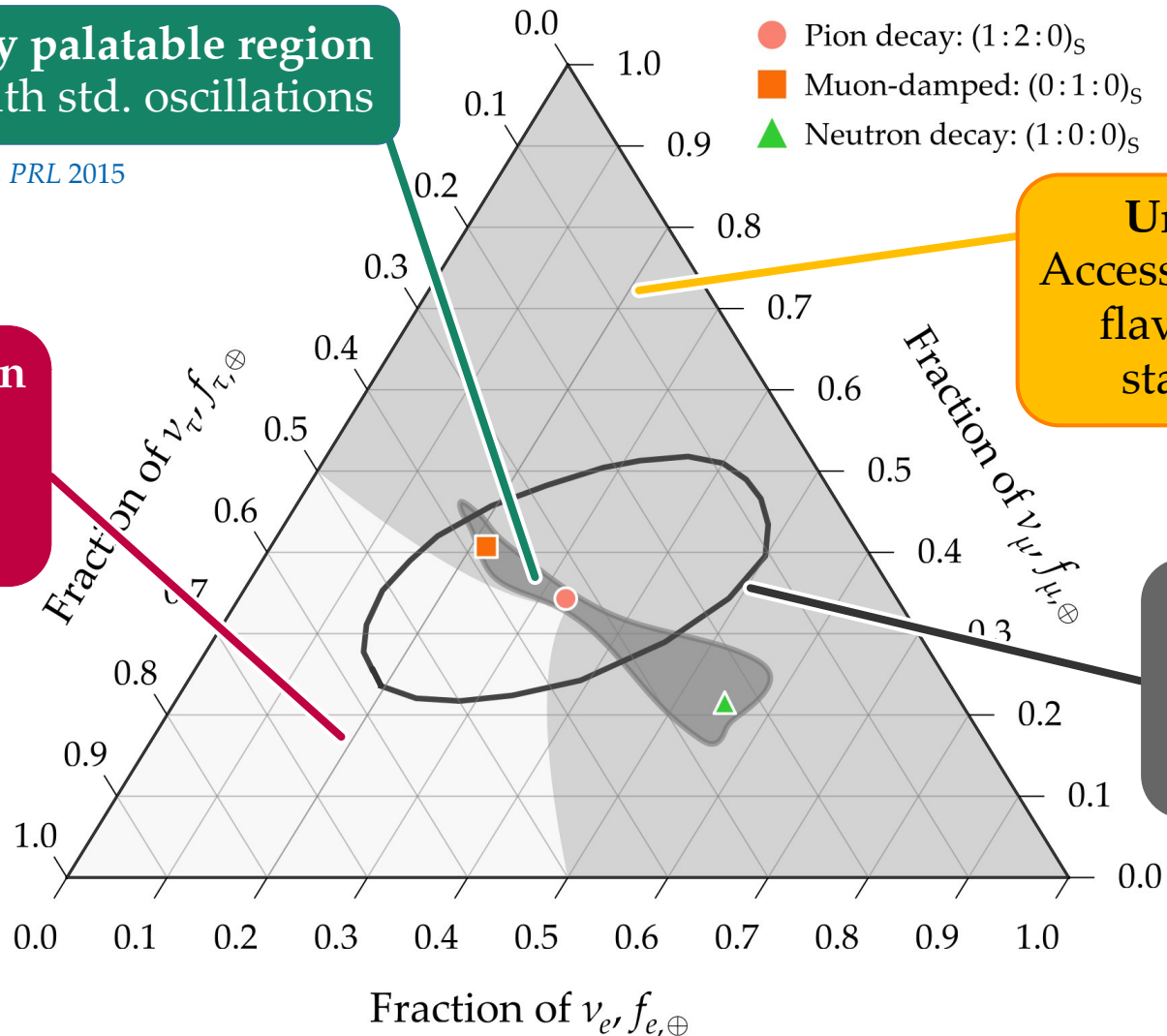
IceCube 2025
Medium-Energy
Starting Events
(MESE) ICRC 2025

Theoretically palatable region
Accessible with std. oscillations

MB, Beacom, Winter, *PRL* 2015

Non-unitary region
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with non-unitary
flavor transitions

What lives here?
Let's see...



Unitary region
Accessible with unitary
flavor transitions,
standard or not

Ahlers, MB, Mu, *PRD* 2018

IceCube 2025
Medium-Energy
Starting Events
(MESE) ICRC 2025

Beyond 3×3 unitarity

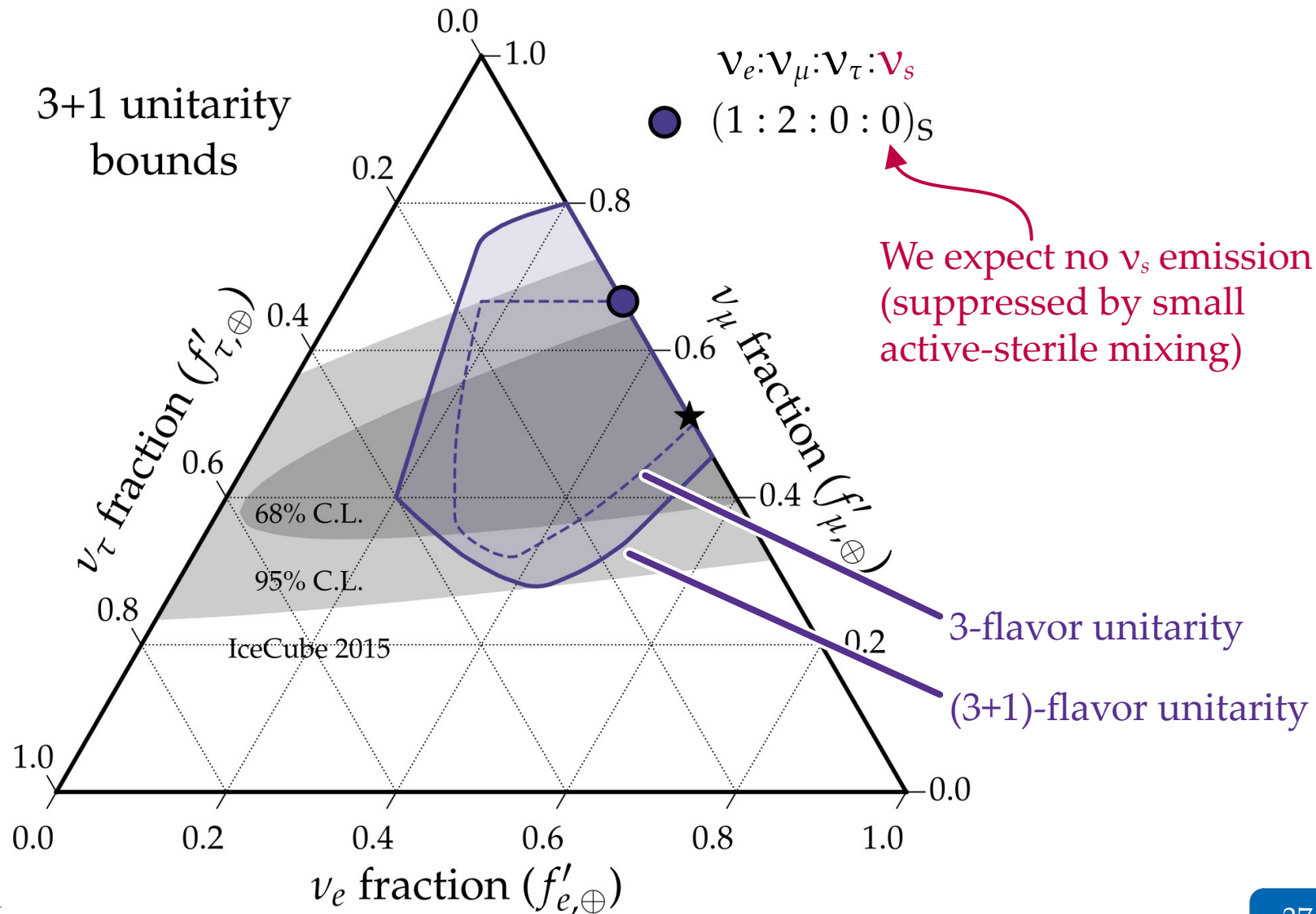
Consider 3+1 neutrino flavors (s: sterile neutrino)

Probabilities for *any kind* of unitary flavor transitions among them:

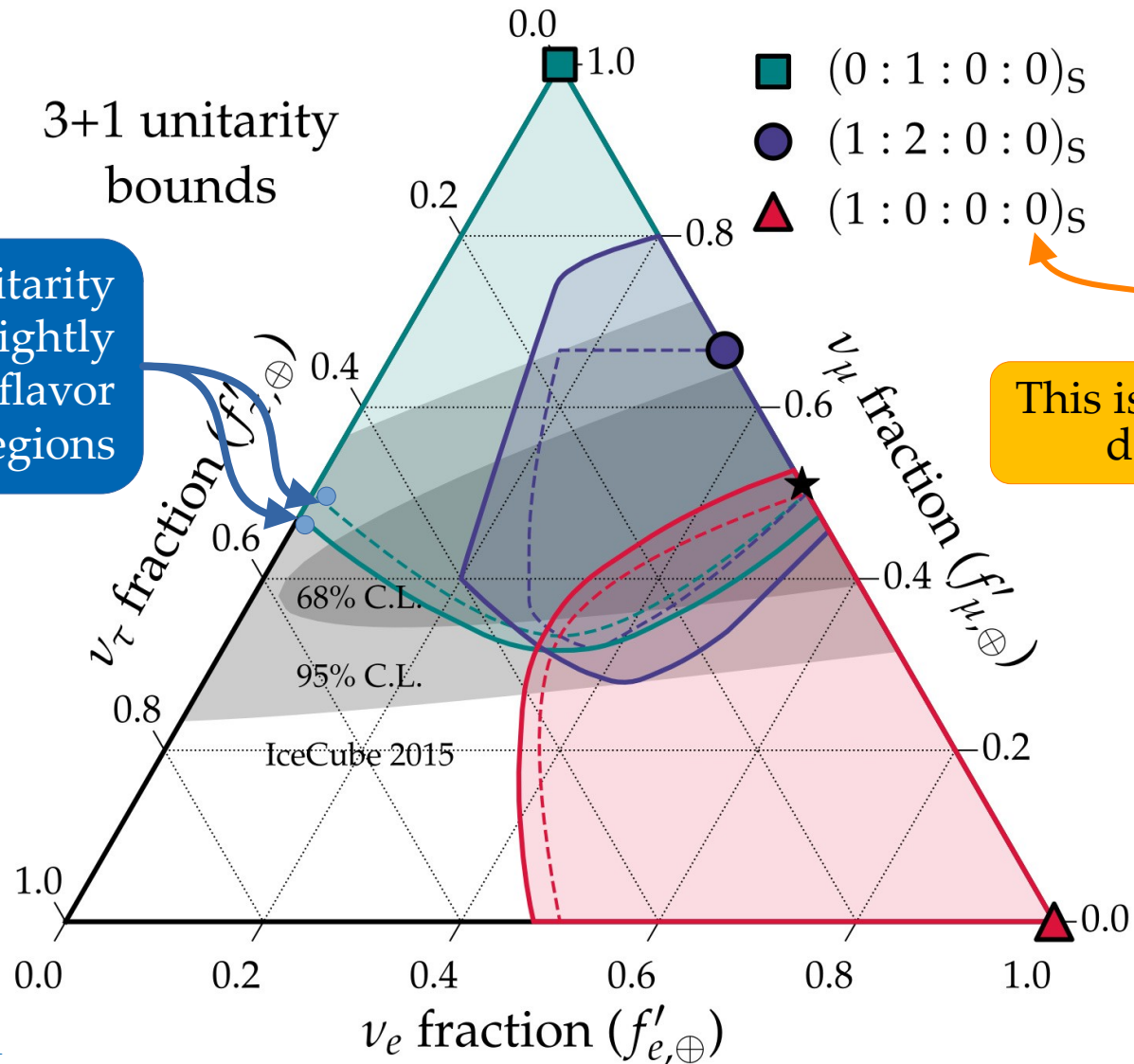
$$P_{\alpha\beta} = \sum_{i=1}^4 |(U_{\text{tot}})_{\alpha i}|^2 |(U_{\text{tot}})_{\beta i}|^2 = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} & P_{es} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} & P_{\mu s} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} & P_{\tau s} \\ P_{es} & P_{\mu s} & P_{\tau s} & P_{ss} \end{pmatrix}$$

6 independent entries

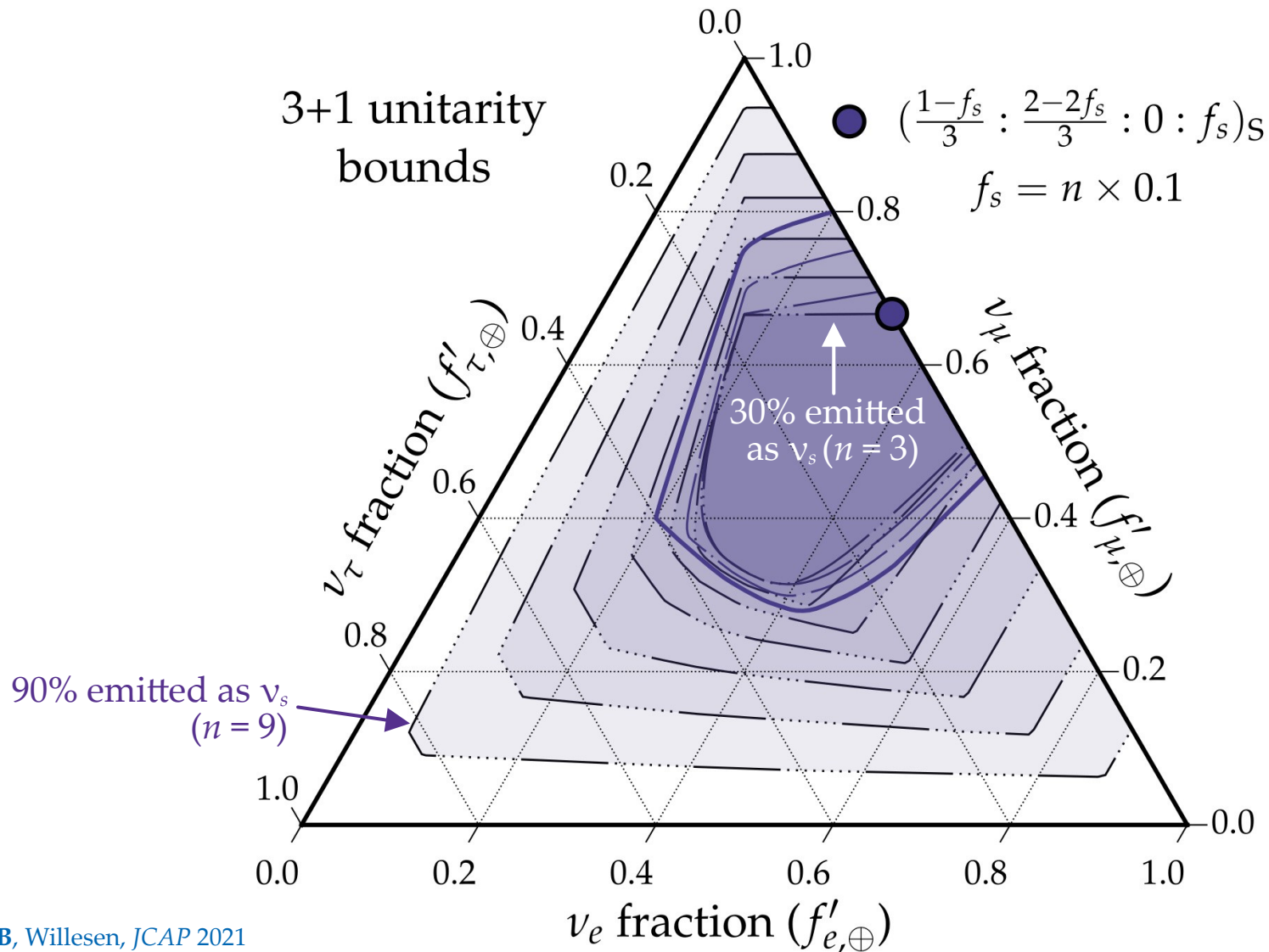
Like before, we find the boundary of the independent entries for any possible value of the components of U_{tot}



The (3+1)-flavor unitarity regions are only slightly larger than the 3-flavor unitarity regions



This is because sources do not emit ν_τ



Measuring high-energy non-unitarity

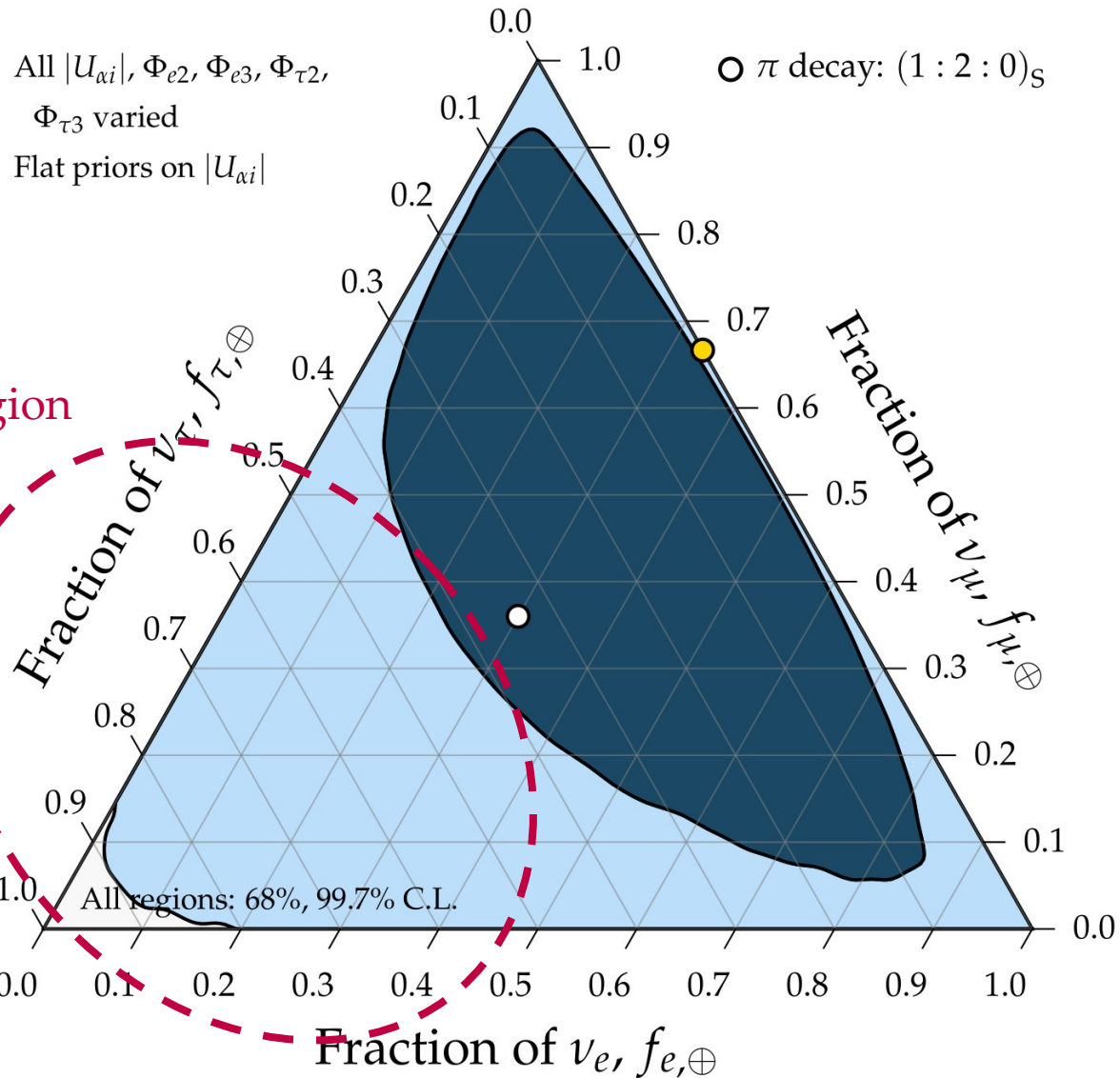
Generic parametrization of a non-unitary 3×3 leptonic mixing matrix:

$$U = \begin{pmatrix} |U_{e1}| & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}|e^{i\phi_{e3}} \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}|e^{i\phi_{\tau2}} & |U_{\tau3}|e^{i\phi_{\tau3}} \end{pmatrix}$$

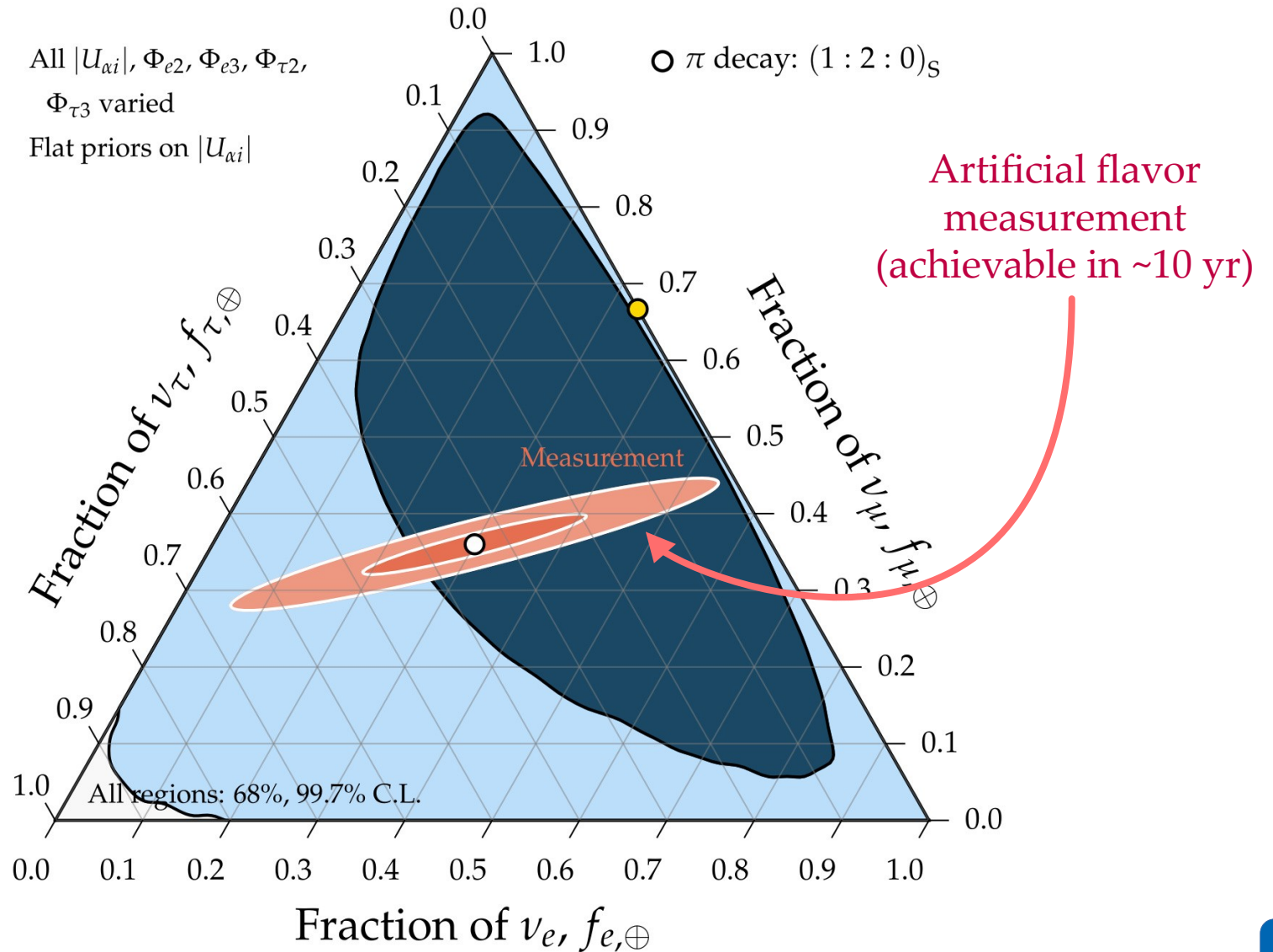
13 parameters = 9 magnitudes + 4 CP-violating phases

To predict the flavor composition at Earth, we also need $f_{e,S}$

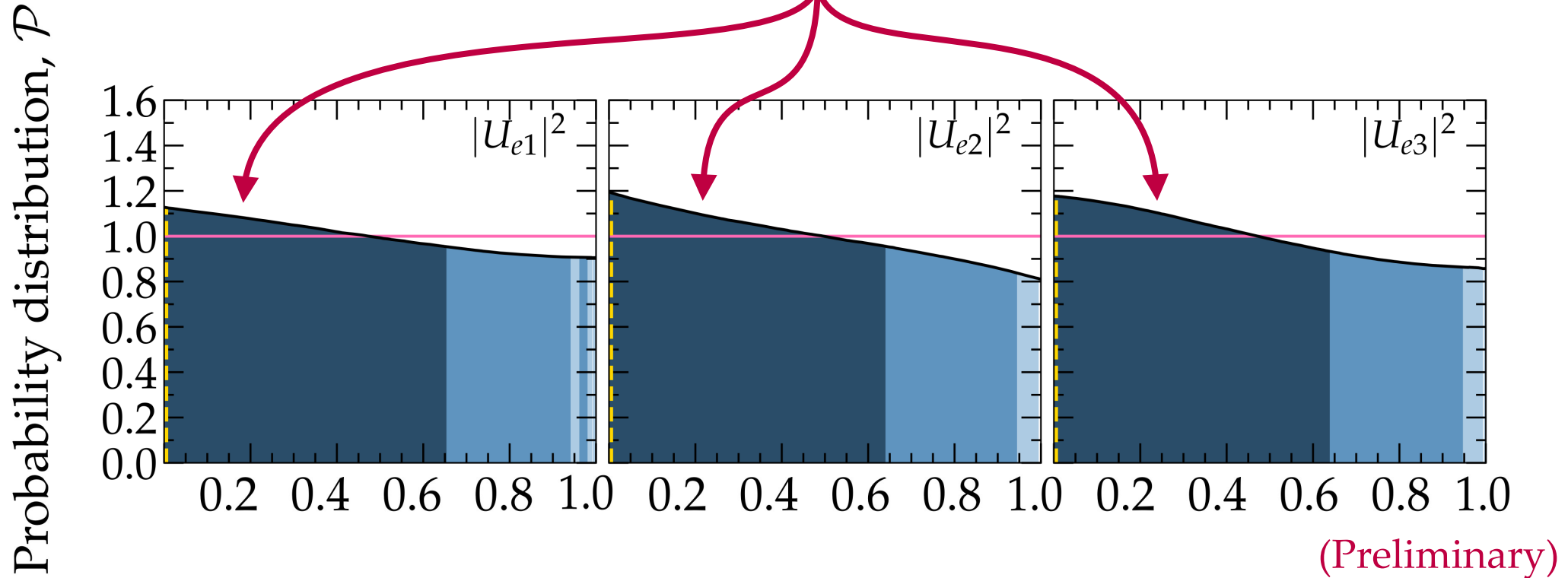
[Start by fixing it to 1/3 (from pion decay); later, we marginalize over it]



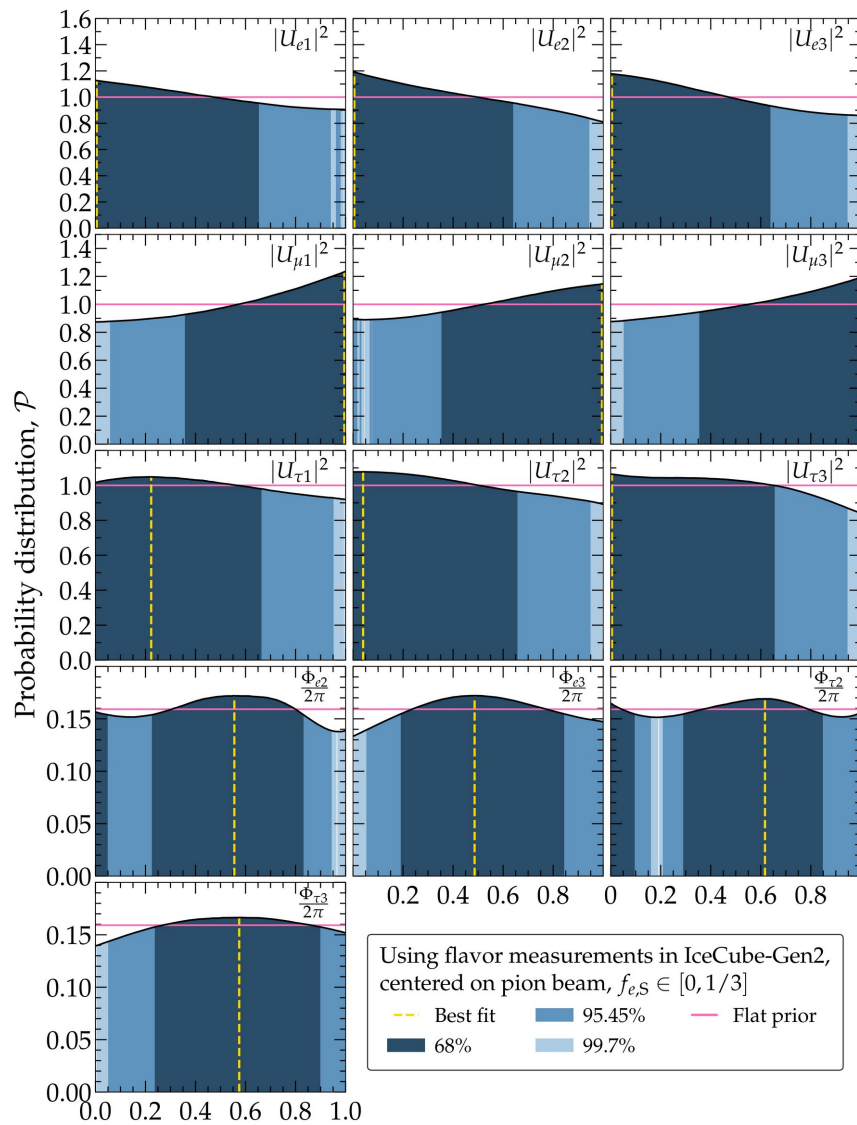
Extends into the region
 accessible only
 with non-unitary
 mixing



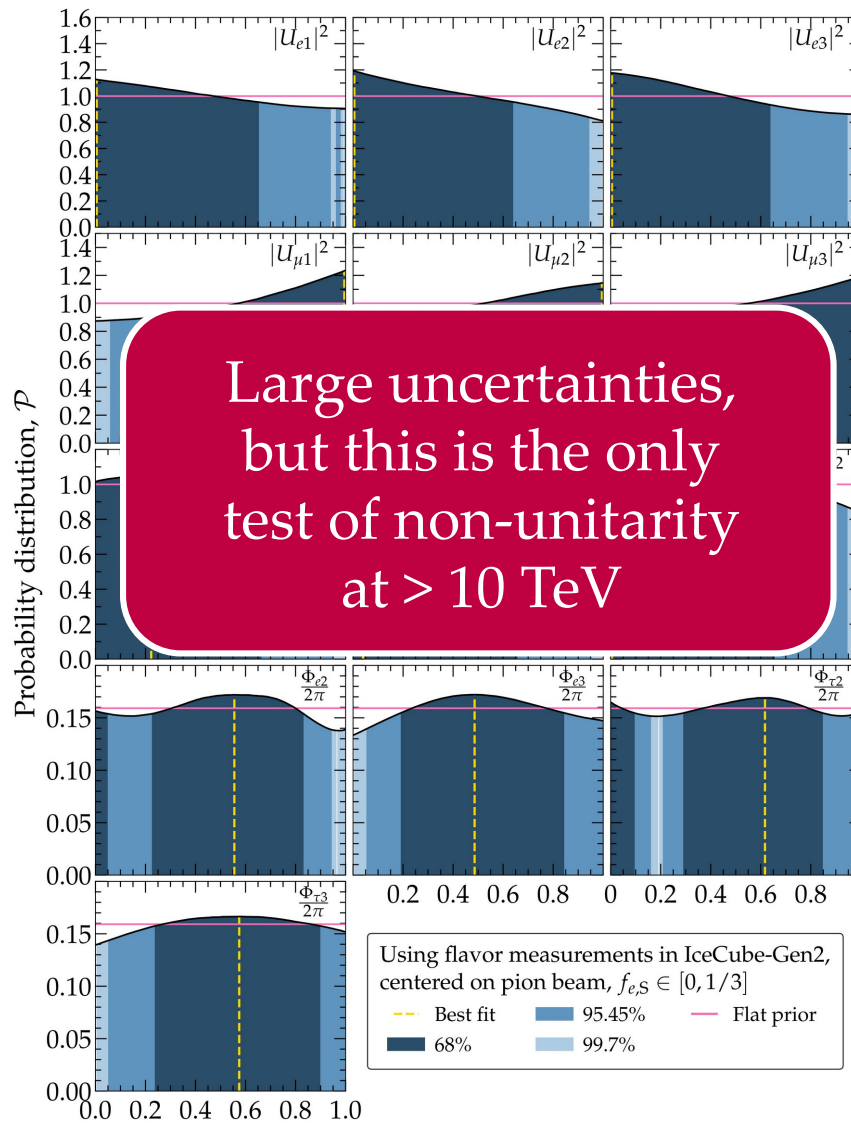
The 68% C.L. allowed parameter regions are wide, *e.g.*,



Main source of uncertainty:
ignorance of the flavor composition at the sources, $f_{e,S}$



(Preliminary)



(Preliminary)

Is ν mixing different
at high energies?

Renormalization group running – the values of the neutrino masses and mixing parameters change with the energy scale, μ

E.g., for the masses, the renormalization-group-running equation (RGE) is

$$16\pi^2 \frac{dm_i}{d \ln \mu} = \left[(\alpha_\kappa + 2\alpha_H) + 2C_\kappa \sum_{\alpha=e,\mu,\tau} |U_{\alpha i}|^2 y_\alpha^2 \right] m_i$$

These depend on the effective field theory adopted (SM or beyond)

Note: the running is negligible in the SM

Yukawa coefficient of charged lepton of flavor α

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and this determines the RGE running of the mixing angles, *i.e.*,

$$\frac{d\theta_{ij}}{d \ln \mu} = F(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\text{CP}}, m_1, m_2, m_3)$$

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We set μ to the transferred momentum, $\mu = Q \equiv \sqrt{-q^2}$.

Two energy scales:

Low-energy scale at neutrino production:
in astrophysical sources, ν are made in π decays

$$Q = m_{\pi} \approx 140 \text{ MeV}$$

Mixing parameters have PMNS values

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Mixing parameters have PMNS values

High-energy scale at neutrino detection:

at Earth, ν are detected in νN deep-inelastic scattering

$$Q = \sqrt{2m_N E_\nu xy} \lesssim \sqrt{m_N E_\nu} \lesssim 10^3 \text{ GeV} \begin{pmatrix} x: \text{Bjorken-}x \\ y: \text{inelasticity} \end{pmatrix}$$

Mixing parameters have potentially non-PMNS values

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Mixing parameters have potentially non-PMNS values

RGE running



In lieu of explicit RGE running we use two lepton mixing matrices:

Low-energy scale at neutrino production:

in astrophysical sources, ν are made in π decays

U_{PMNS}

$$Q = m_\pi \approx 140 \text{ MeV}$$

Mixing parameters have PMNS values

High-energy scale at neutrino detection:

at Earth, ν are detected in νN deep-inelastic scattering

U'

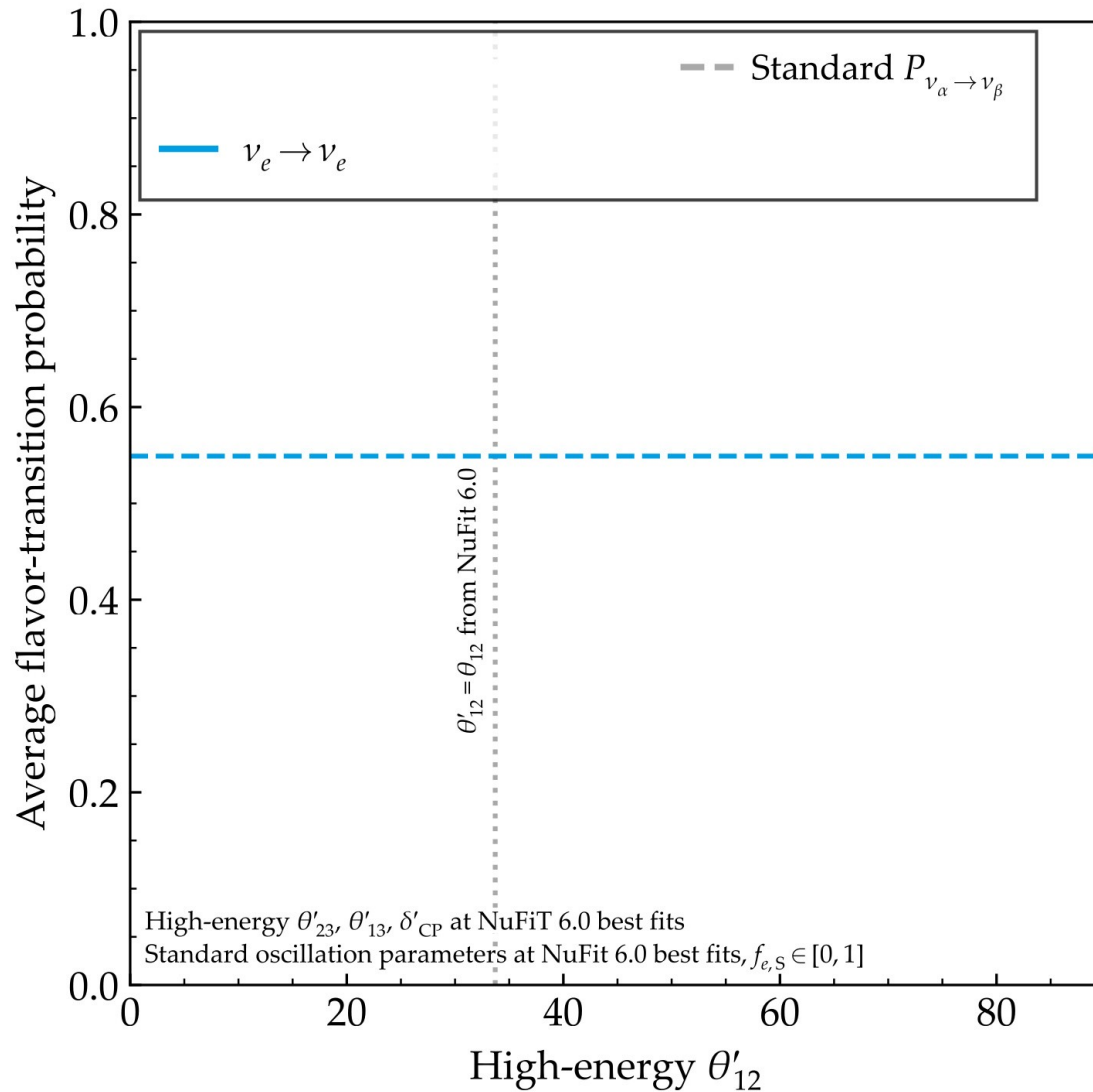
$$Q = \sqrt{2m_N E_\nu xy} \lesssim \sqrt{m_N E_\nu} \lesssim 10^3 \text{ GeV} \begin{pmatrix} x: \text{Bjorken-}x \\ y: \text{inelasticity} \end{pmatrix}$$

Mixing parameters have potentially non-PMNS values

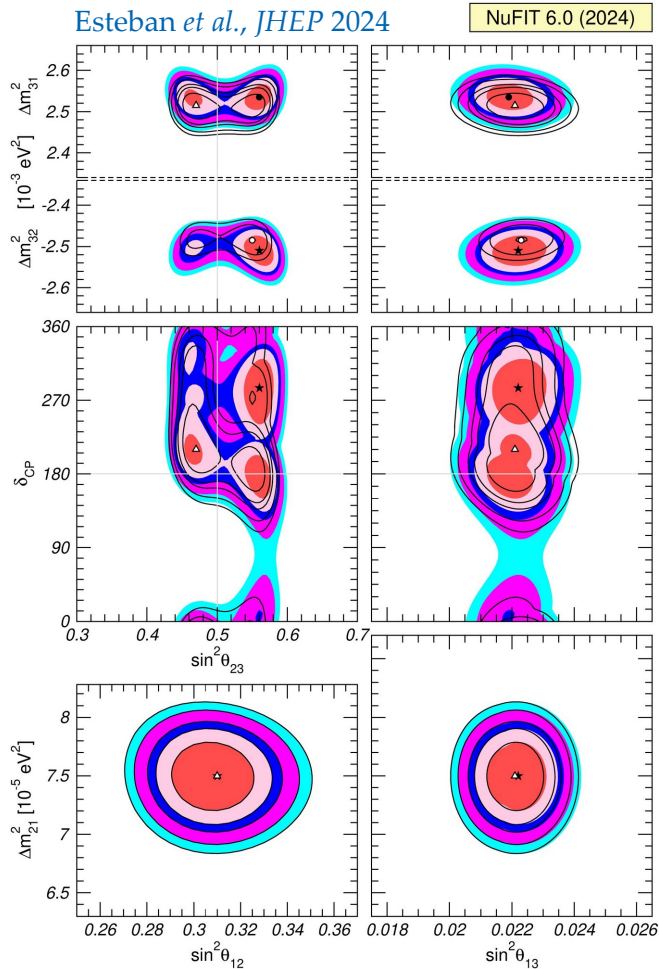
Average flavor-transition probability:

They now depend on **low-energy** & **high-energy** mixing parameters

$$P_{\alpha\beta} = \sum_{i=1}^3 \underbrace{|(U_{\text{PMNS}})_{\alpha i}|^2}_{\substack{\text{At} \\ \text{neutrino production}}} \underbrace{|U'_{\beta i}(\theta'_{12}, \theta'_{23}, \theta'_{13}, \delta'_{\text{CP}})|^2}_{\substack{\text{At} \\ \text{neutrino detection}}}$$



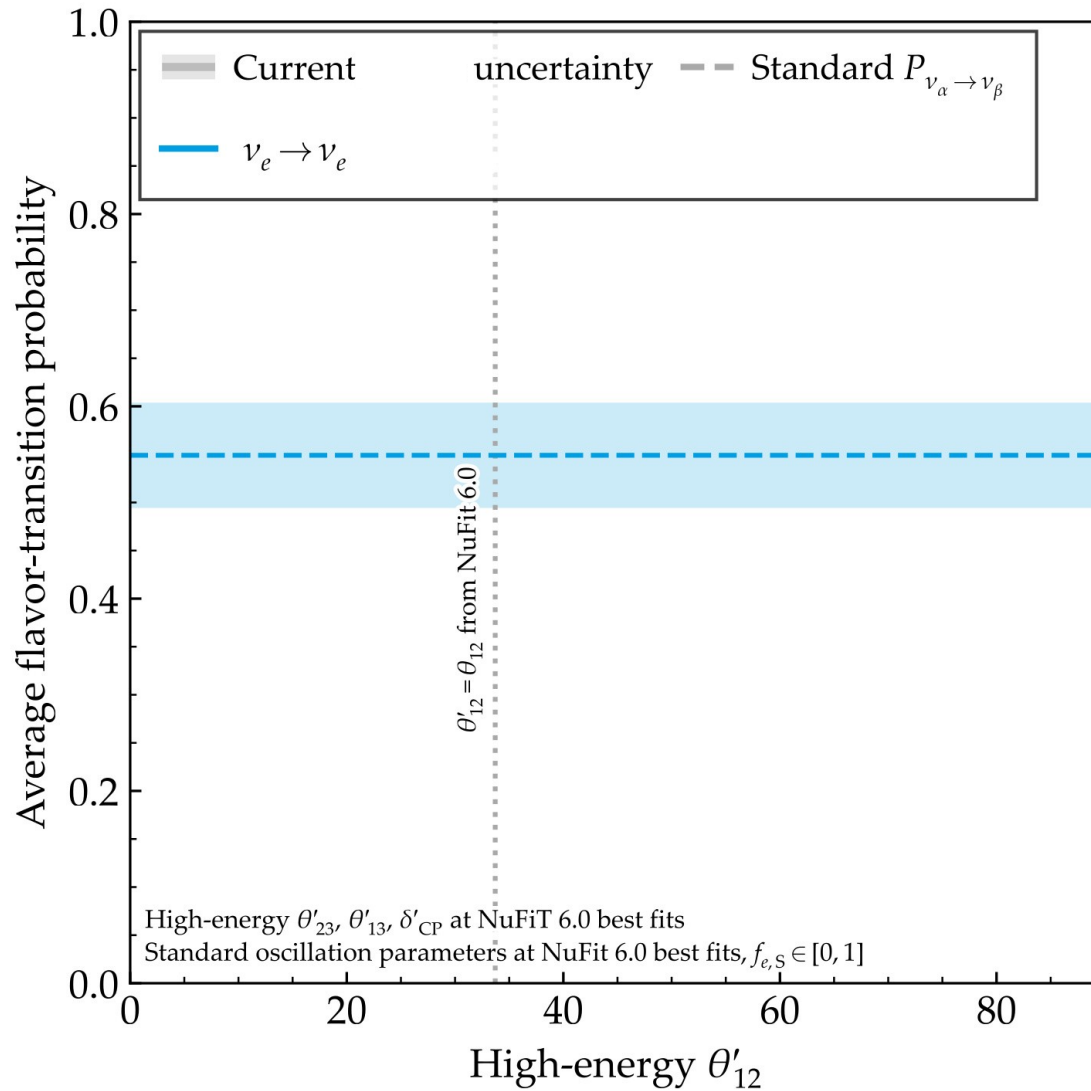
Best-fit & current uncertainties on mixing angles and CP-violation phase



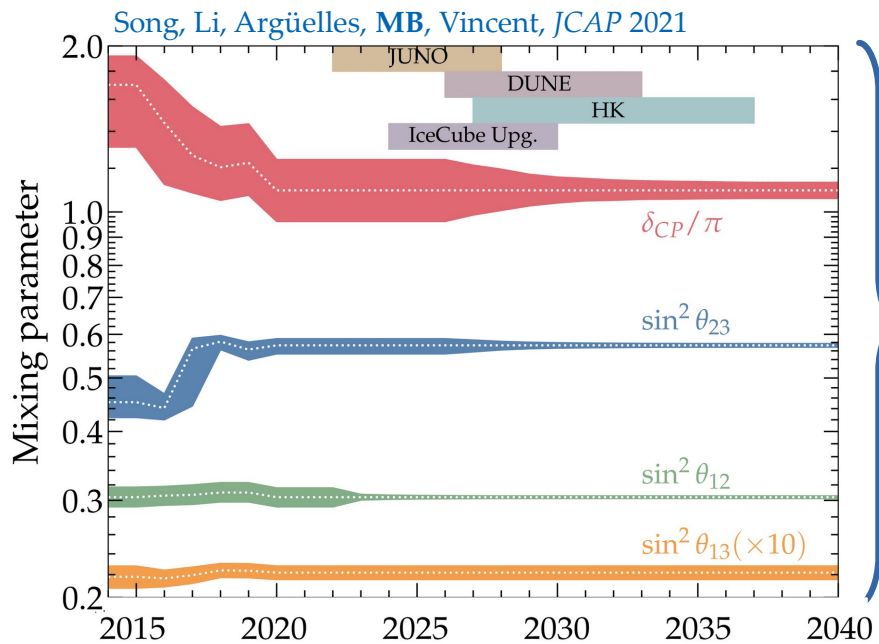
$$P_{\alpha\beta} = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2$$

PMNS

Best-fit & current uncertainties on the standard probabilities



Future uncertainties on mixing angles and CP-violation phase

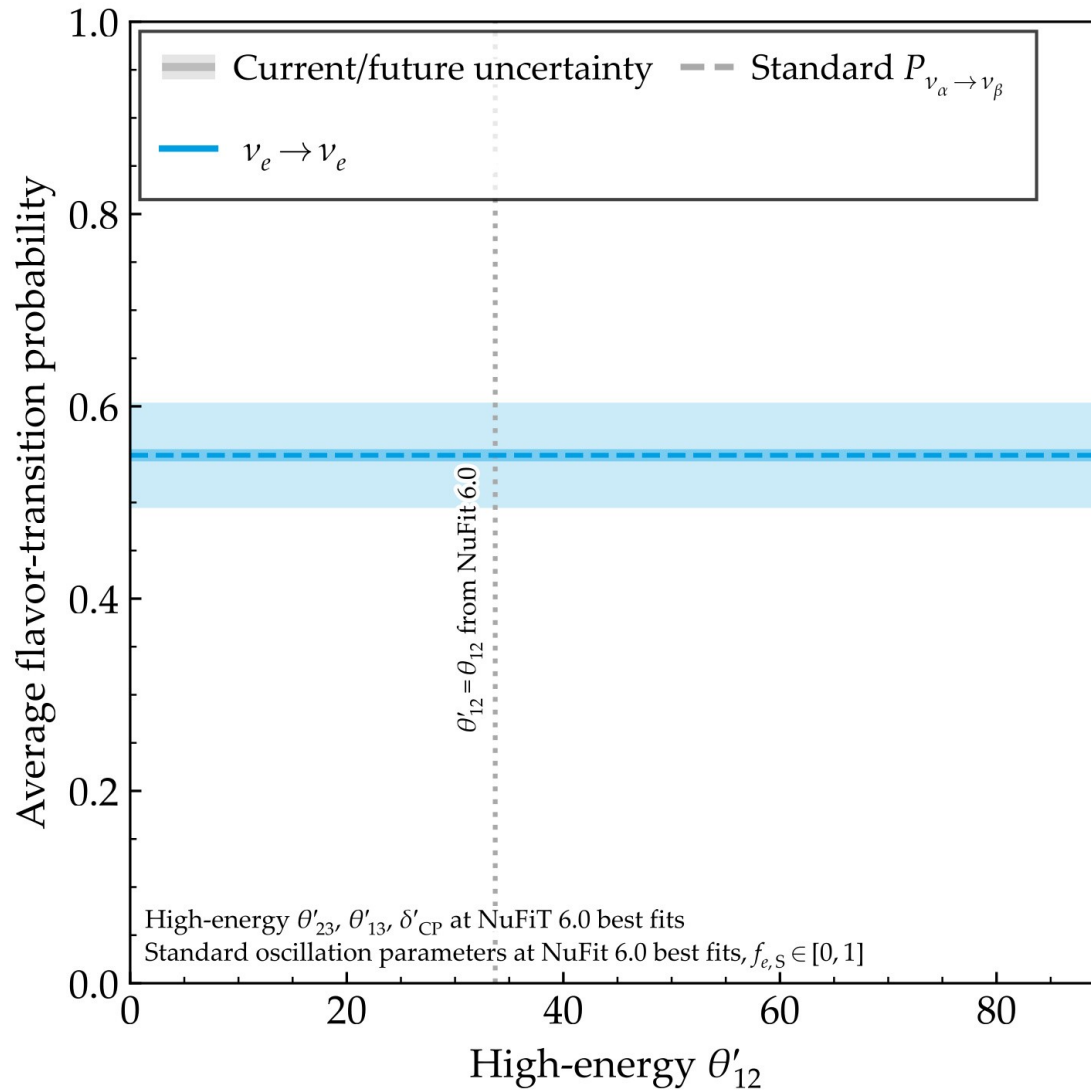


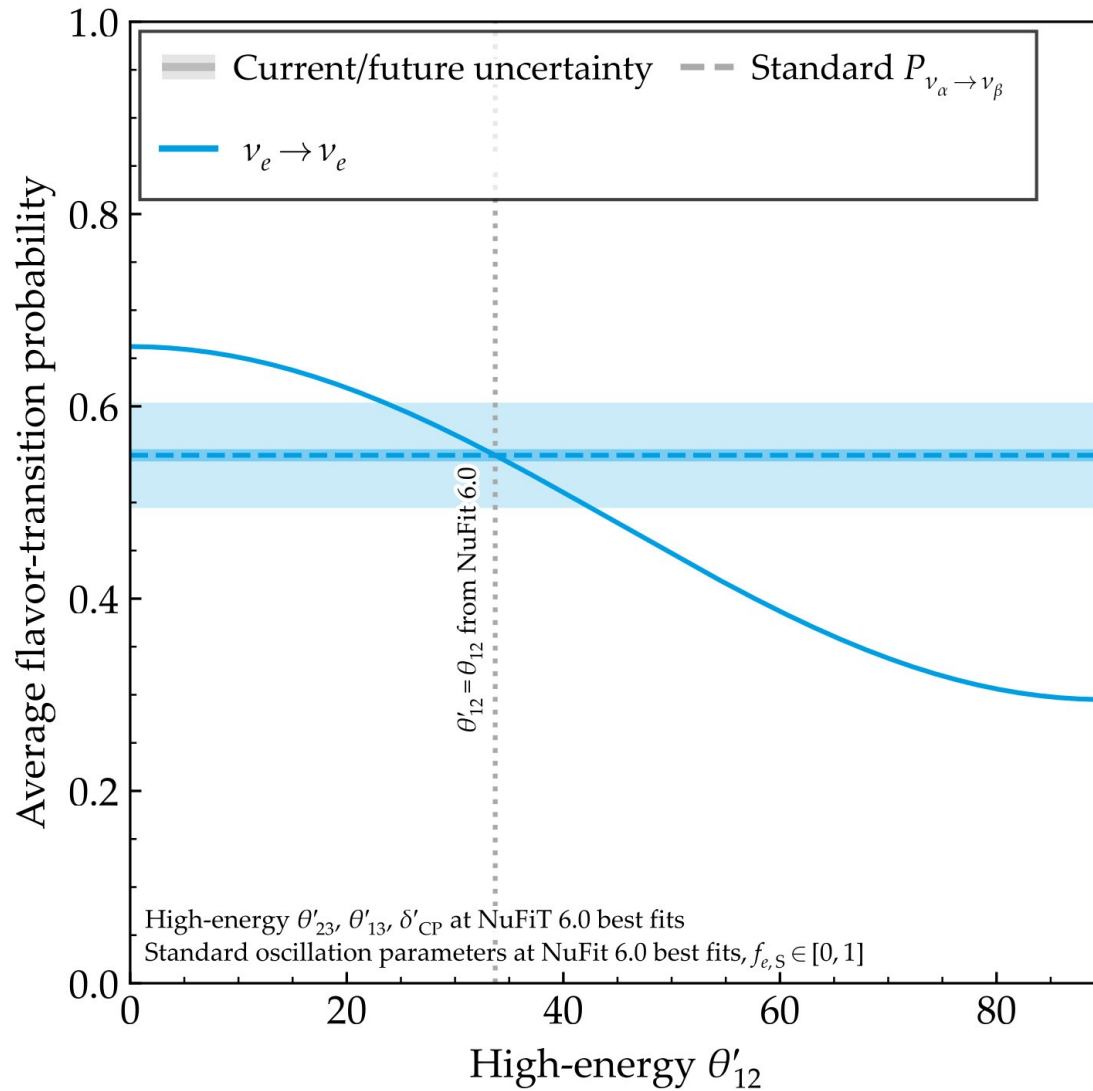
(Central values fixed at current NuFIT best fits)

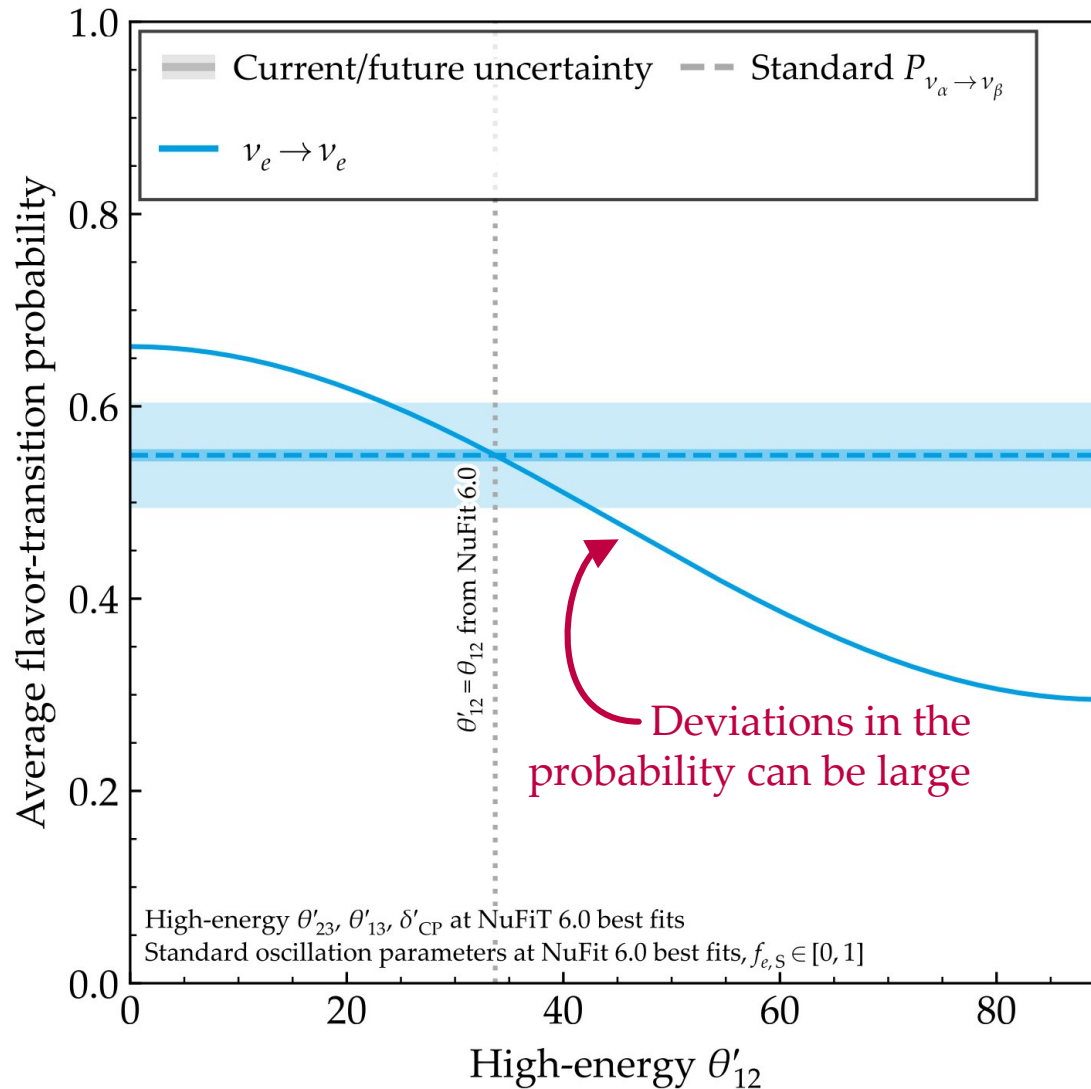
$$P_{\alpha\beta} = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2$$

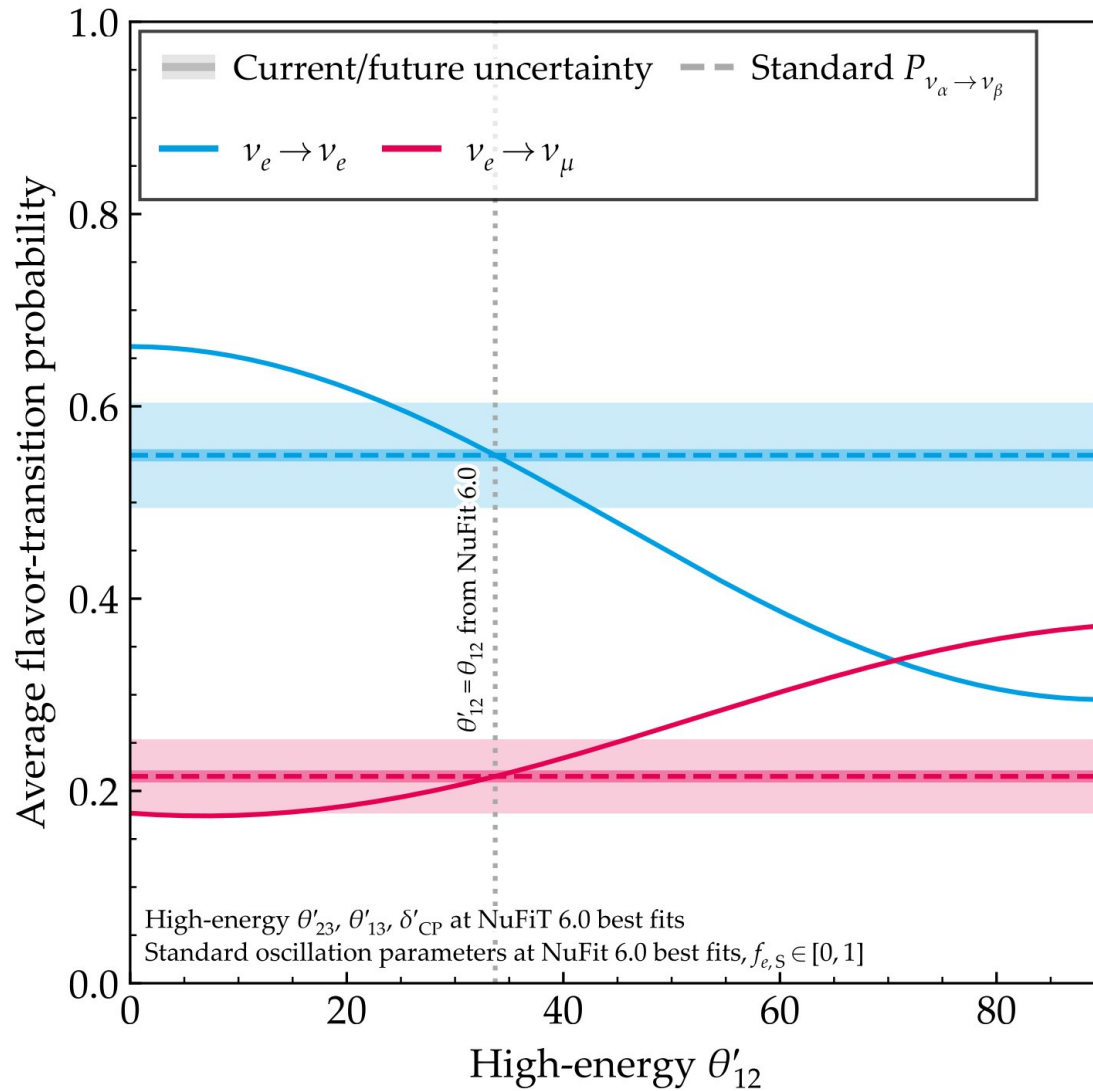
PMNS

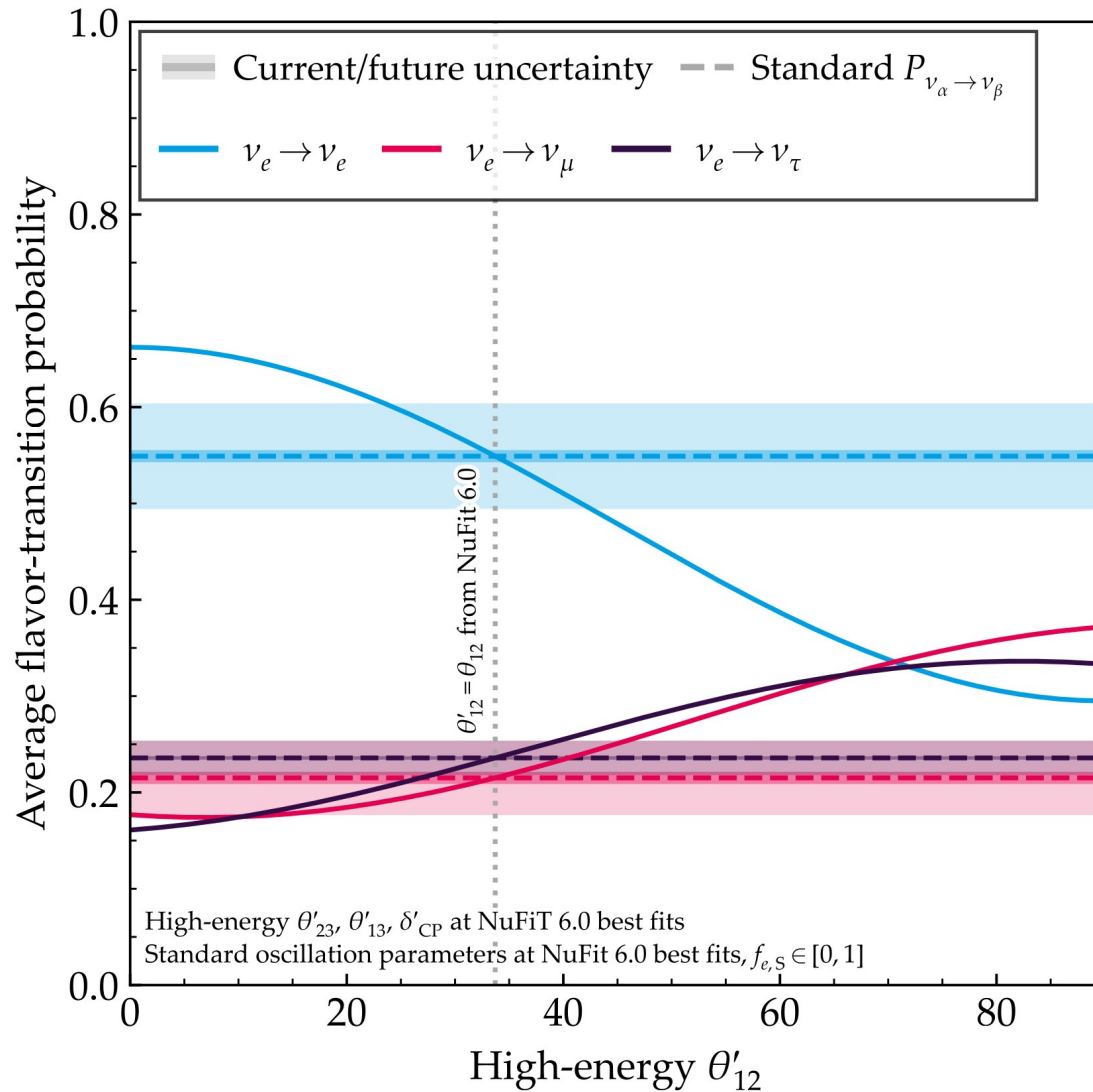
Future uncertainties on the standard probabilities

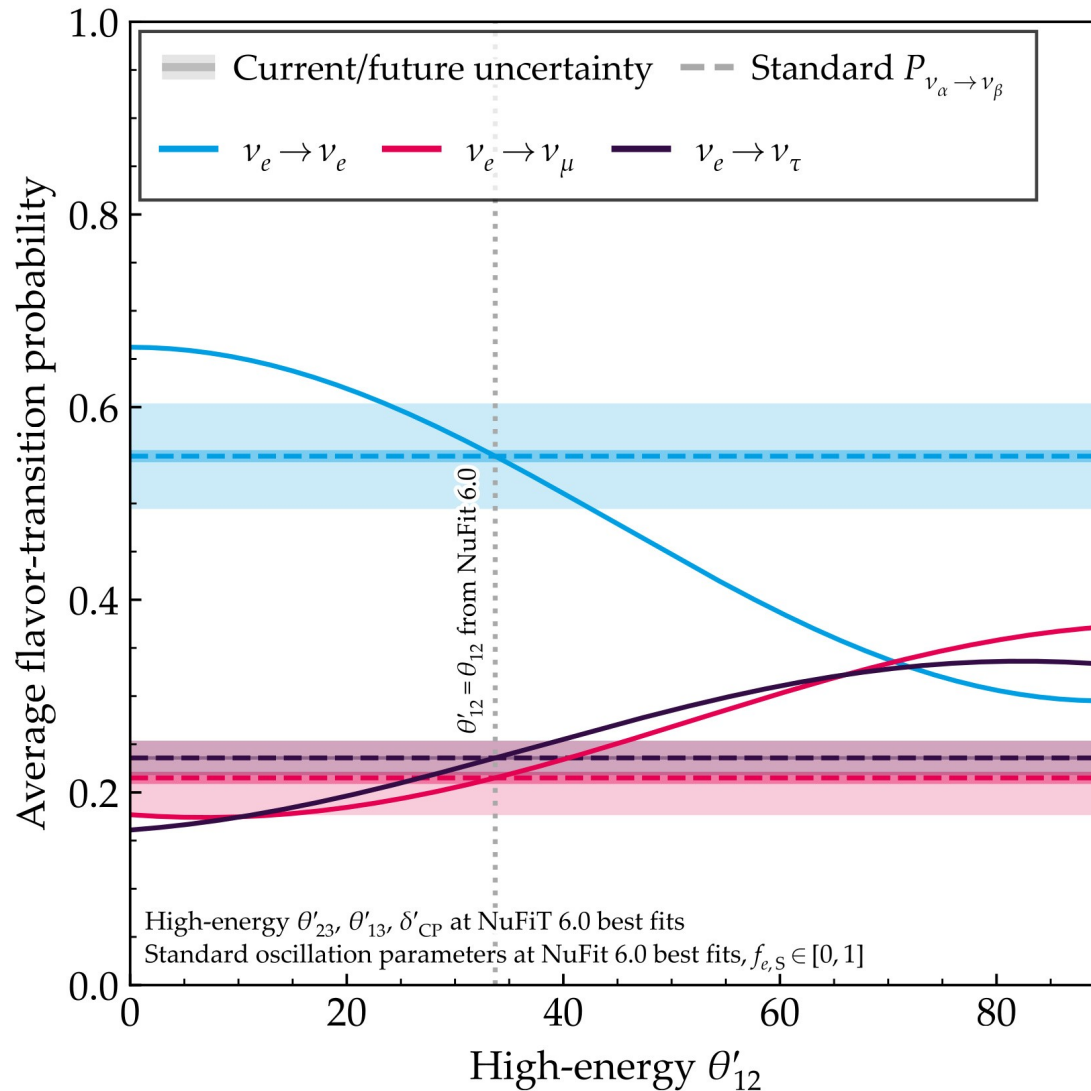












Results for the other high-energy mixing parameters are similar (not shown here)

Flavor composition at the Earth:

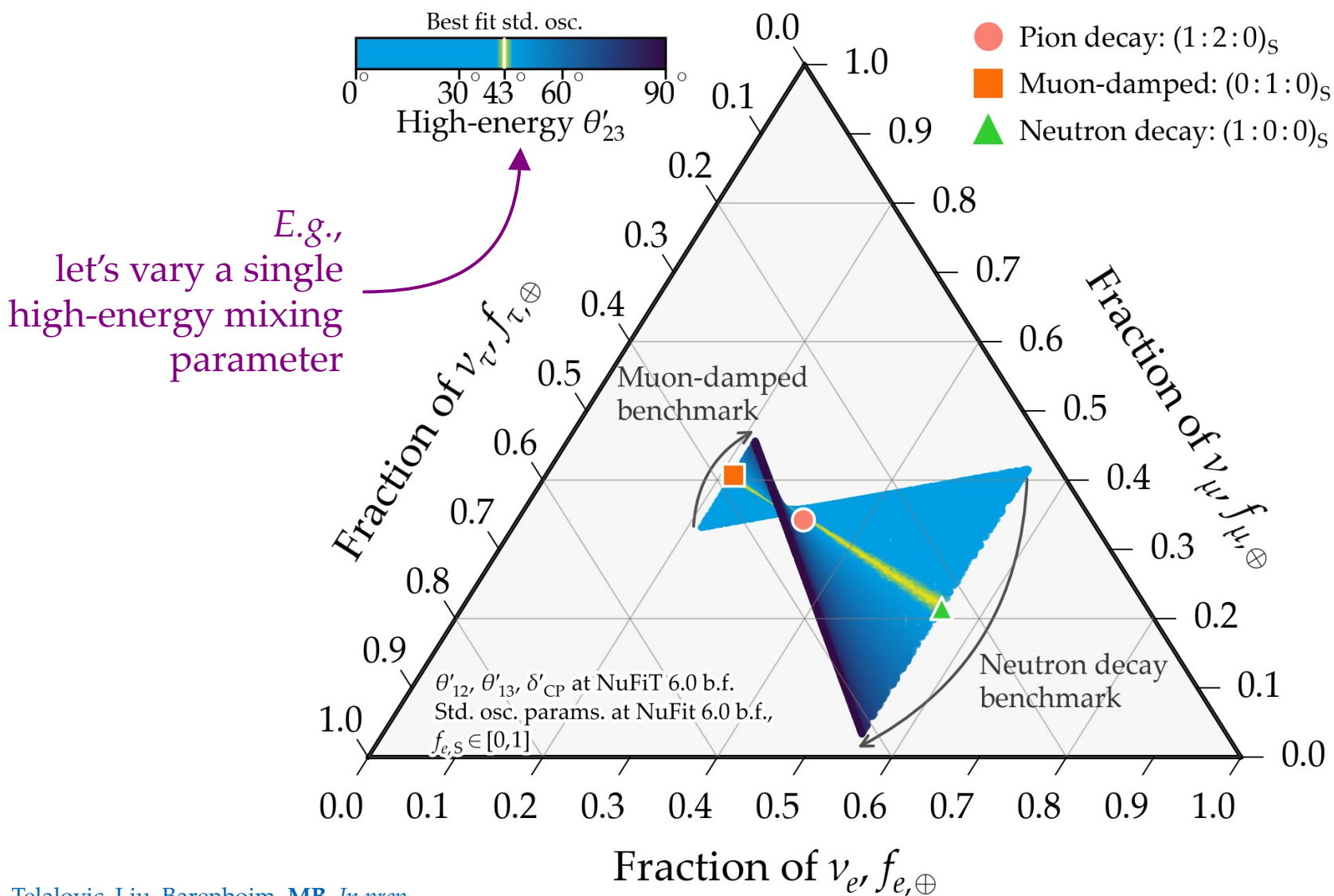
$$\begin{pmatrix} f_{e,\oplus} \\ f_{\mu,\oplus} \\ f_{\tau,\oplus} \end{pmatrix} = \underbrace{\begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix}}_{\text{Probabilities}} \begin{pmatrix} f_{e,S} \\ 1 - f_{e,S} \\ 0 \end{pmatrix}$$

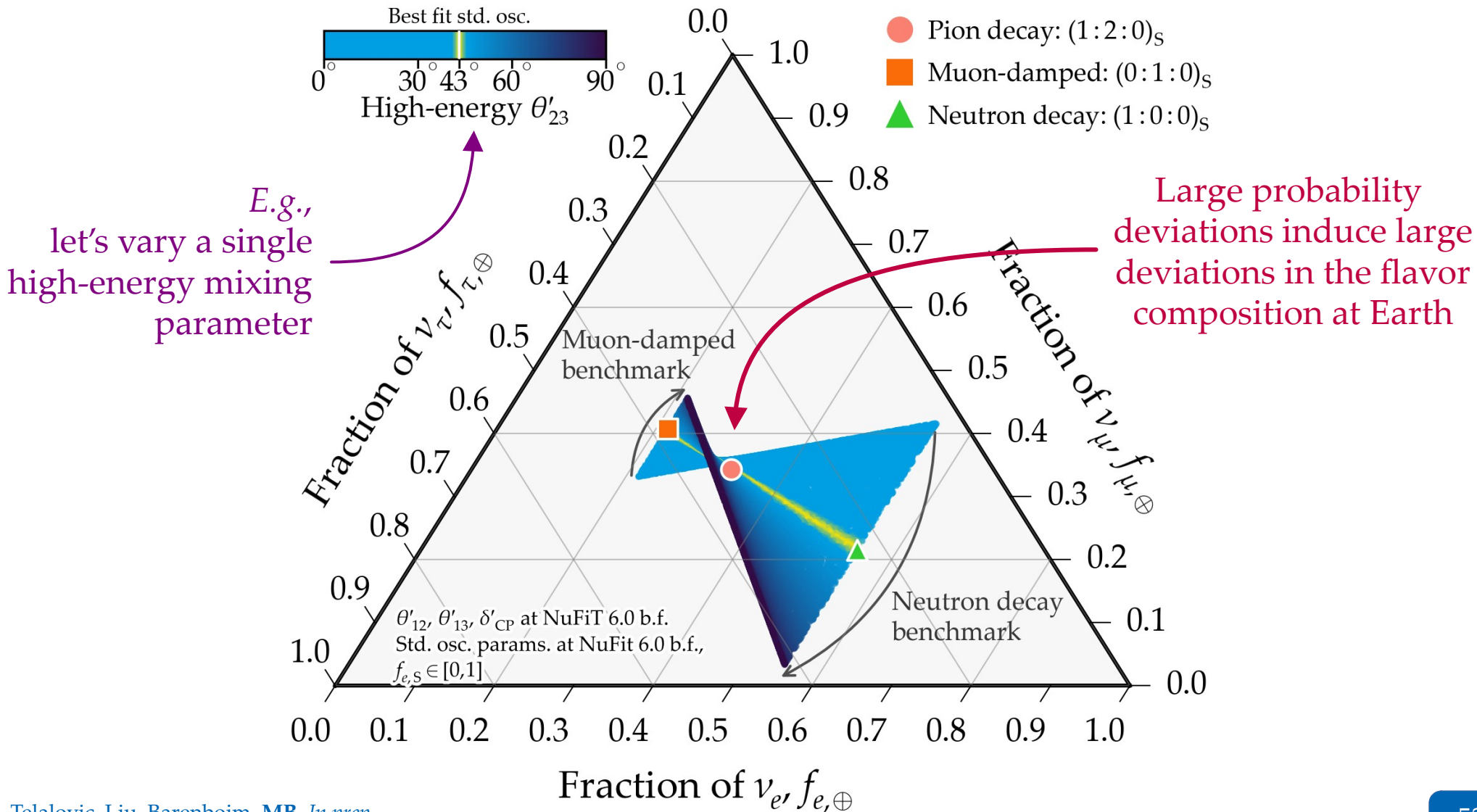
Probabilities now depend on **low-energy** & **high-energy** mixing parameters

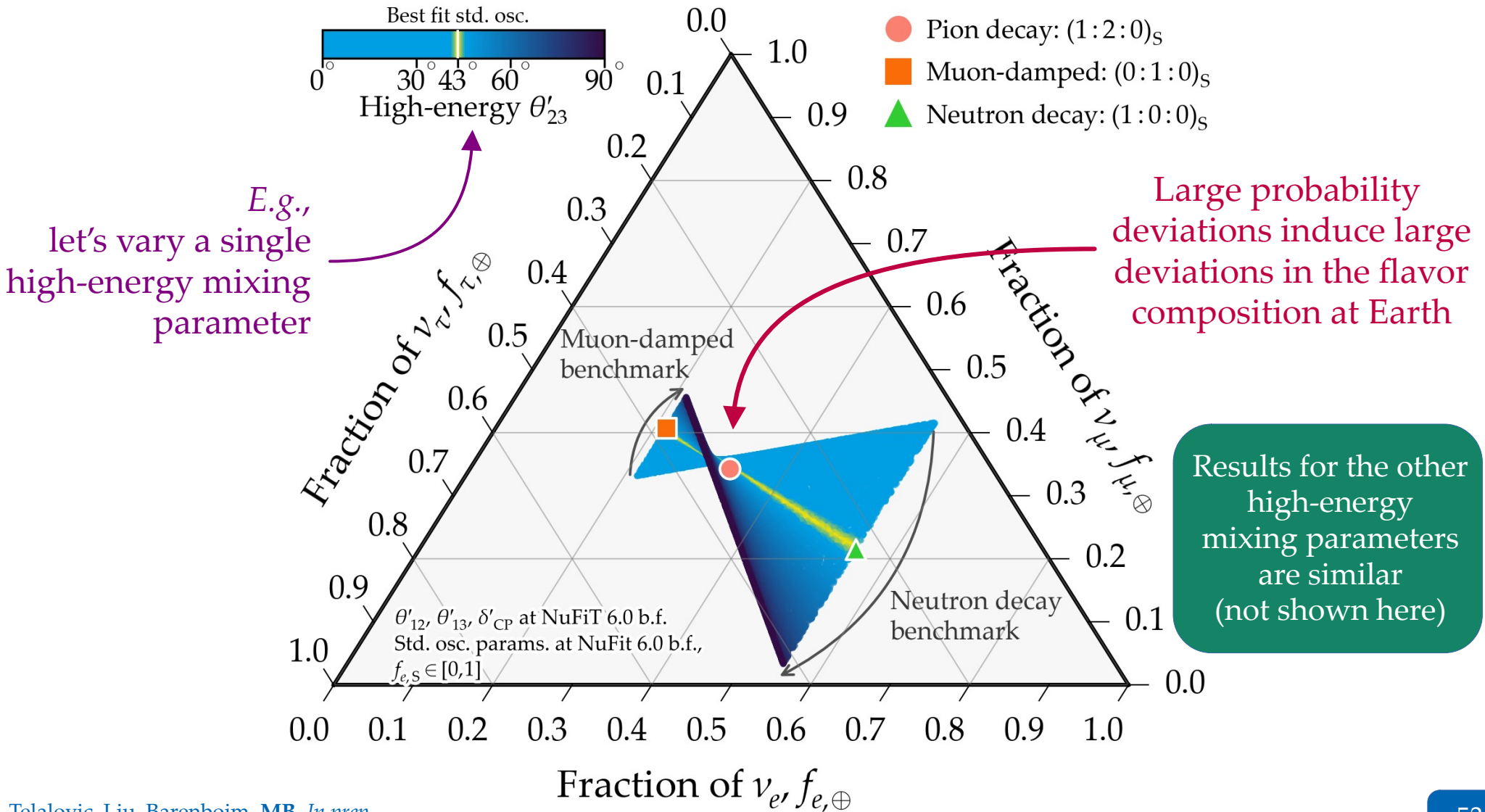
$$P_{\alpha\beta} = \sum_{i=1}^3 |(U_{\text{PMNS}})_{\alpha i}|^2 |U'_{\beta i}(\theta'_{12}, \theta'_{23}, \theta'_{13}, \delta'_{\text{CP}})|^2$$

Varied within current
or future uncertainties

Varied with uninformed priors
(flat or Haar measure on U')



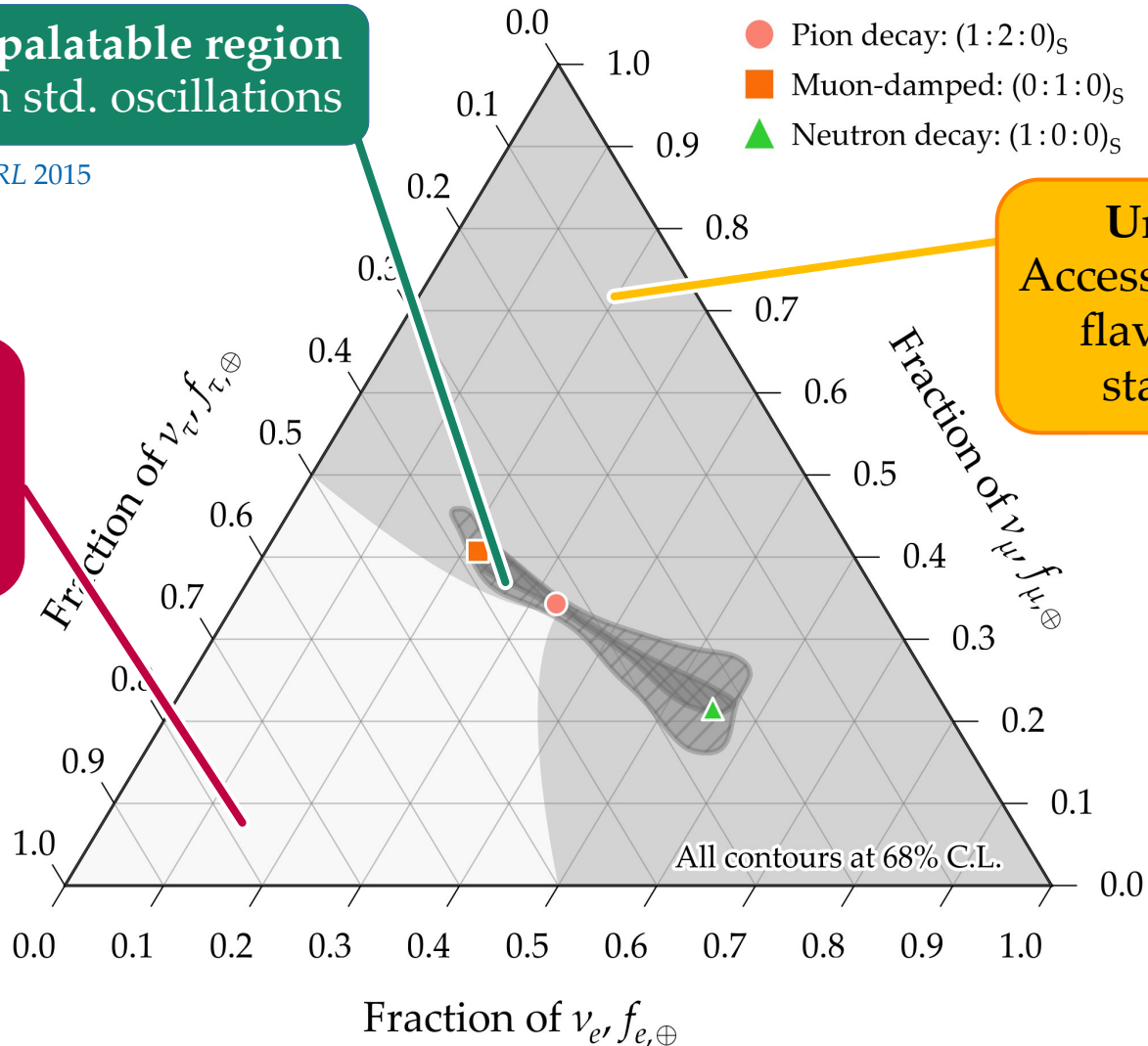




Theoretically palatable region
 Accessible with std. oscillations

MB, Beacom, Winter, *PRL* 2015

Non-unitary region
 Accessible only
 with non-unitary
 flavor transitions



Theoretically palatable region
 Accessible with std. oscillations

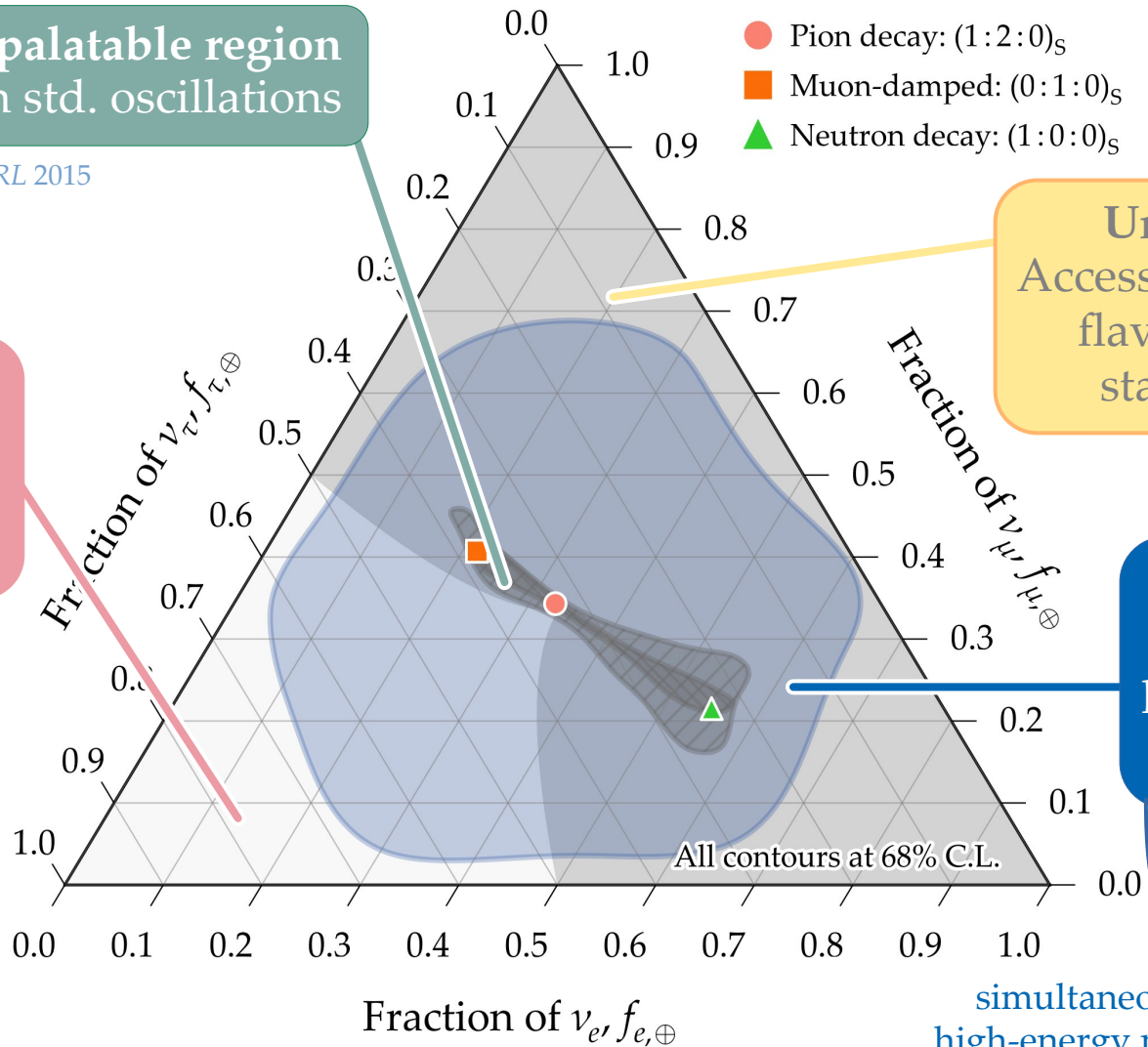
MB, Beacom, Winter, *PRL* 2015

- Pion decay: $(1:2:0)_S$
- Muon-damped: $(0:1:0)_S$
- ▲ Neutron decay: $(1:0:0)_S$

Unitary region
 Accessible with unitary
 flavor transitions,
 standard or not

Ahlers, MB, Mu, *PRD* 2018

Non-unitary region
 Accessible only
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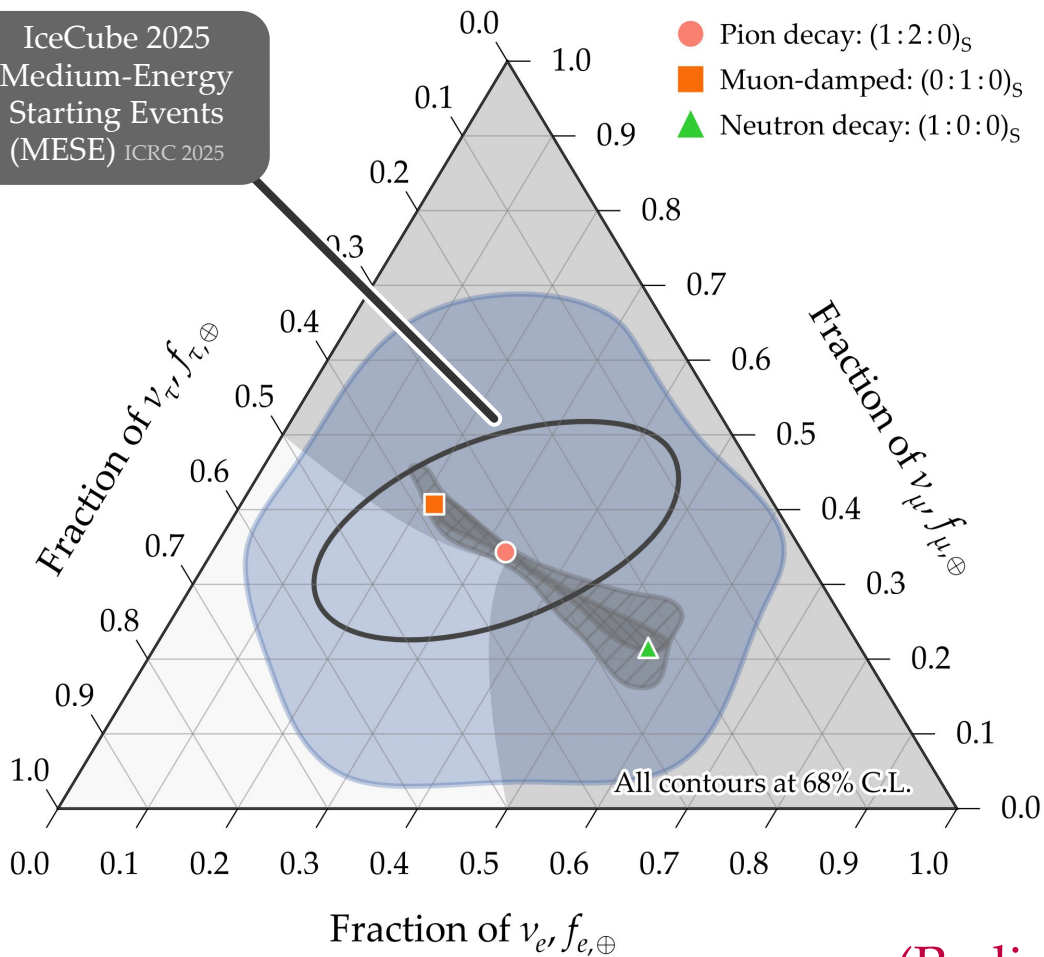
RGE region
 Accessible when
 low- & high-energy
 mixing differ

Telalovic, Liu, Barenboim, MB,
In prep.

Region generated by
 simultaneously varying all low- and
 high-energy mixing parameters, and $f_{e,S}$

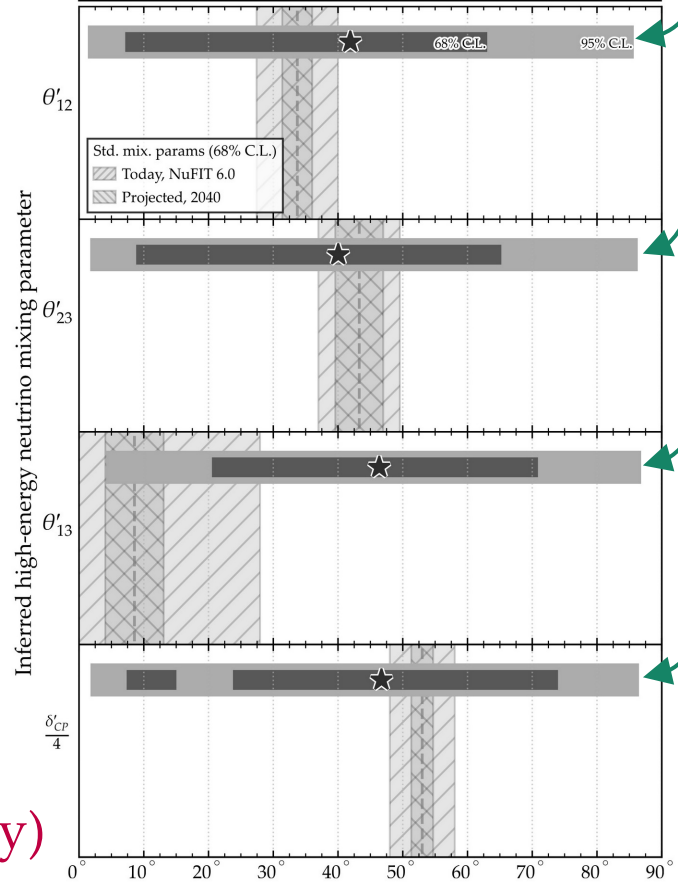
Measurement power *today* —

IceCube 2025
Medium-Energy
Starting Events
(MESE) ICRC 2025



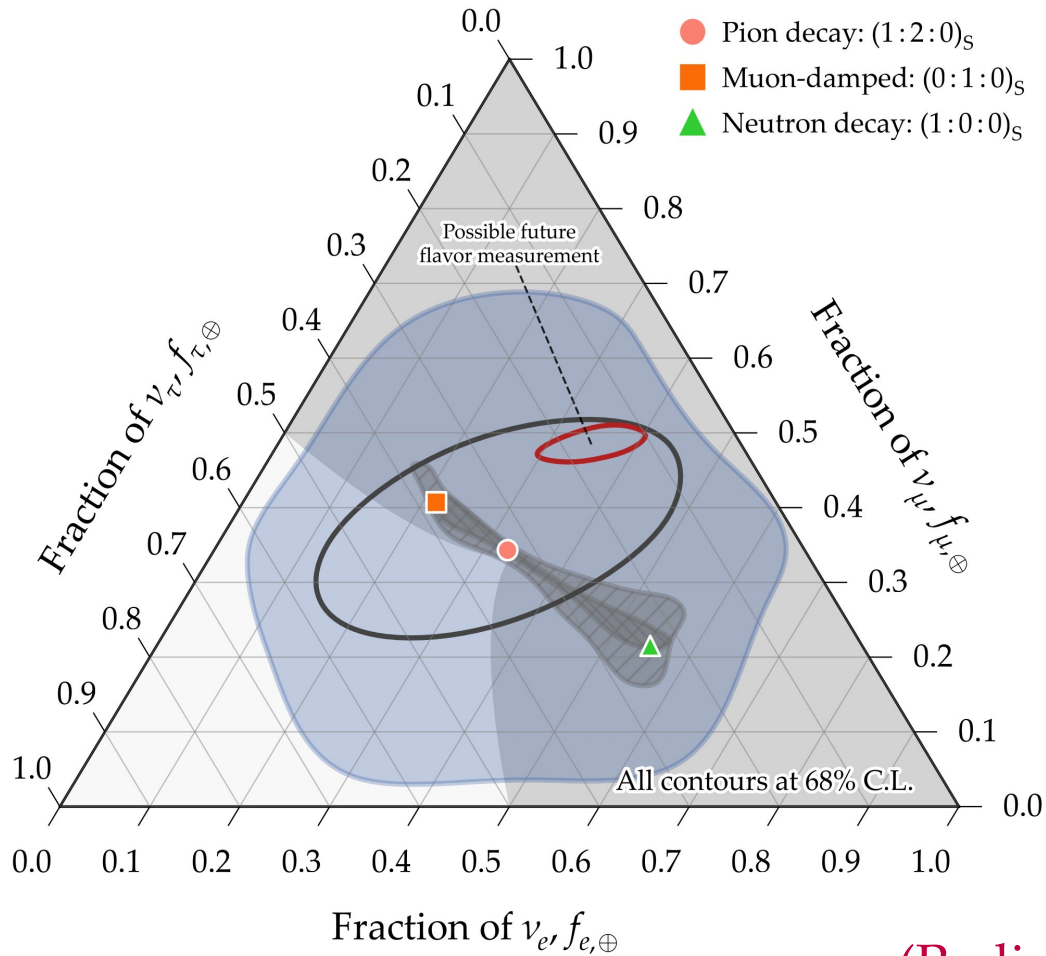
Present constraints:
 ★ IceCube 2025 Medium-Energy Starting Events (MESE)

There is no power *today* to measure the high-energy neutrino mixing parameters

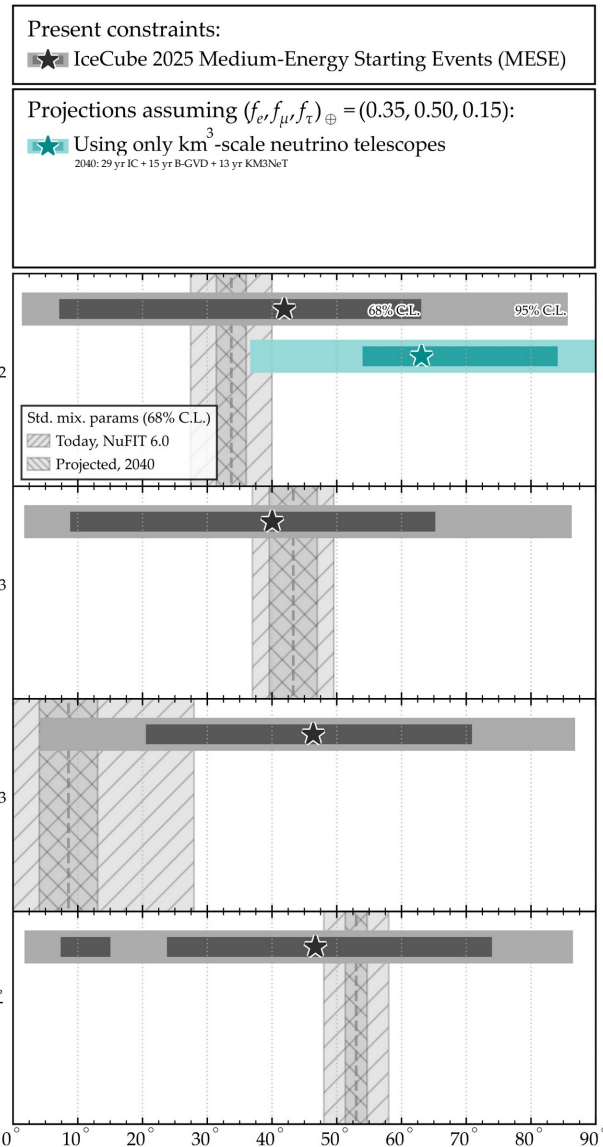


(Preliminary)

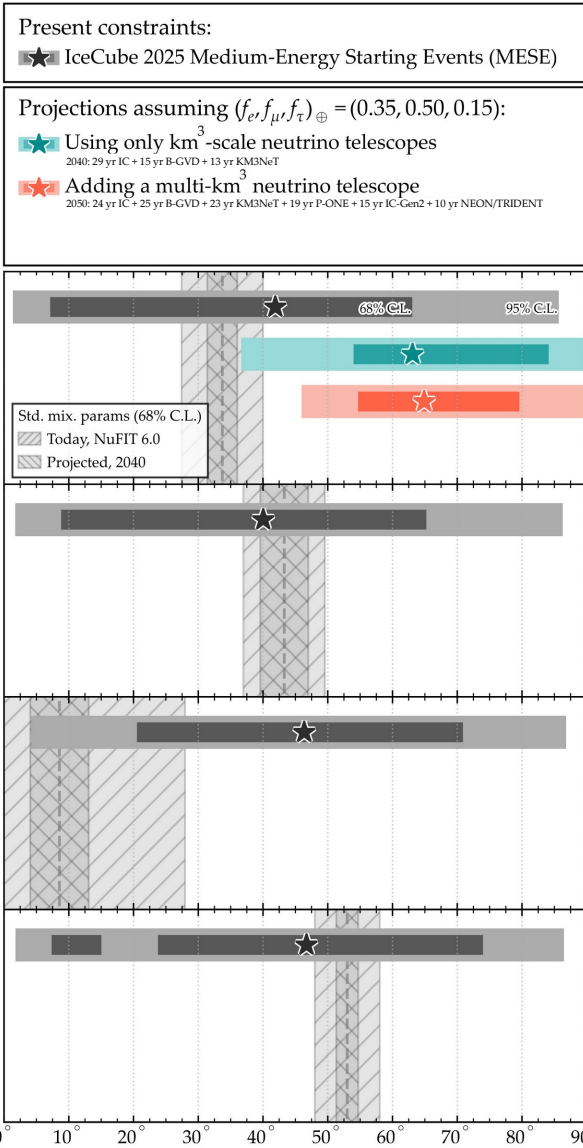
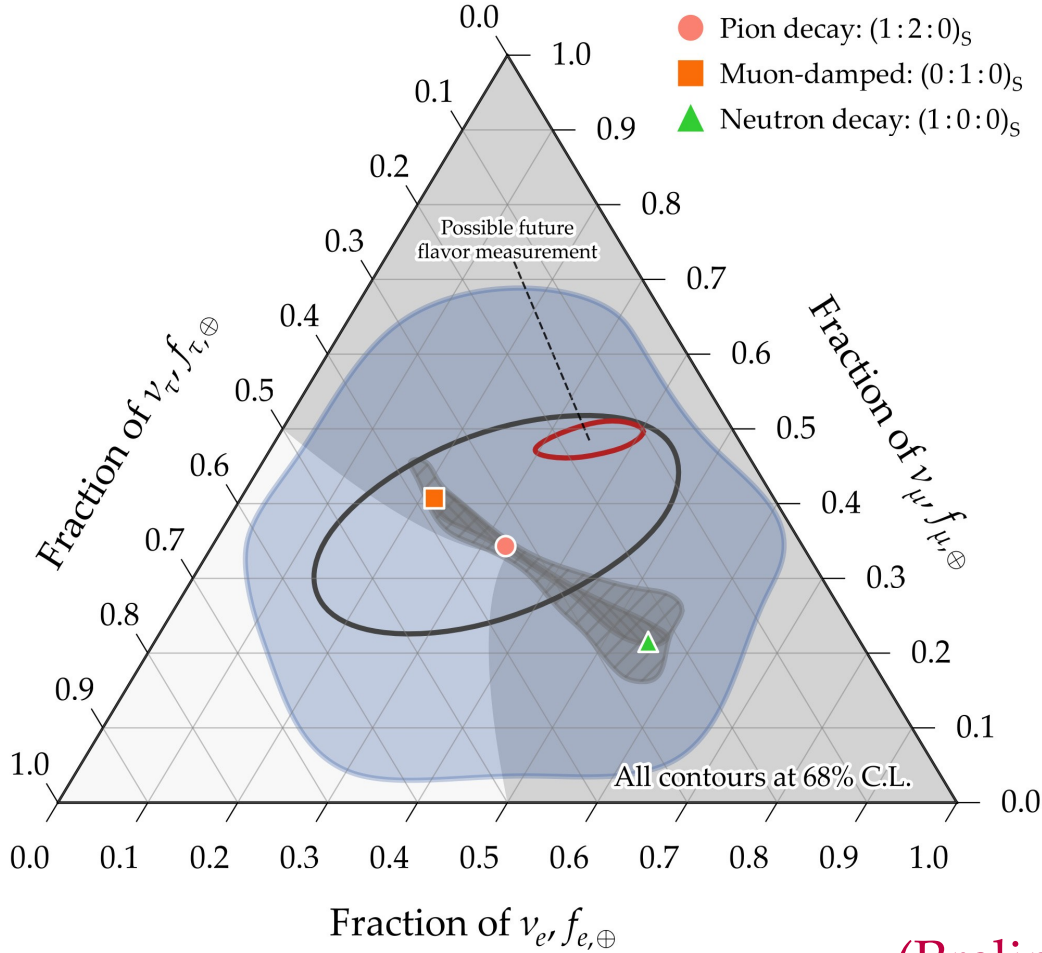
Measurement power *in the future* —



(Preliminary)

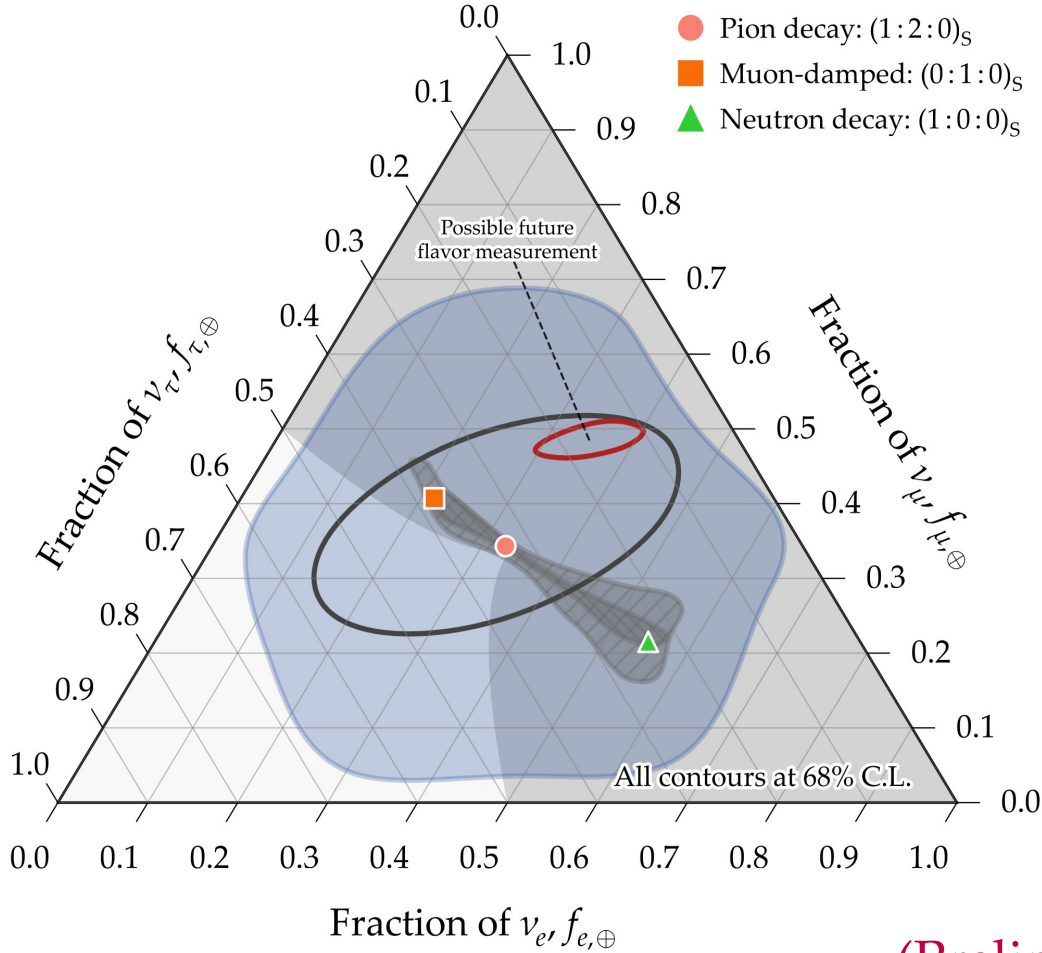


Measurement power *in the future* —

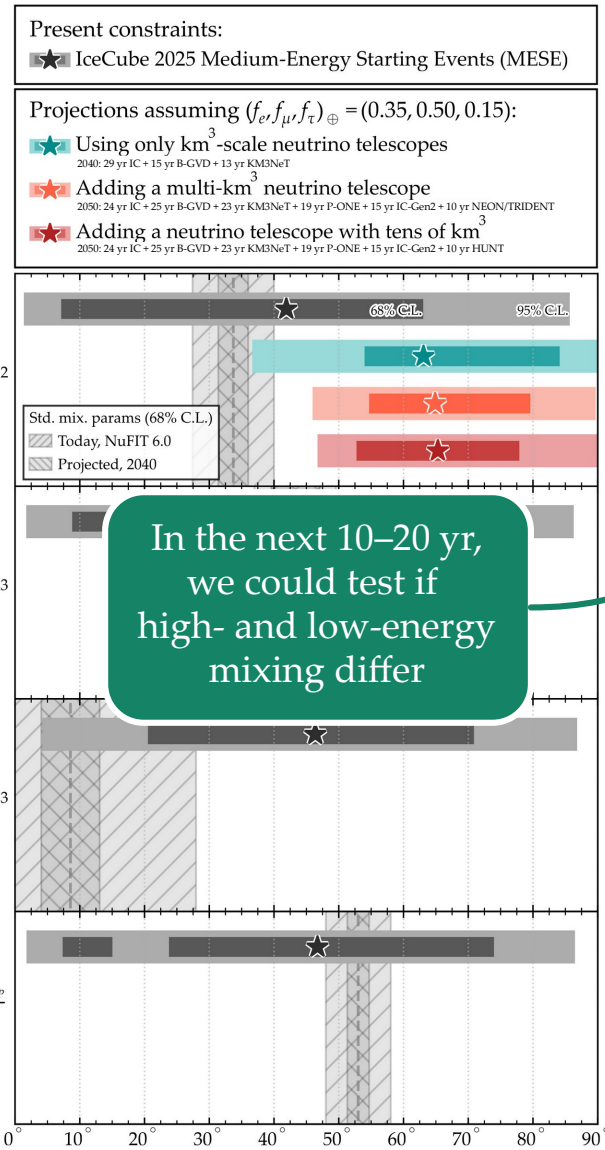


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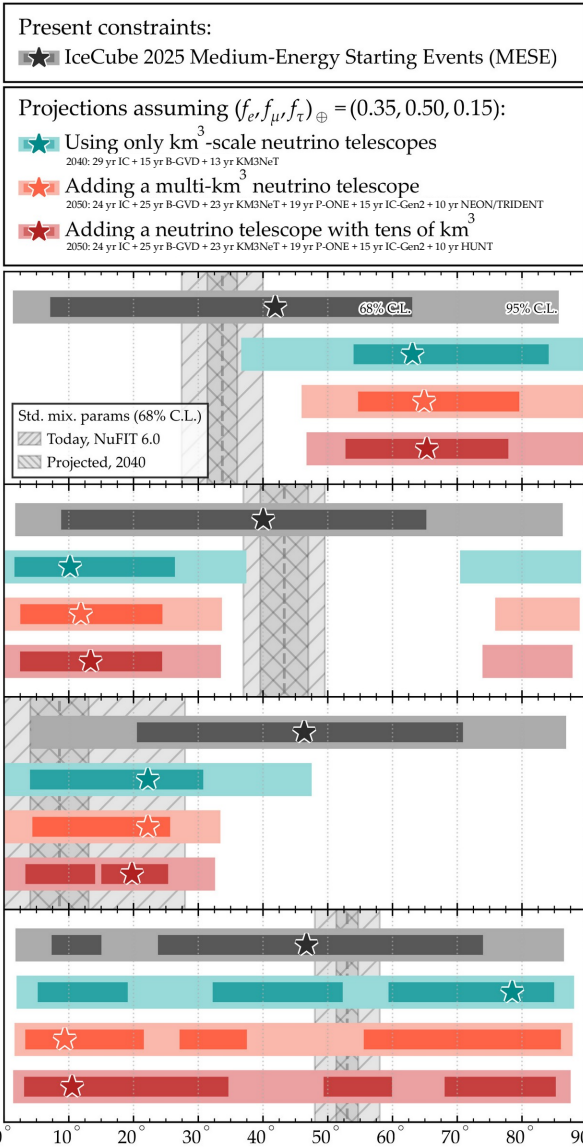
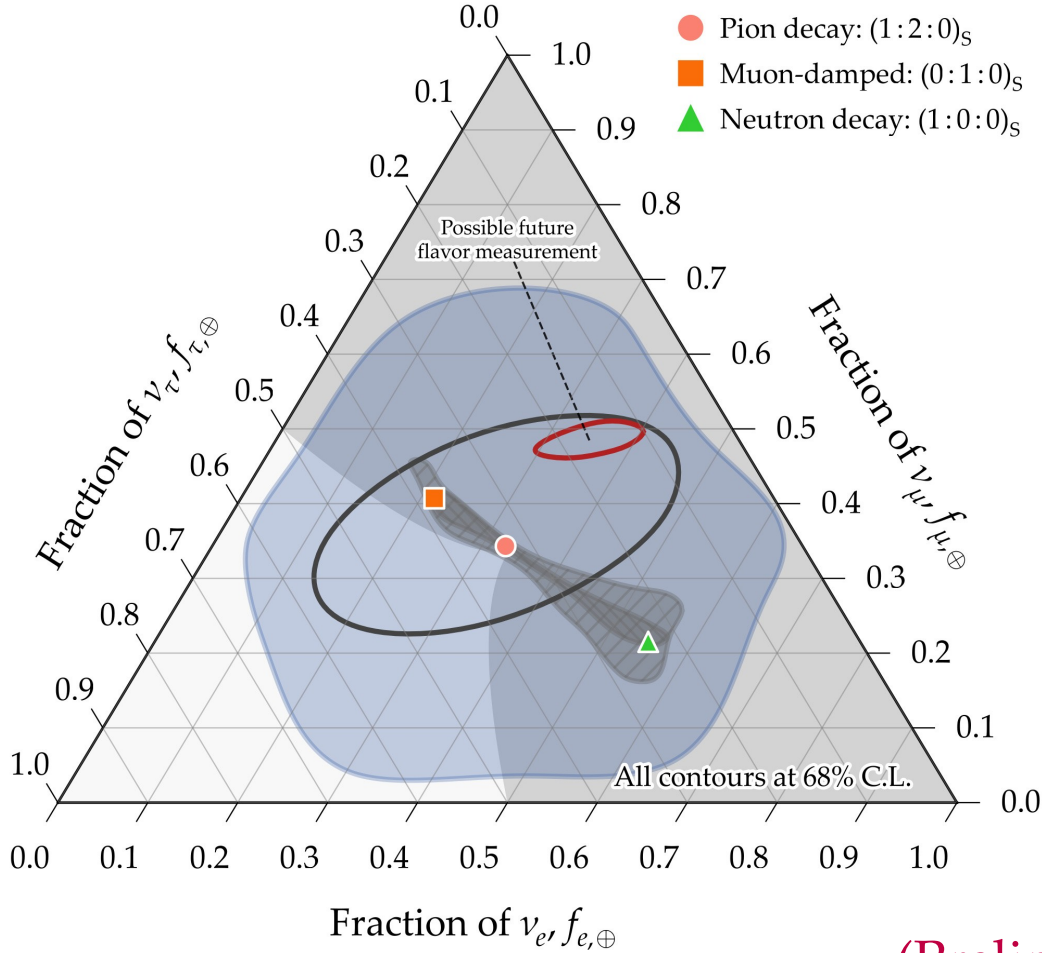
Measurement power *in the future* —



(Preliminary)

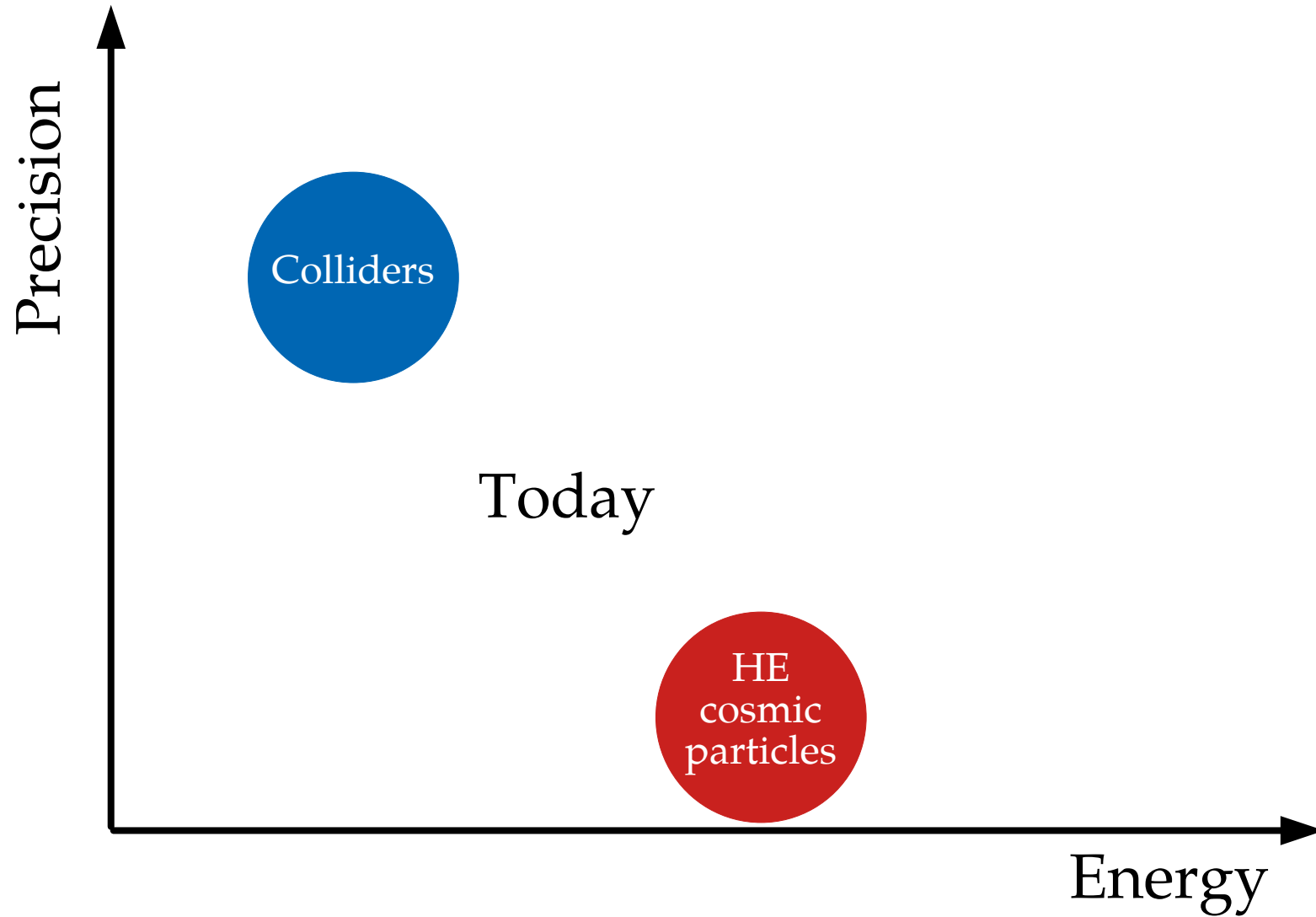


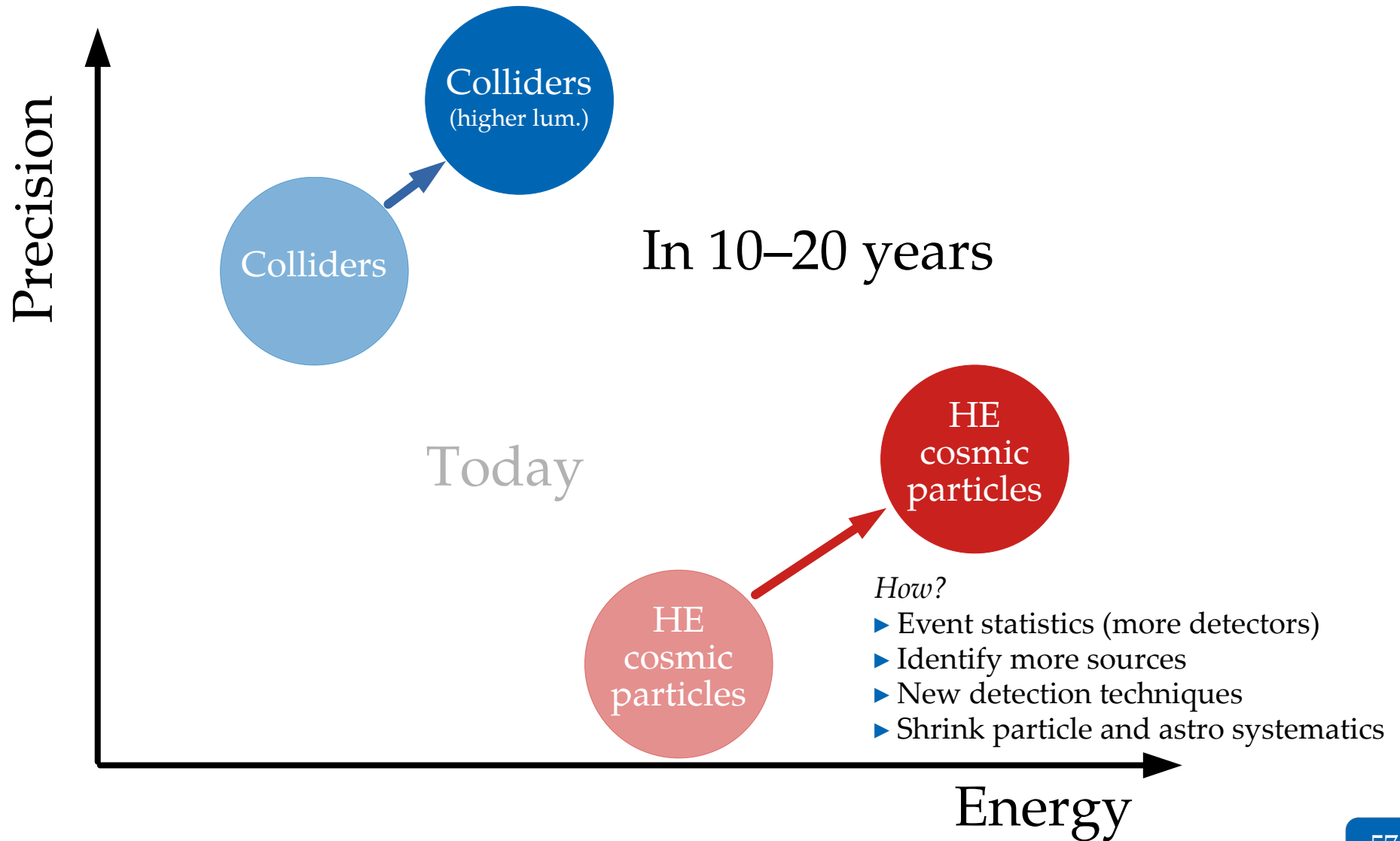
Measurement power *in the future* —



(Preliminary)

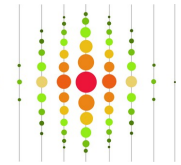

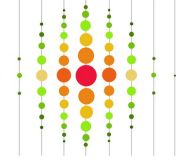

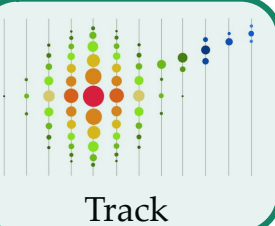
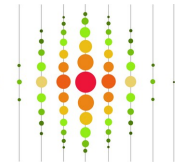
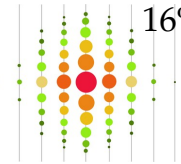
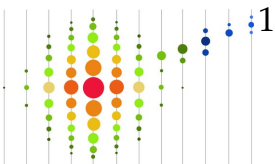
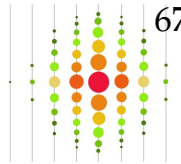
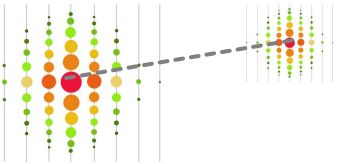
So...





Thank you!

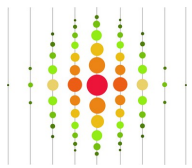
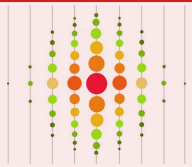
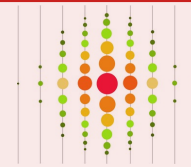
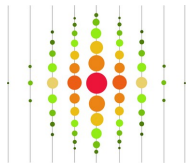

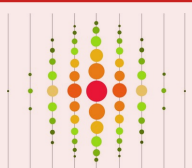
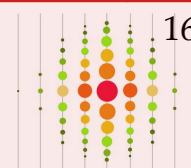
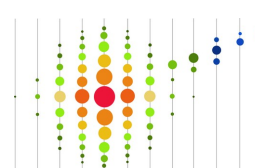
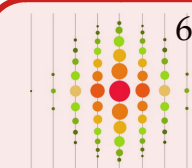
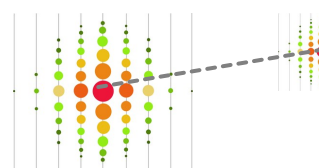
Backup slides

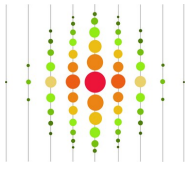
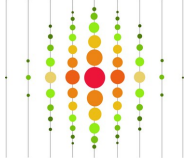
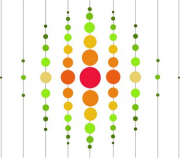
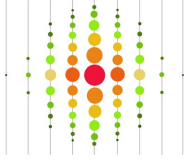
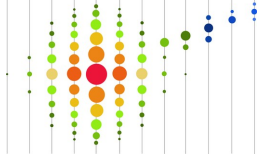
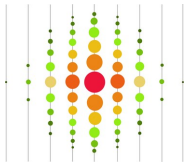
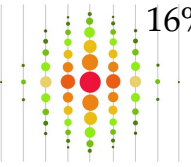
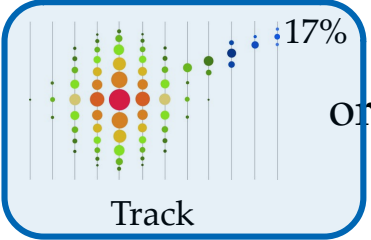
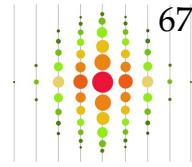
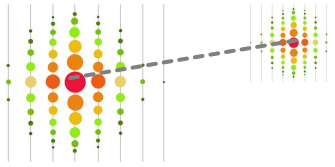
$\nu_x + \bar{\nu}_x$ NC	 <p>Hadronic X shower</p>			
$\nu_e + \bar{\nu}_e$ CC	 <p>Hadronic X shower</p>	+  <p>E.m. shower</p>	<div style="border: 2px solid green; padding: 10px; display: inline-block;"> ν_μ: easy to identify the outgoing track </div>	
$\nu_\mu + \bar{\nu}_\mu$ CC	 <p>Hadronic X shower</p>	+ <div style="border: 2px solid green; border-radius: 15px; padding: 5px; display: inline-block;">  <p>Track</p> </div>		
$\nu_\tau + \bar{\nu}_\tau$ CC	 <p>Hadronic X shower</p>	+  <p>E.m. shower</p>	or  <p>Track</p>	or  <p>Hadronic shower</p> or  <p>Double pulse/bang</p>

16%

17%

67%

$\nu_x + \bar{\nu}_x$ NC	 <p>Hadronic X shower</p>
$\nu_e + \bar{\nu}_e$ CC	<div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 2px solid red; padding: 5px; display: flex; align-items: center; gap: 10px;">  +  </div> <div style="border: 2px solid red; padding: 5px; text-align: center;"> ν_e and ν_τ: difficult to distinguish, both make showers </div> </div> <p>Hadronic X shower E.m. shower</p>
$\nu_\mu + \bar{\nu}_\mu$ CC	<div style="display: flex; align-items: center; justify-content: space-around;">  +  </div> <p>Hadronic X shower Track</p>
$\nu_\tau + \bar{\nu}_\tau$ CC	<div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 2px solid red; padding: 5px; display: flex; align-items: center; gap: 10px;">  +  16% </div> or  17% or <div style="border: 2px solid red; padding: 5px; display: flex; align-items: center; gap: 10px;">  67% </div> or  </div> <p>Hadronic X shower E.m. shower Track Hadronic shower Double pulse/bang</p>

$\nu_x + \bar{\nu}_x$ NC	 <p>Hadronic X shower</p>				
$\nu_e + \bar{\nu}_e$ CC	 <p>Hadronic X shower</p>	+  <p>E.m. shower</p>	<div style="border: 2px solid blue; padding: 5px; width: fit-content; margin: auto;"> The occasional track (weakly) breaks the ν_e / ν_τ degeneracy </div>		
$\nu_\mu + \bar{\nu}_\mu$ CC	 <p>Hadronic X shower</p>	+  <p>Track</p>			
$\nu_\tau + \bar{\nu}_\tau$ CC	 <p>Hadronic X shower</p>	+  <p>E.m. shower</p>	or  <p>Track</p>	or  <p>Hadronic shower</p>	or  <p>Double pulse/bang</p>

The occasional track
(weakly) breaks the
 ν_e / ν_τ degeneracy

16%
17%

67%

Double pulse/bang

Fundamental physics with high-energy cosmic neutrinos

Numerous new ν physics effects grow as $\sim \kappa_n \cdot E^n \cdot L$

So we can probe $\kappa_n \sim 4 \cdot 10^{-47} (E/\text{PeV})^{-n} (L/\text{Gpc})^{-1} \text{PeV}^{1-n}$

Improvement over limits using atmospheric ν : $\kappa_0 < 10^{-29} \text{PeV}$, $\kappa_1 < 10^{-33}$

Fundamental physics with high-energy cosmic neutrinos

Numerous new ν physics effects grow as $\sim \kappa_n \cdot E^n \cdot L$ } *E.g.,*
 $n = -1$: neutrino decay
 $n = 0$: CPT-odd Lorentz violation
 $n = +1$: CPT-even Lorentz violation

So we can probe $\kappa_n \sim 4 \cdot 10^{-47} (E/\text{PeV})^{-n} (L/\text{Gpc})^{-1} \text{PeV}^{1-n}$

Improvement over limits using atmospheric ν : $\kappa_0 < 10^{-29} \text{PeV}$, $\kappa_1 < 10^{-33}$

An example:
Lorentz violation

Flavor-dependent
interactions
between neutrinos
and a fundamental
Lorentz-violating tensor




Standard oscillations:

$$H_{\text{std}} = \frac{1}{2E} U_{\text{PMNS}}^\dagger \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) U_{\text{PMNS}}$$

Lorentz-violating interactions (Standard Model Extension):

Kostelecky, Mewes, PRD 2004

$$H_{\text{new}} = \sum_{n \geq 0} \left(\frac{E}{\Lambda_n} \right)^n U_n^\dagger (\mathcal{O}_{n,1}, \mathcal{O}_{n,2}, \mathcal{O}_{n,3}) U_n$$


U_n has the same shape as U_{PMNS} ,
but its entries are a priori undetermined

Total Hamiltonian:

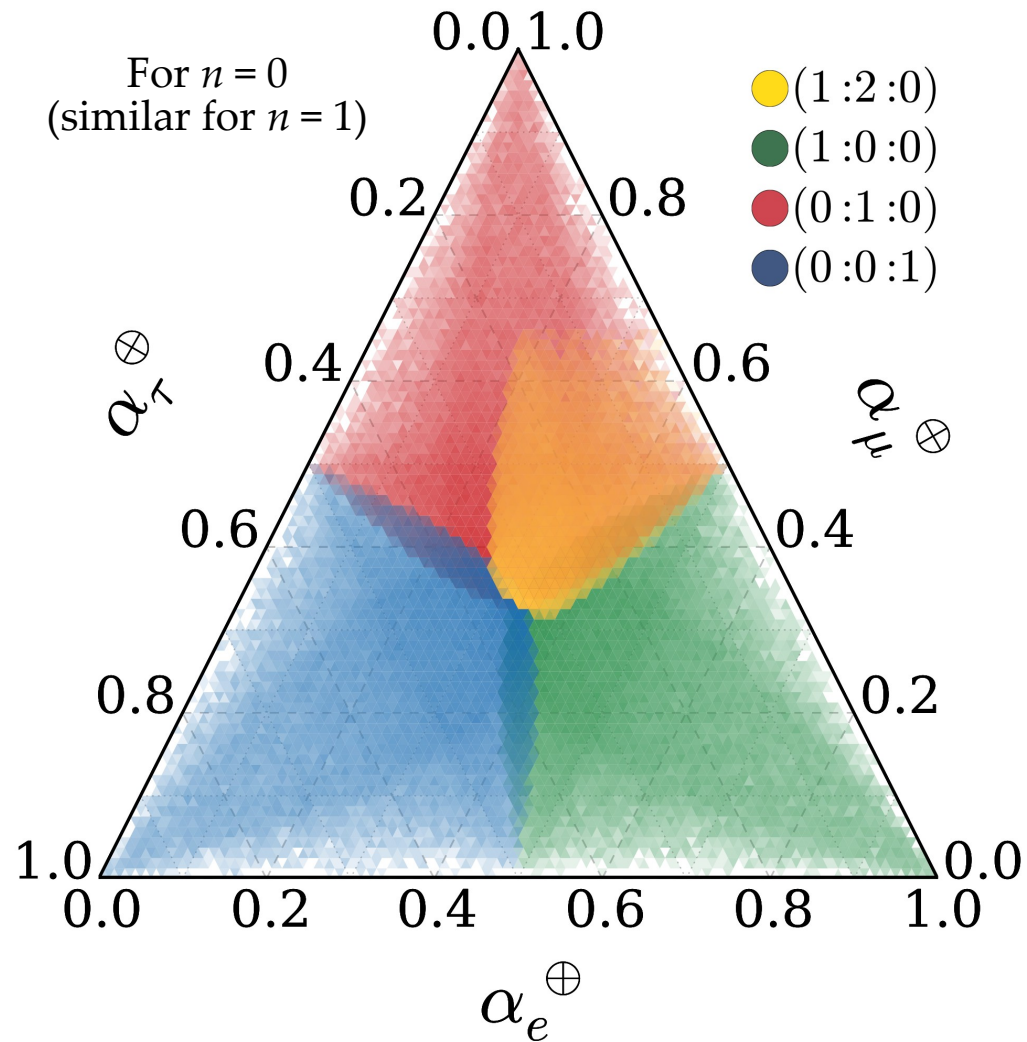
$$H_{\text{tot}} = H_{\text{std}} + H_{\text{new}}$$

The flavor-transition probabilities are calculated as before

$$P_{\alpha\beta} = \sum_{i=1}^3 |(\mathbf{U}_{\text{tot}})_{\alpha i}|^2 |(\mathbf{U}_{\text{tot}})_{\beta i}|^2 ,$$

Depends on standard & new parameters

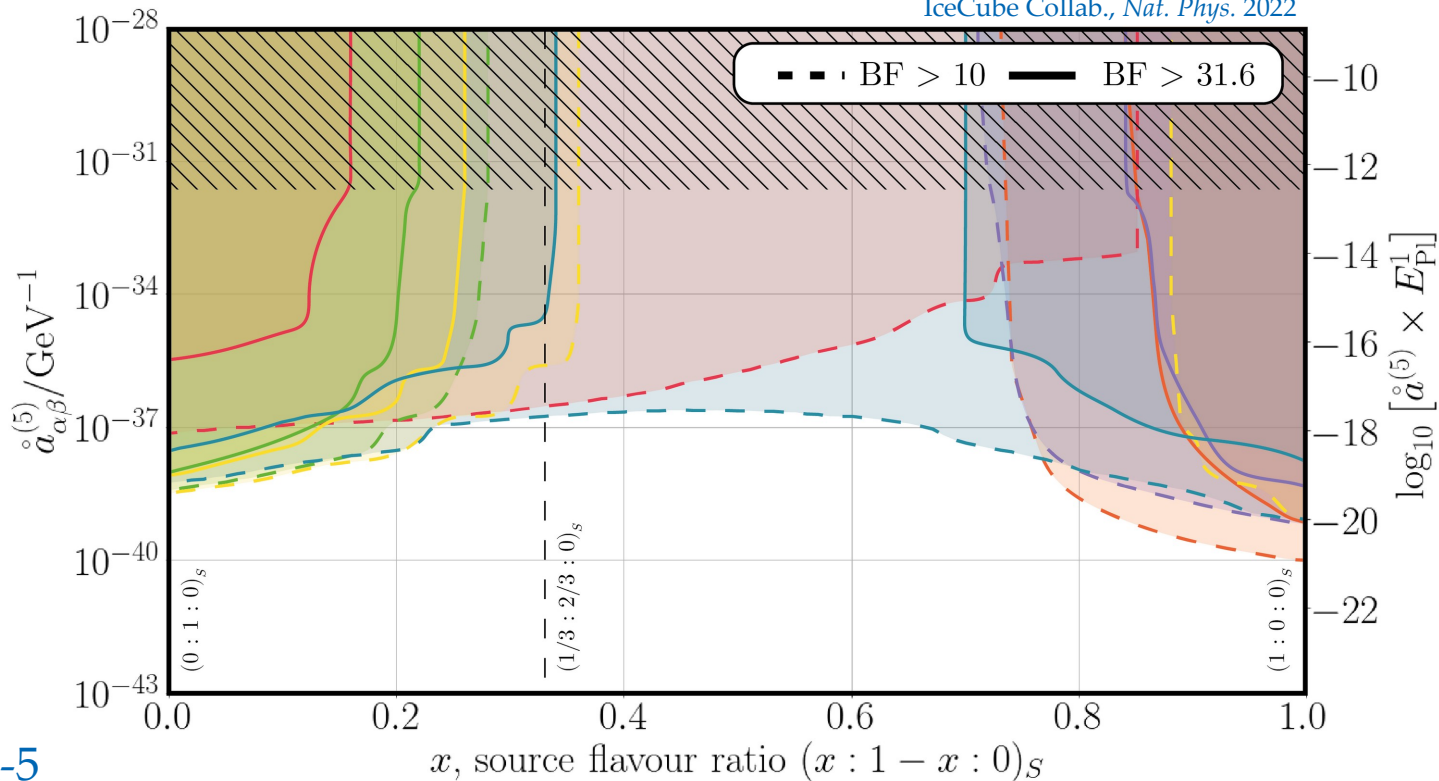
but now the lepton mixing matrix, \mathbf{U}_{tot} , is the one that diagonalizes H_{tot}



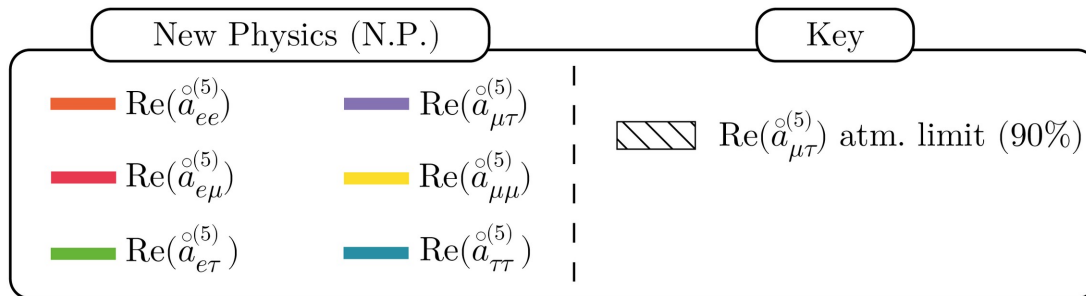
Argüelles, Katori, Salvadó, *PRL* 2015

See also Ahlers, **MB**, Mu, *PRD* 2018; Rasmussen *et al.*, *PRD* 2017; **MB**, Beacom, Winter *PRL* 2015;

MB, Gago, Peña-Garay *JCAP* 2010; Bazo, **MB**, Gago, Miranda *IJMPA* 2009; + many others

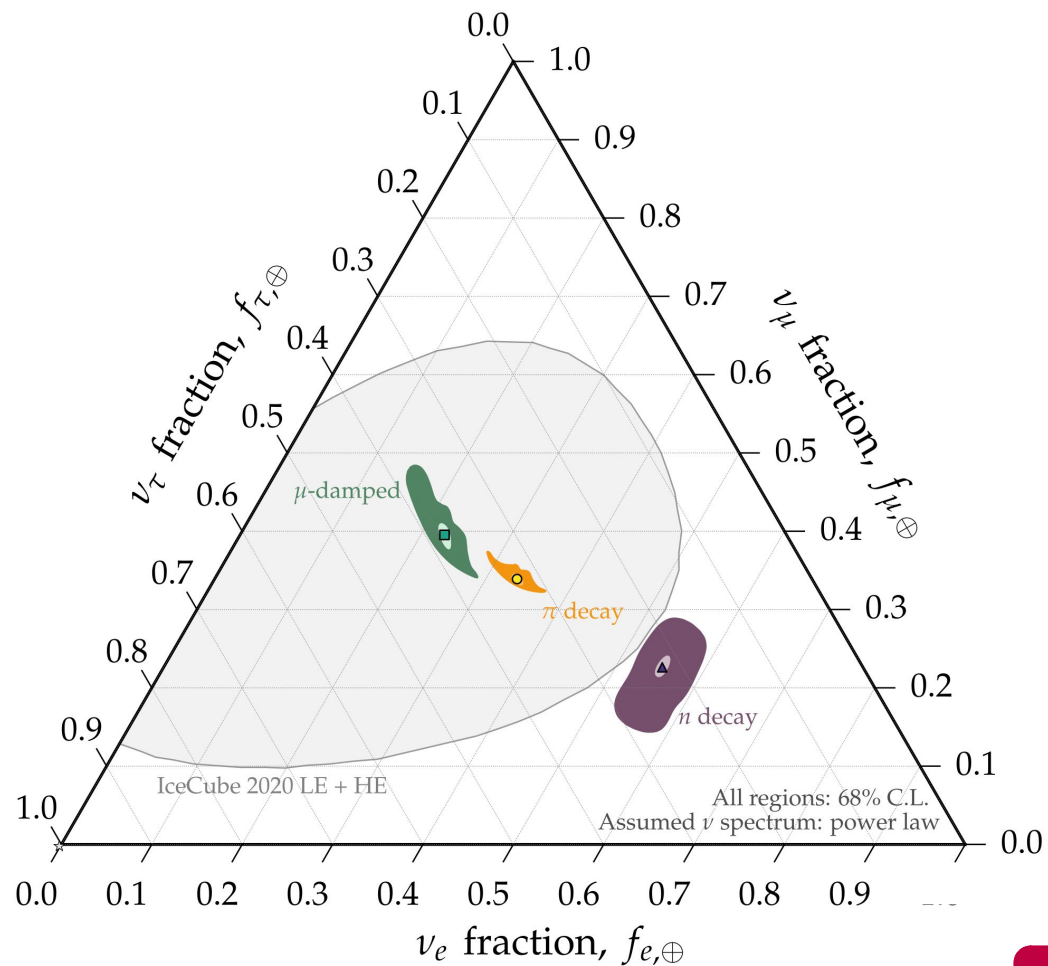


Dimension-5
CPT-odd
isotropic
Lorentz-invariance
-violating
coefficient



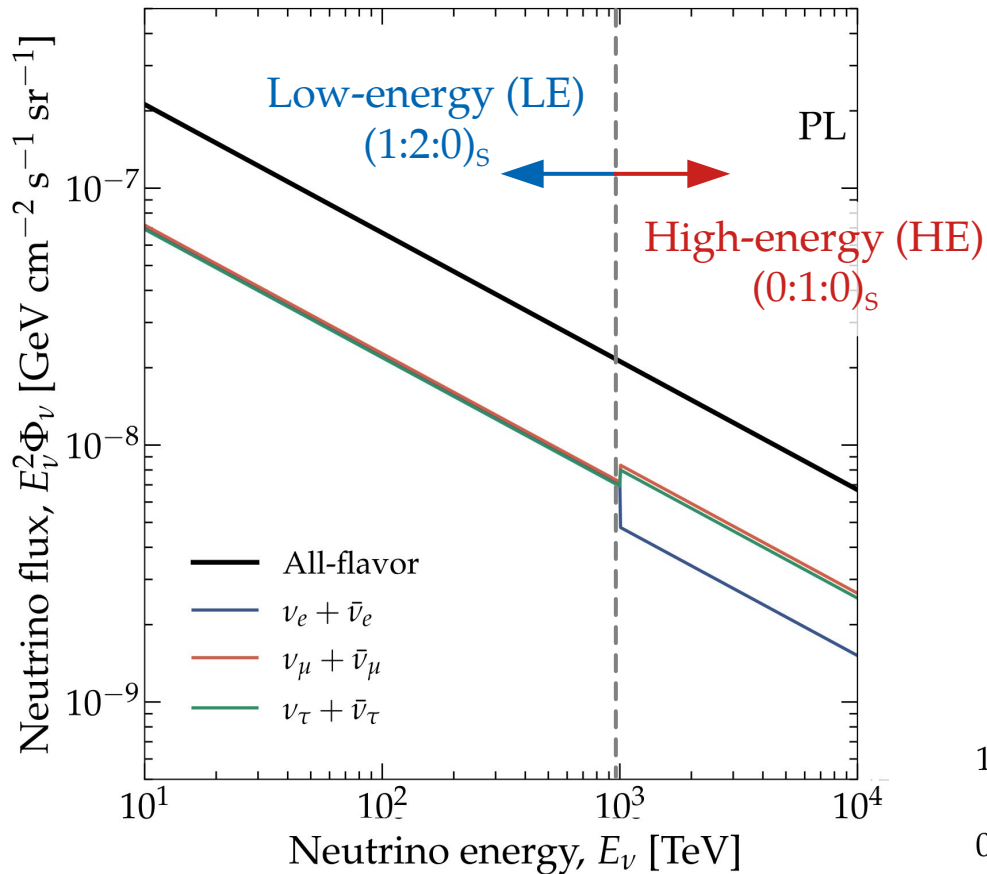
Energy-dependent flavor composition

Flavor composition: measuring the energy dependence

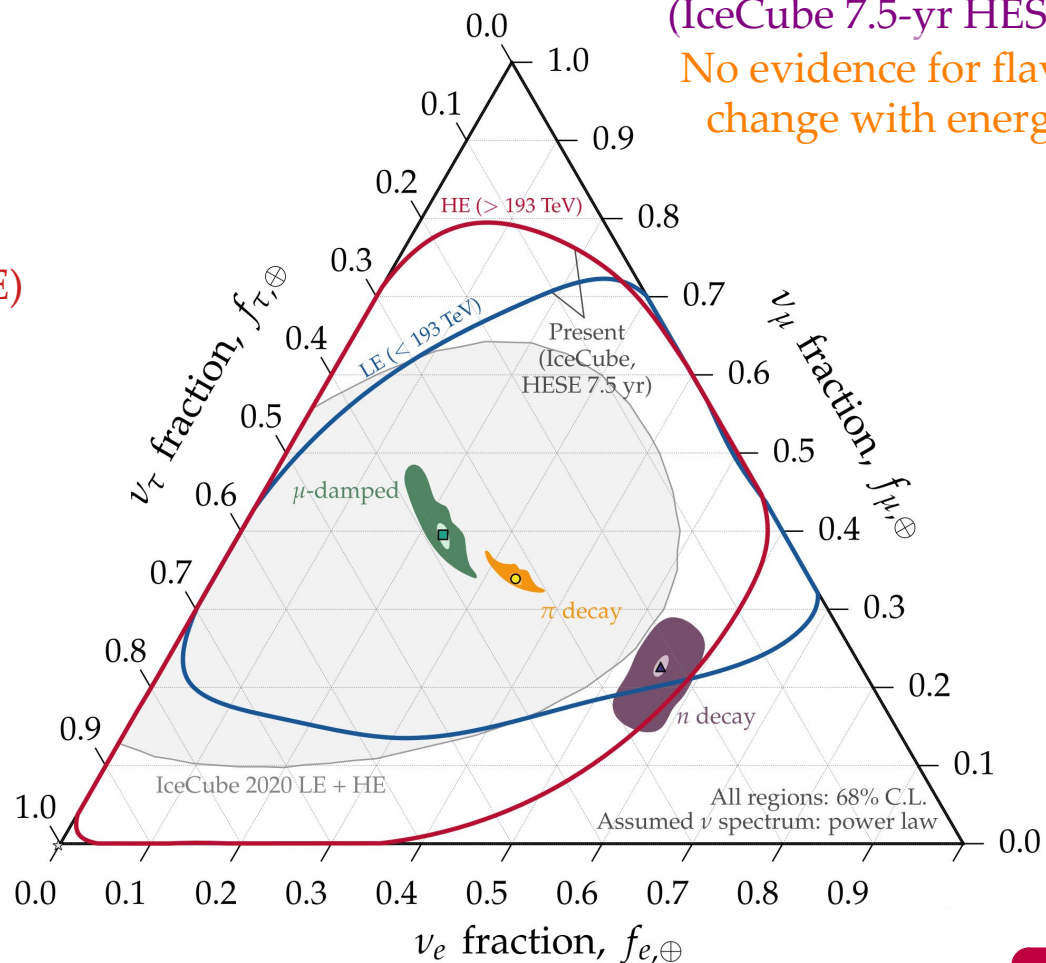


Flavor composition: measuring the energy dependence

Power-law (PL) diffuse ν flux

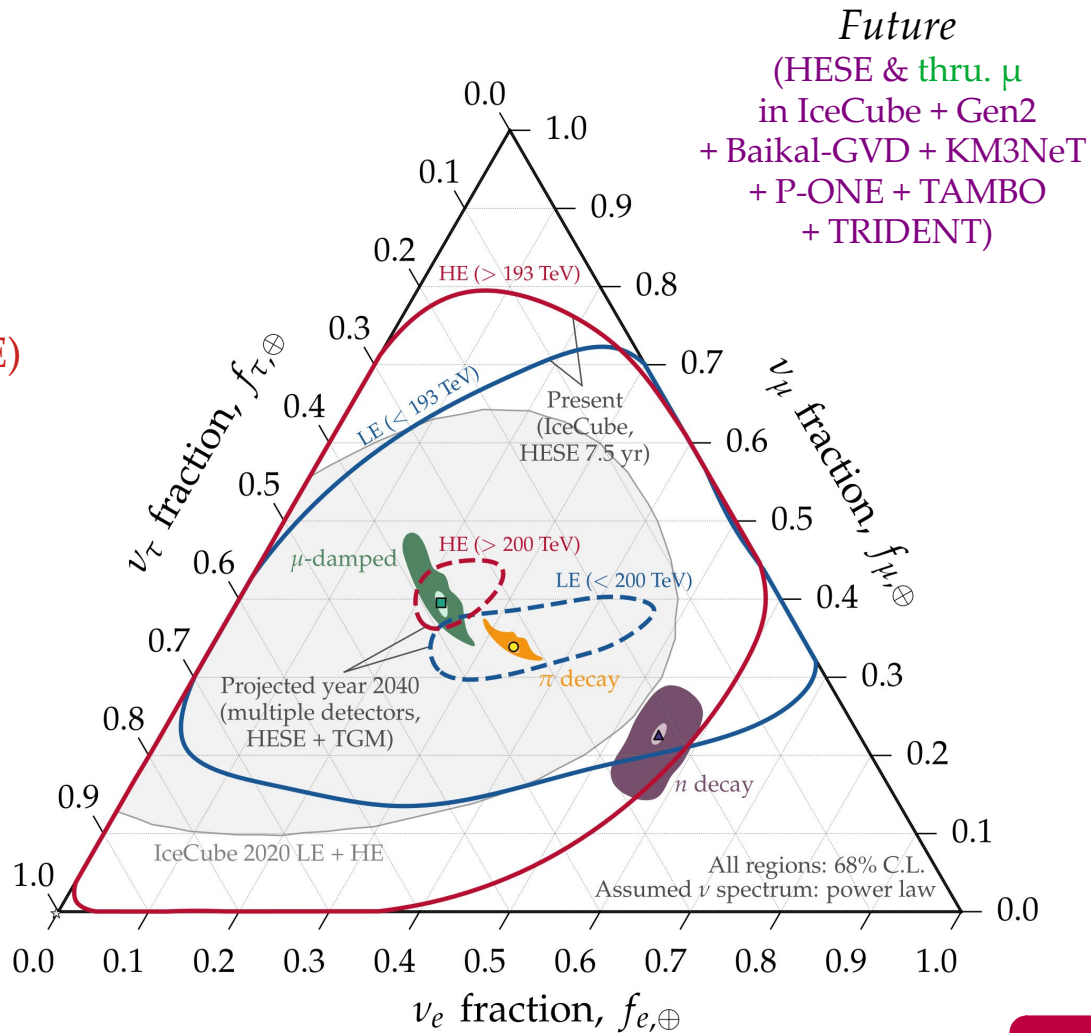
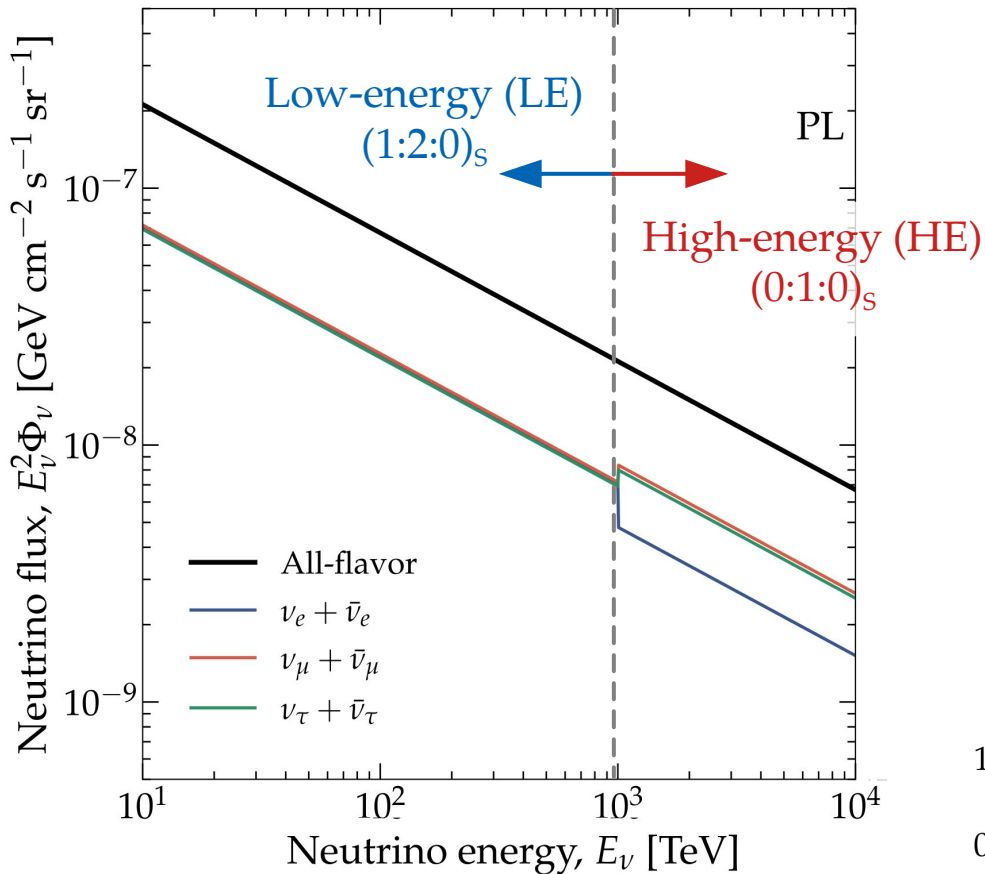


Today
(IceCube 7.5-yr HESE):
No evidence for flavor
change with energy



Flavor composition: measuring the energy dependence

Power-law (PL) diffuse ν flux



Flavor composition: measuring the energy dependence

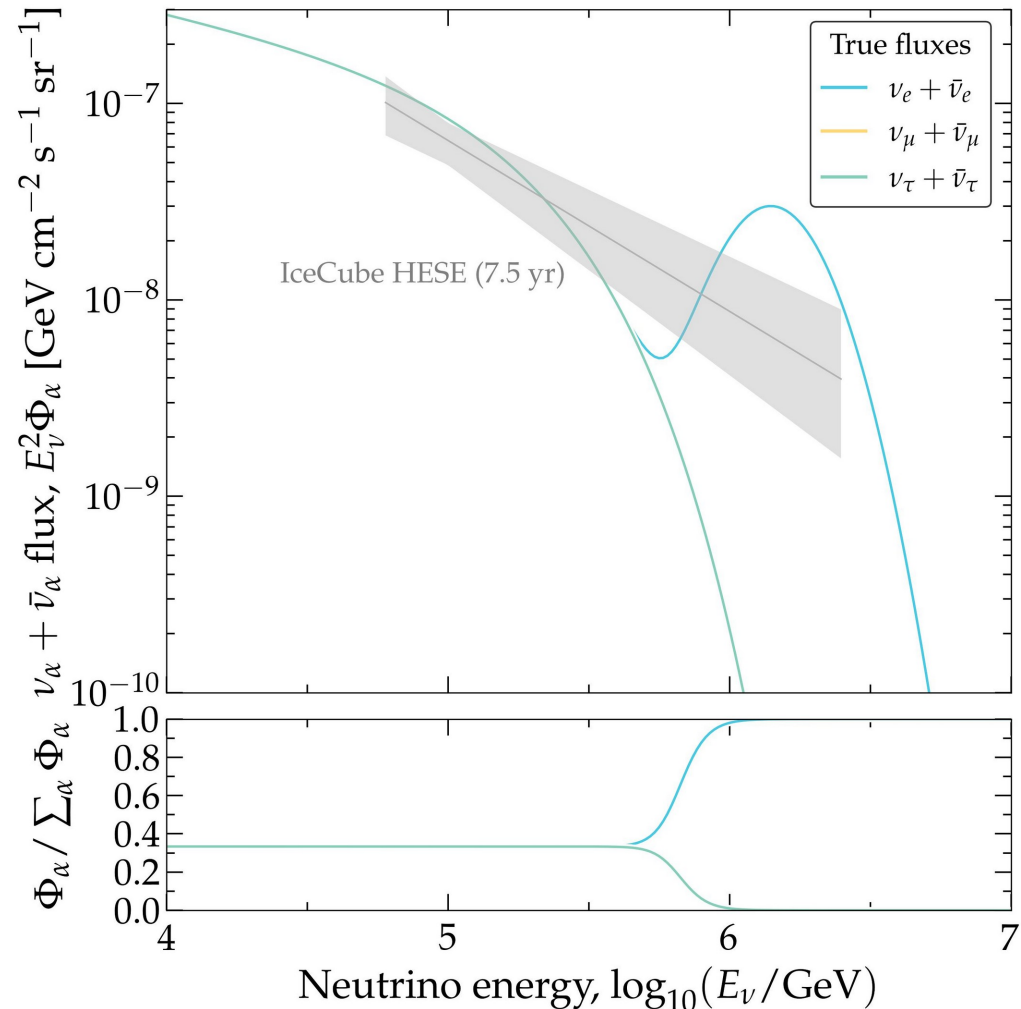
Can we do better?

Maybe

- If we do not try to pinpoint the energy of flavor transition

How?

- Infer the spectrum of ν_e , ν_μ , ν_τ separately



Flavor composition: measuring the energy dependence

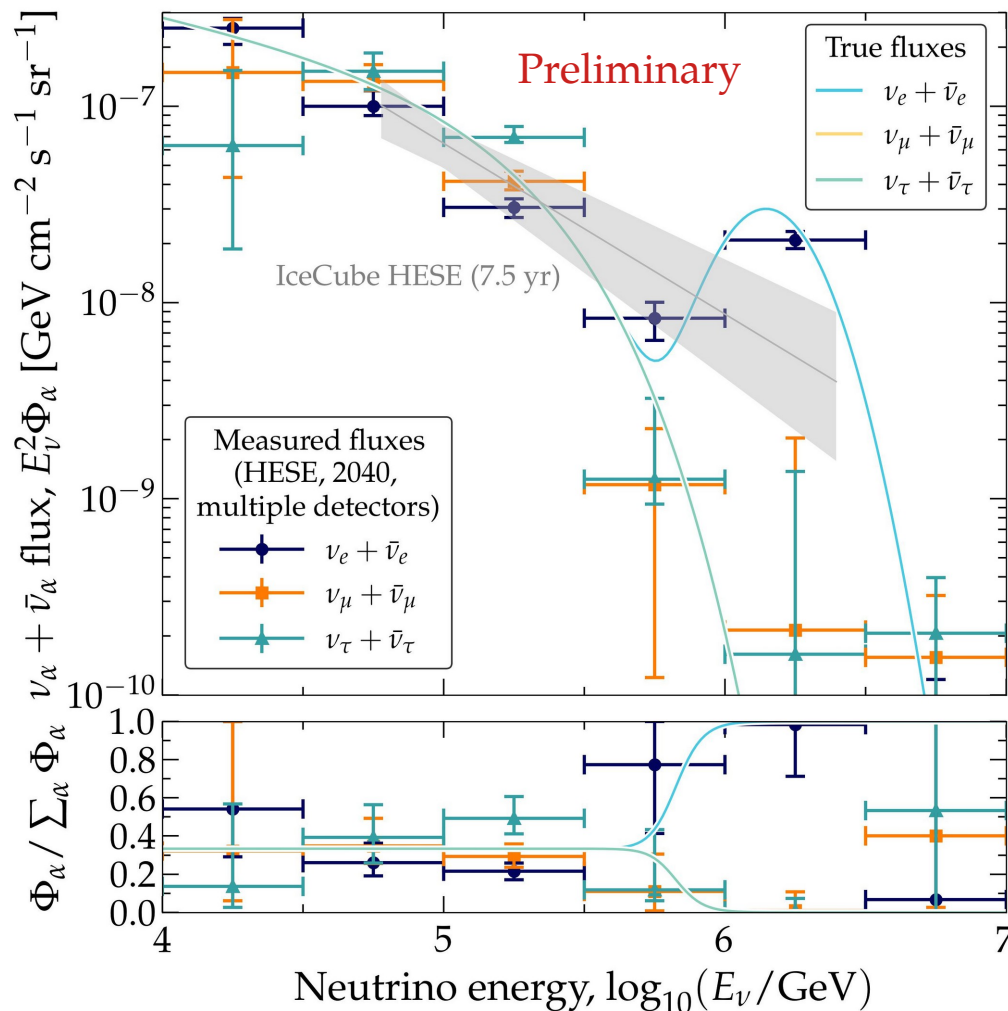
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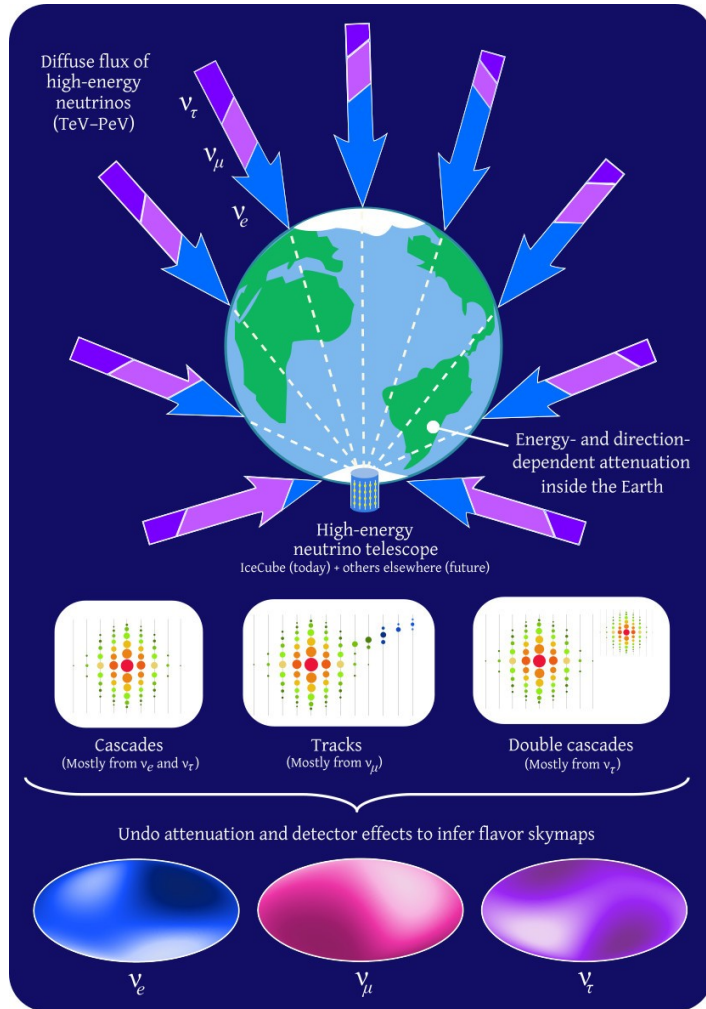
- Infer the spectrum of ν_e , ν_μ , ν_τ separately



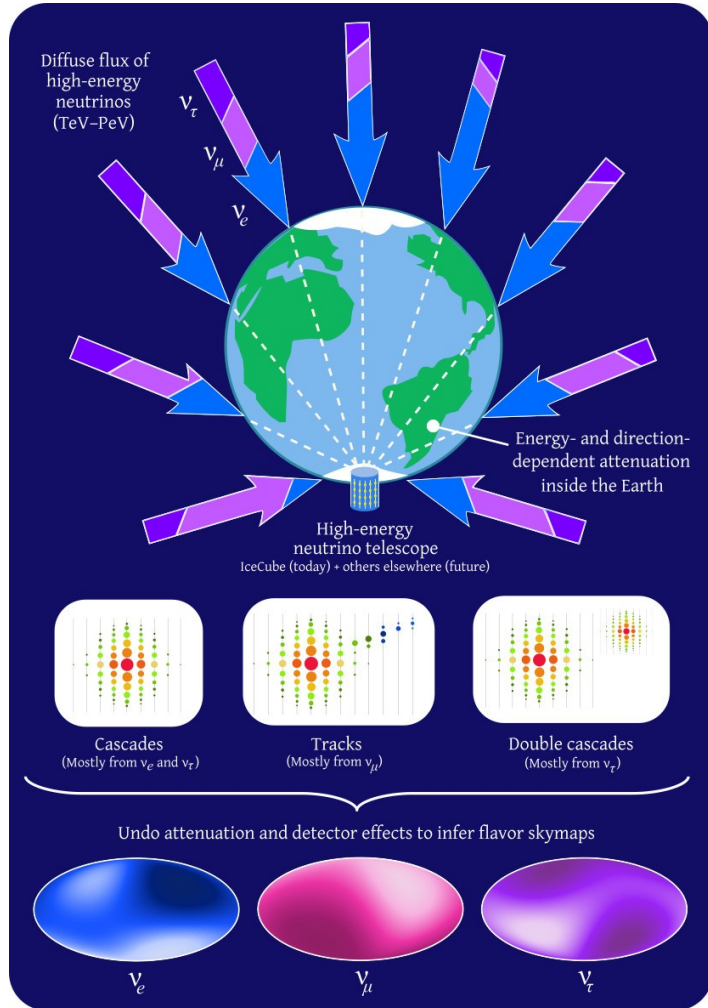
Direction-dependent flavor composition

Flavor anisotropy in the high-energy neutrino sky

*Does the high-energy sky shine equally brightly
In neutrinos of all flavors?*

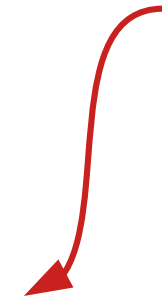


Flavor anisotropy in the high-energy neutrino sky

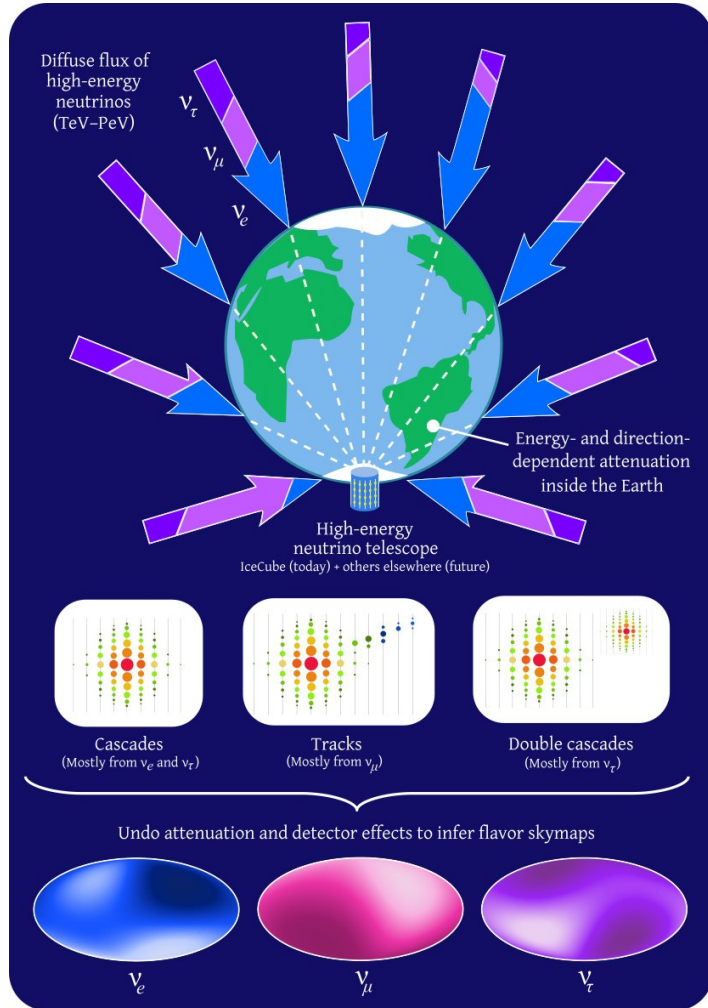


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From the angular distribution of detected events in neutrino telescopes (HESE cascades, tracks, double cascades) ...



Flavor anisotropy in the high-energy neutrino sky

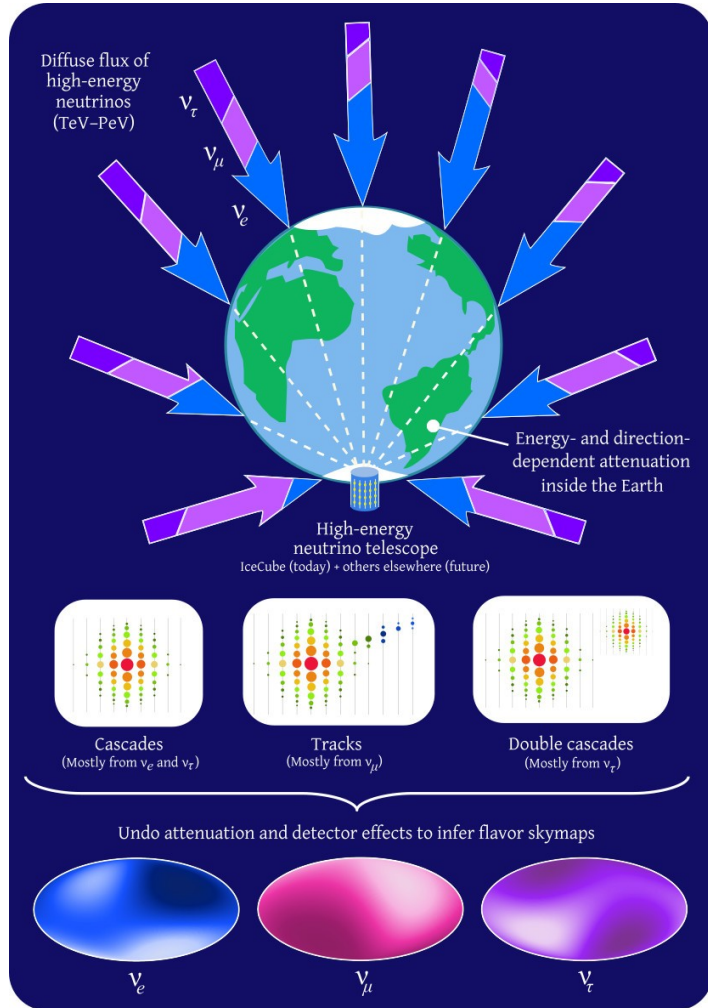


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From the angular distribution of detected events in neutrino telescopes (HESE cascades, tracks, double cascades) ...

... we infer the directional dependence of the diffuse fluxes of ν_e , ν_μ , ν_τ

Flavor anisotropy in the high-energy neutrino sky



*Does the high-energy sky shine equally brightly
In neutrinos of all flavors?*

*From the angular distribution of detected
events in neutrino telescopes
(HESE cascades, tracks, double cascades) ...*

*How? Undo detection effects
(use public IceCube
HESE Monte Carlo)*

*... we infer the directional dependence of
the diffuse fluxes of ν_e , ν_μ , ν_τ*

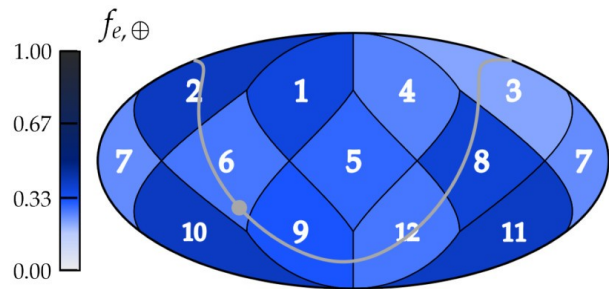
Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

Real, public data

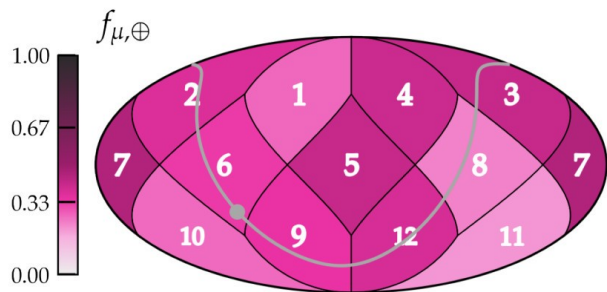
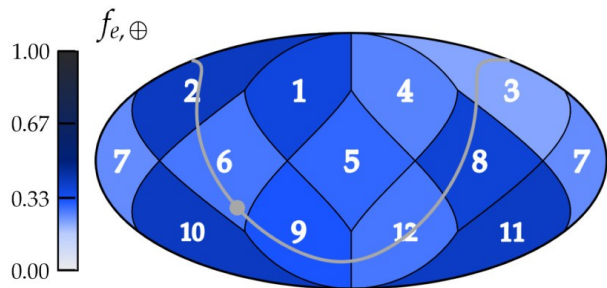


Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

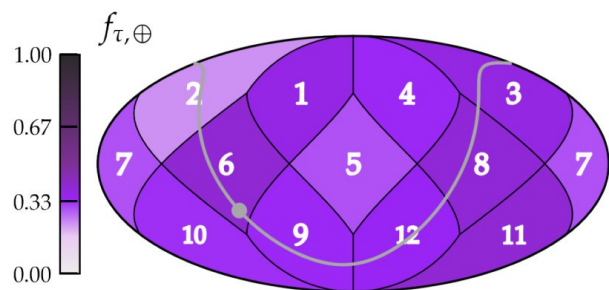
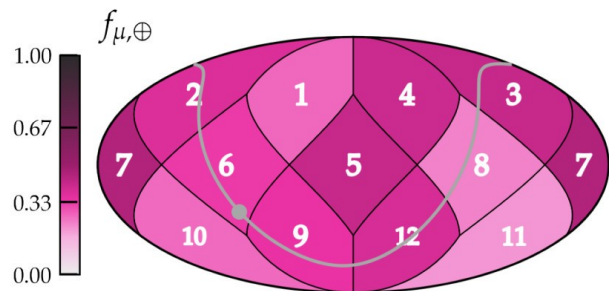
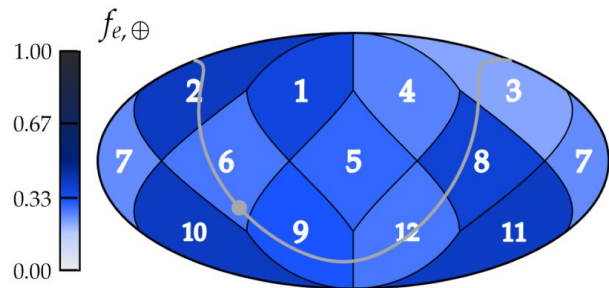
Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)



Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)



Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)



Equatorial

Telalovic, MB, JCAP 2025

Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

This work:

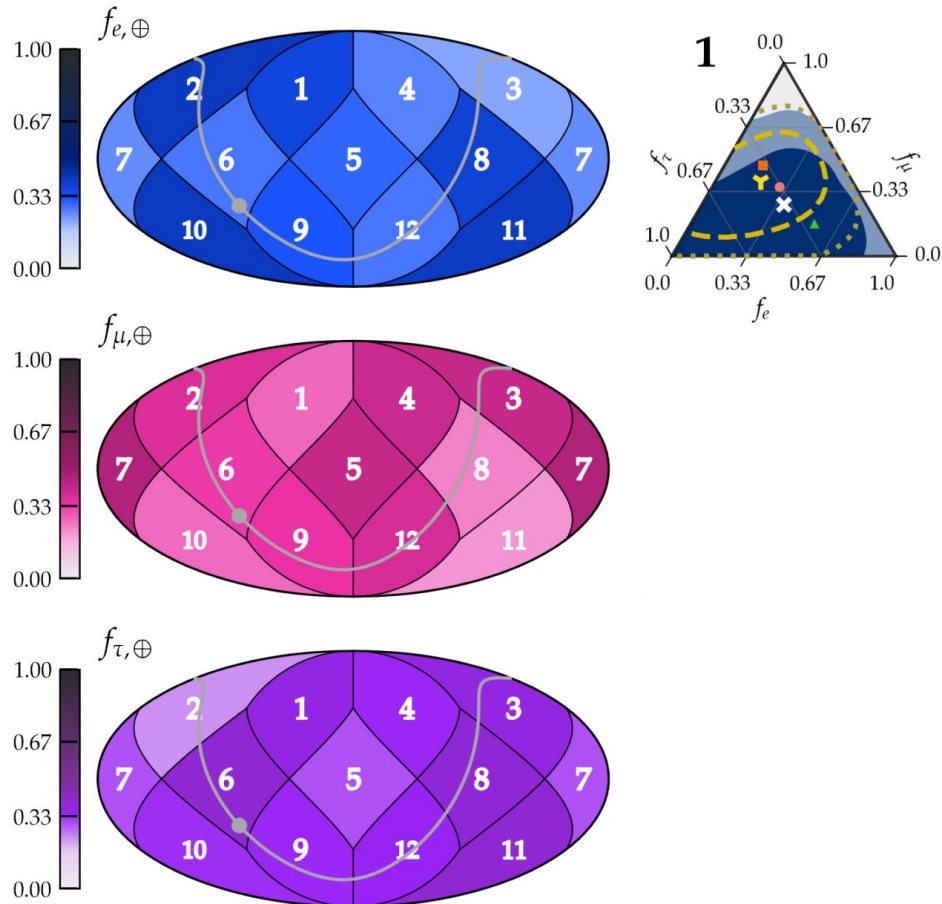
⊗ Best fit ■ 1σ ■ 2σ □ 3σ

IceCube 2020 all-sky:

⊗ Best fit - - 1σ ··· 2σ

Benchmarks:

● π^\pm decay: (1:2:0)_S ■ μ -damped: (0:1:0)_S ▲ n decay: (1:0:0)_S



Equatorial

Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

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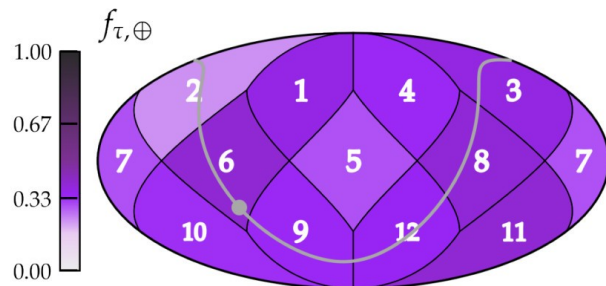
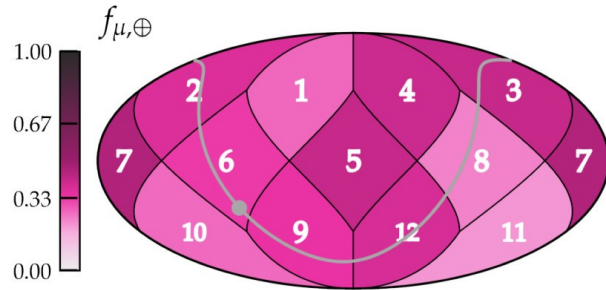
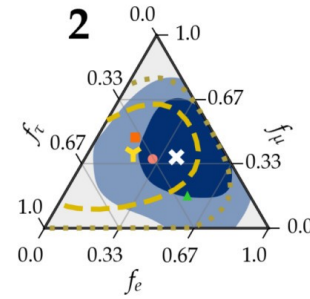
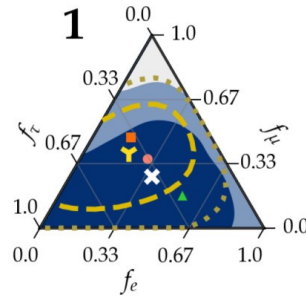
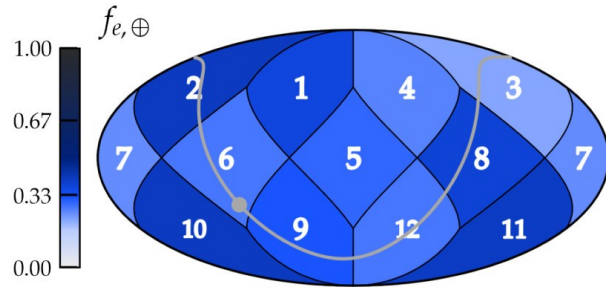
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Equatorial

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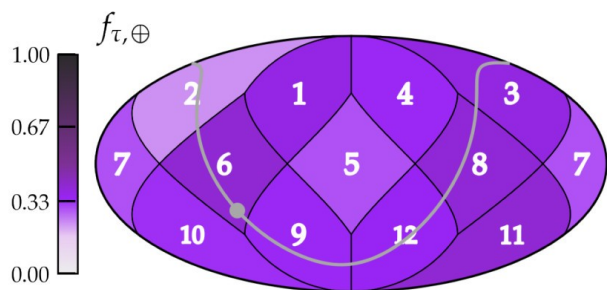
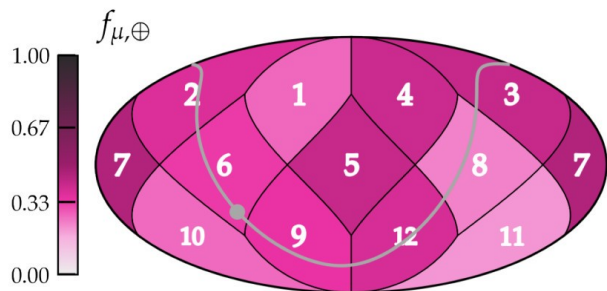
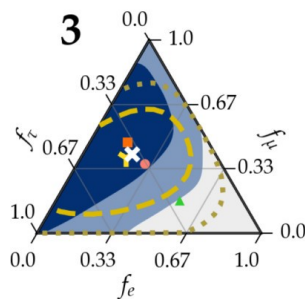
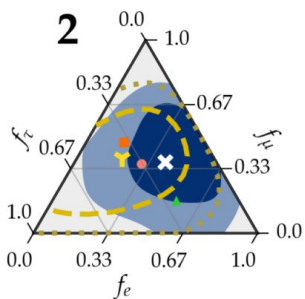
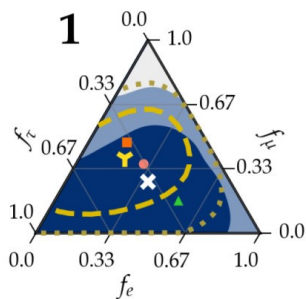
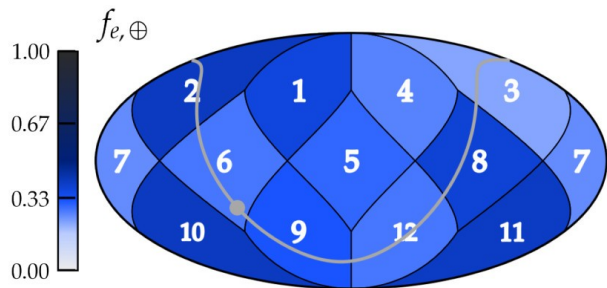
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Equatorial

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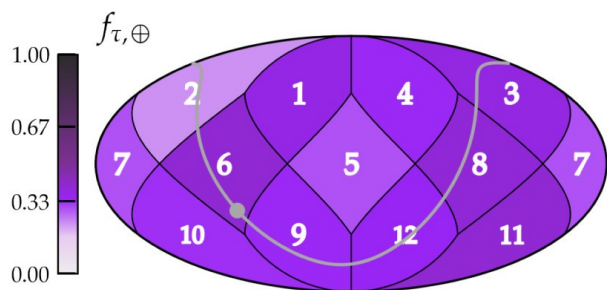
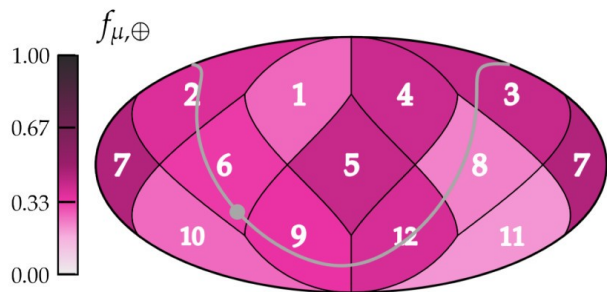
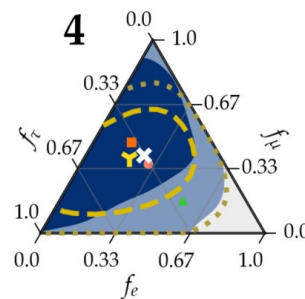
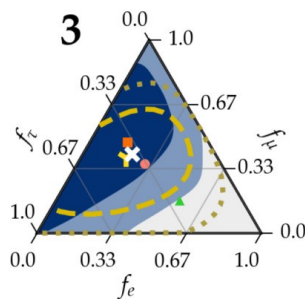
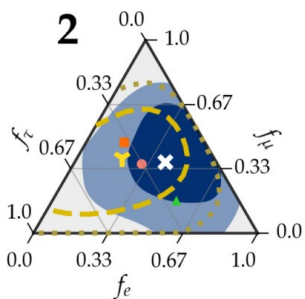
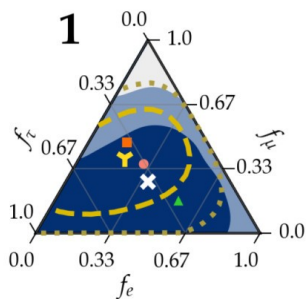
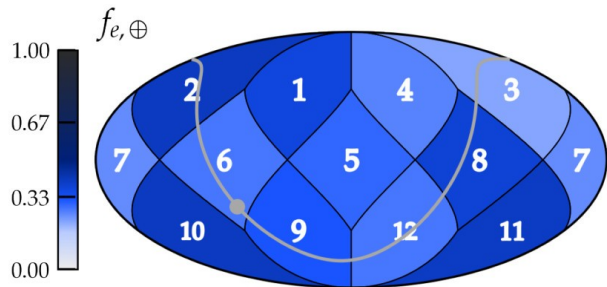
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Equatorial

Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

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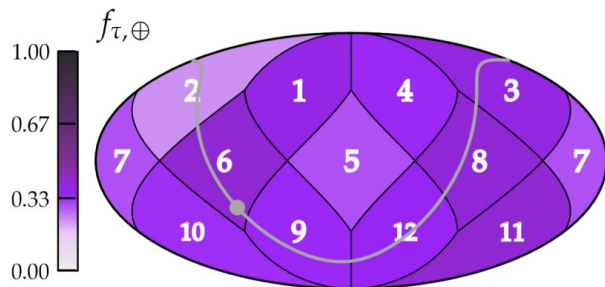
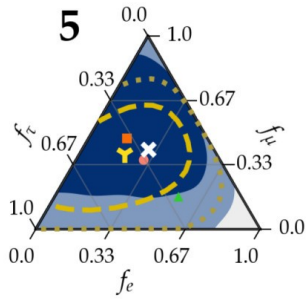
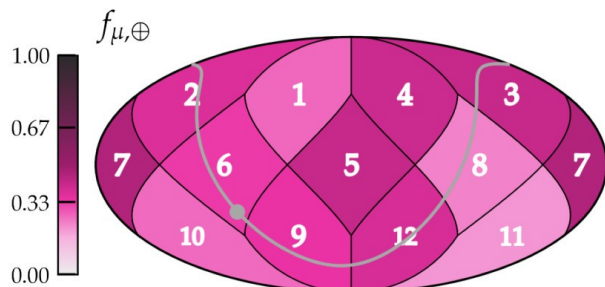
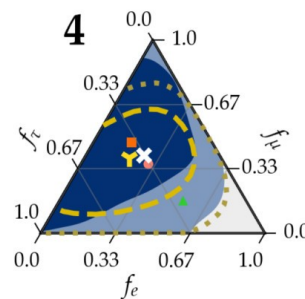
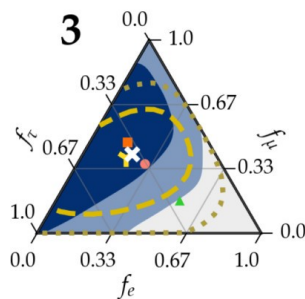
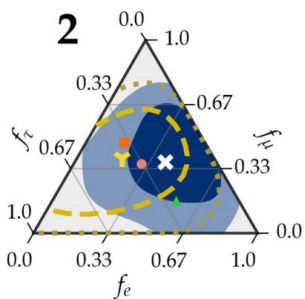
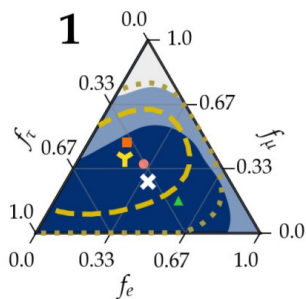
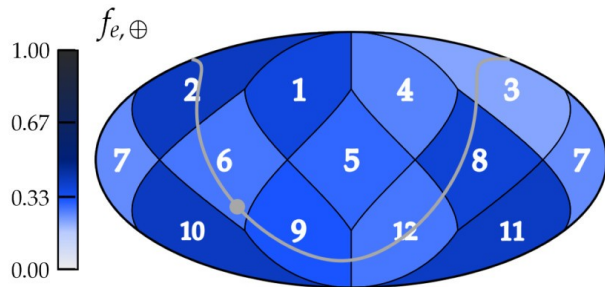
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Equatorial

Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

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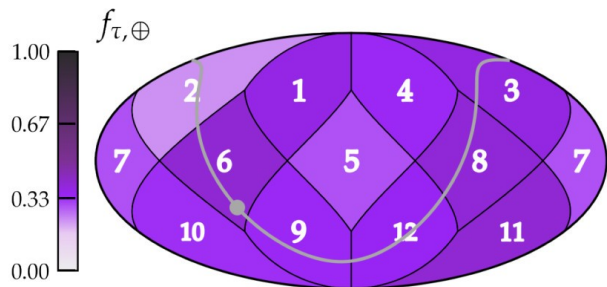
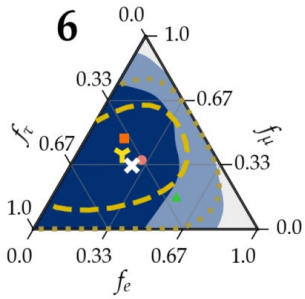
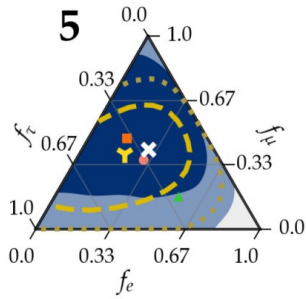
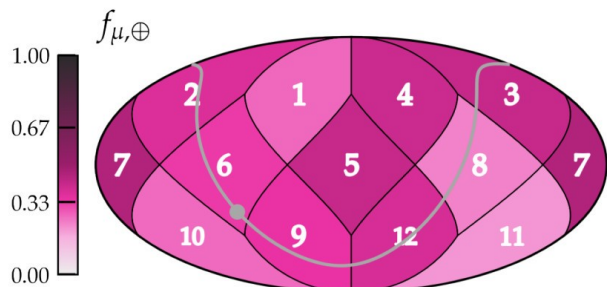
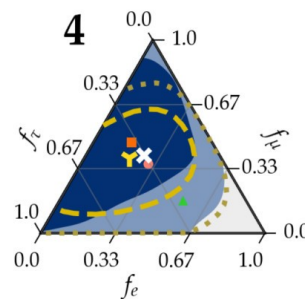
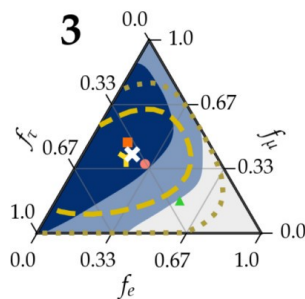
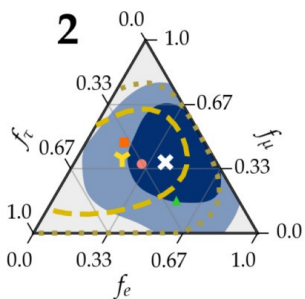
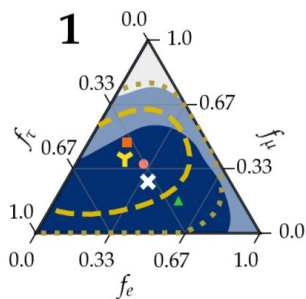
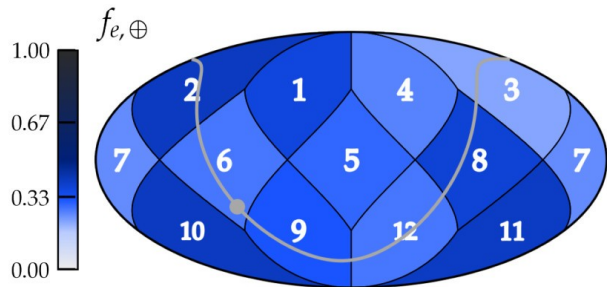
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IceCube 2020 all-sky:

⊗ Best fit - - 1σ ··· 2σ

Benchmarks:

● π^\pm decay: (1:2:0)_S ■ μ -damped: (0:1:0)_S ▲ n decay: (1:0:0)_S



Equatorial

Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

This work:

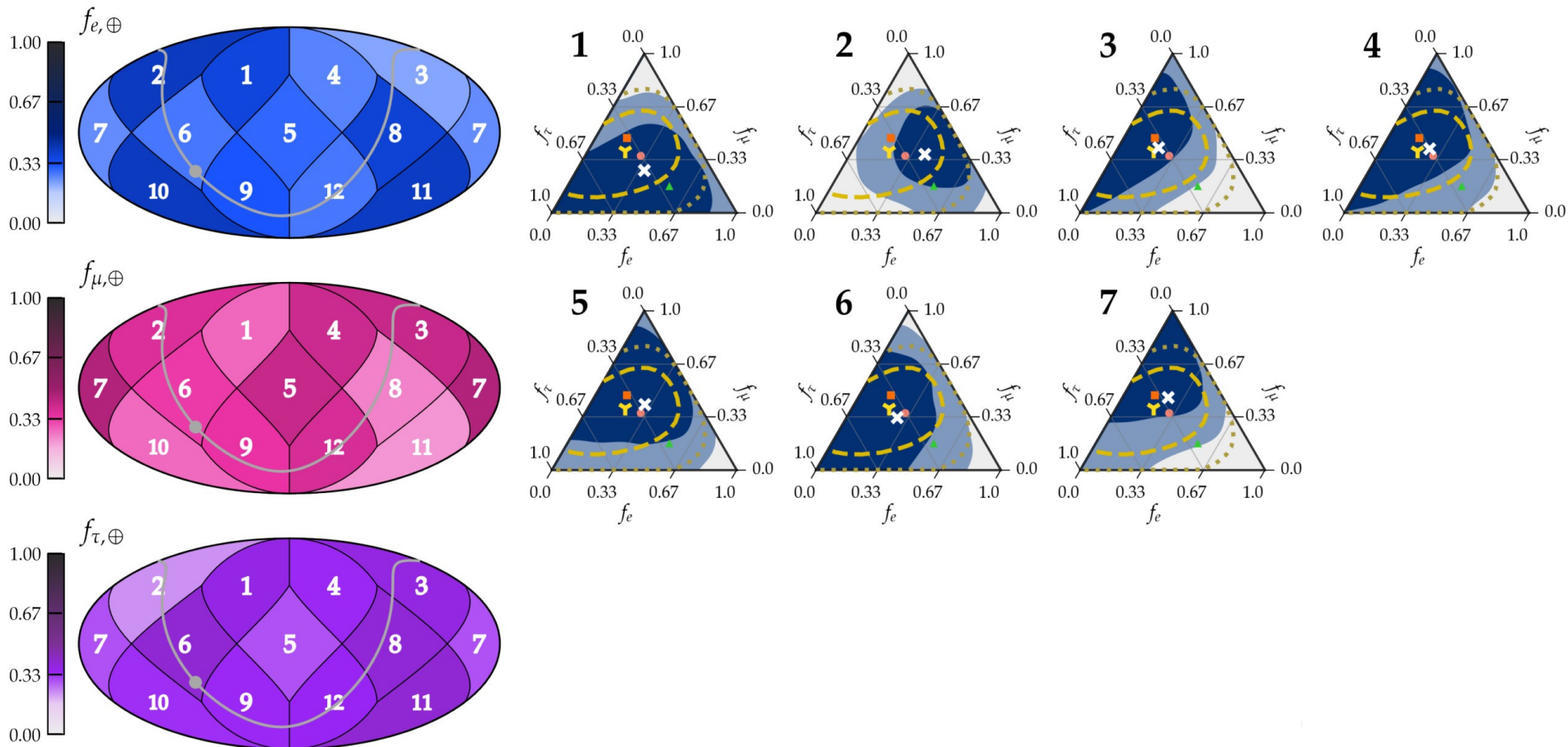
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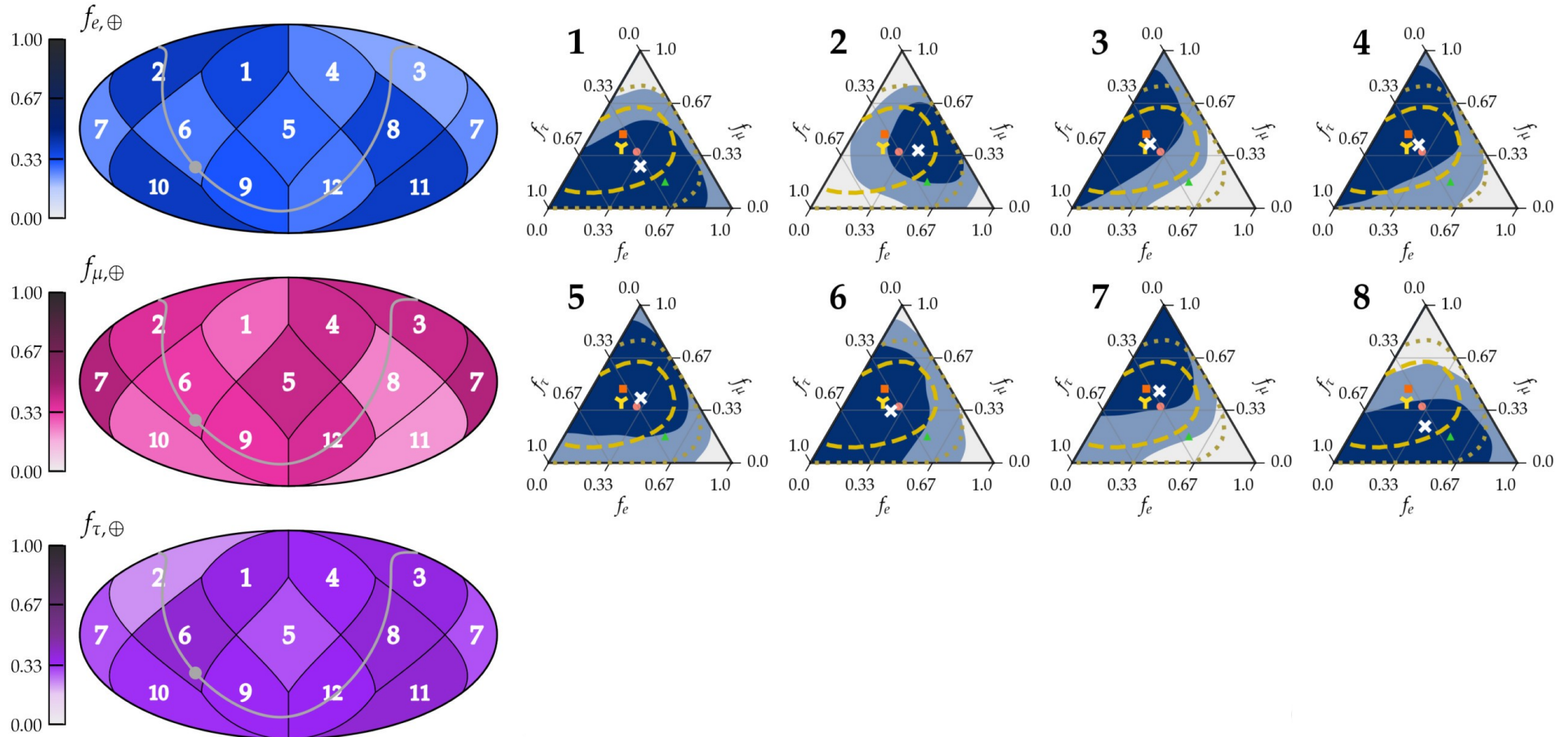
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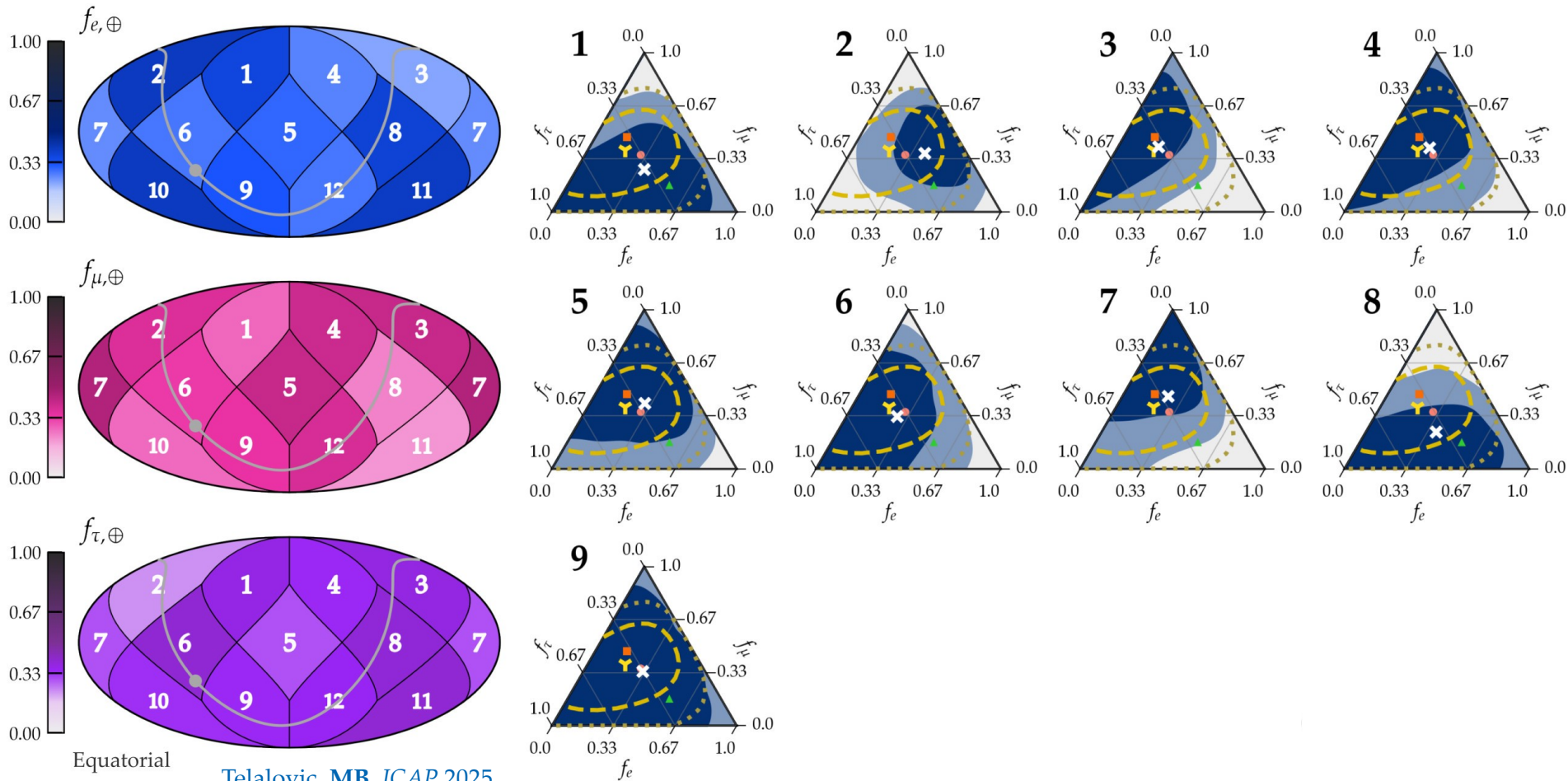
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Telalovic, MB, JCAP 2025

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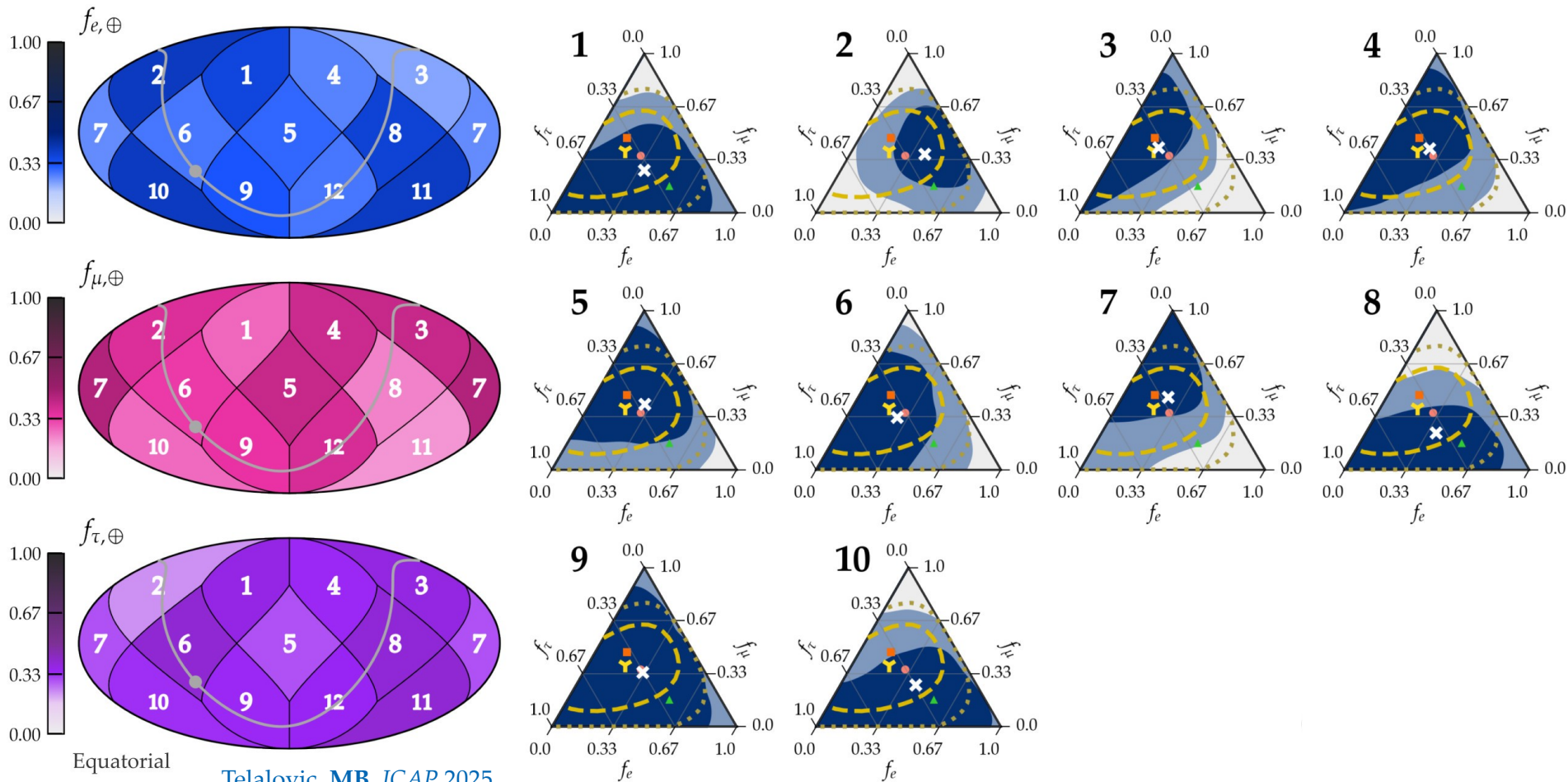
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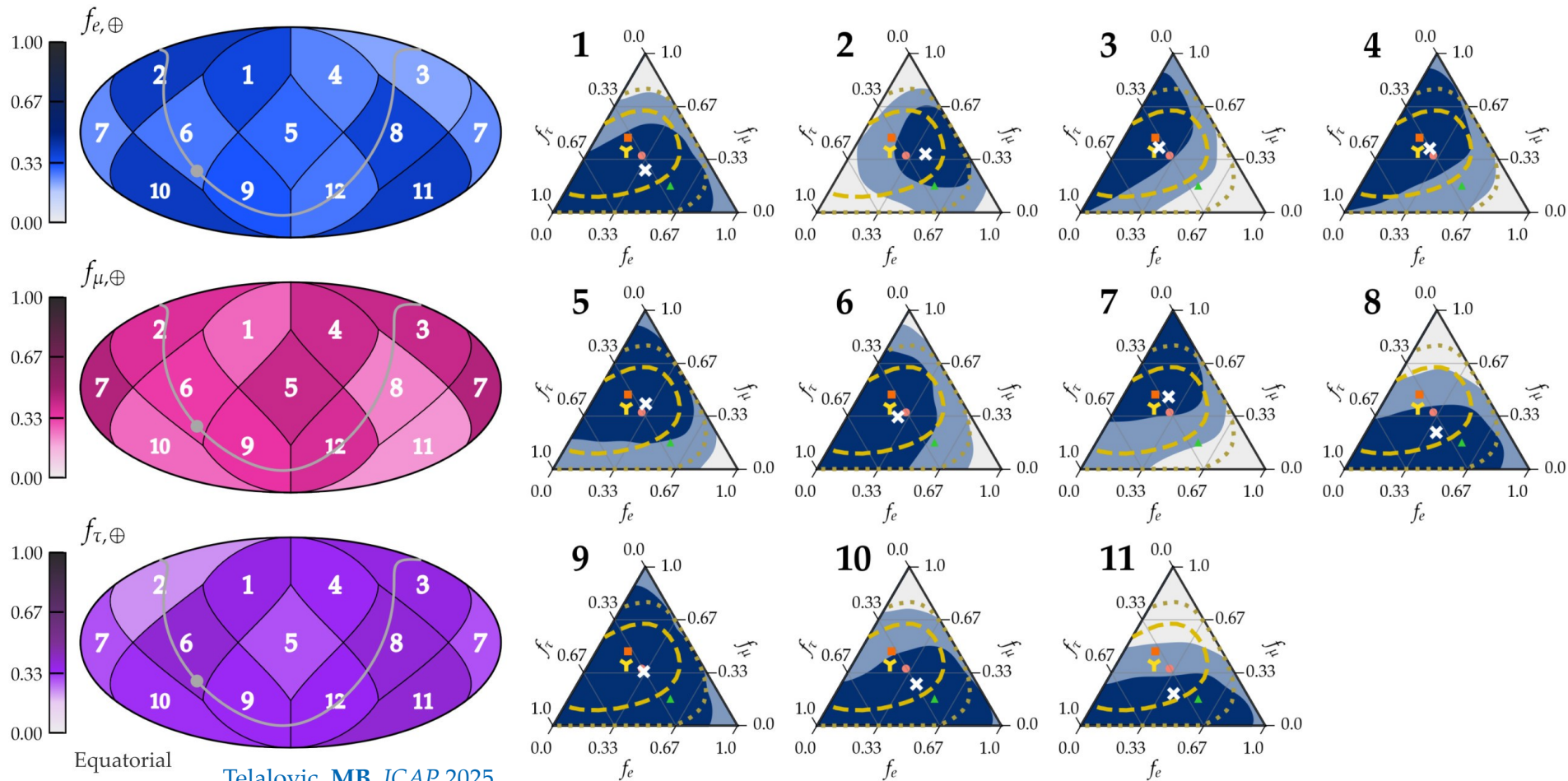
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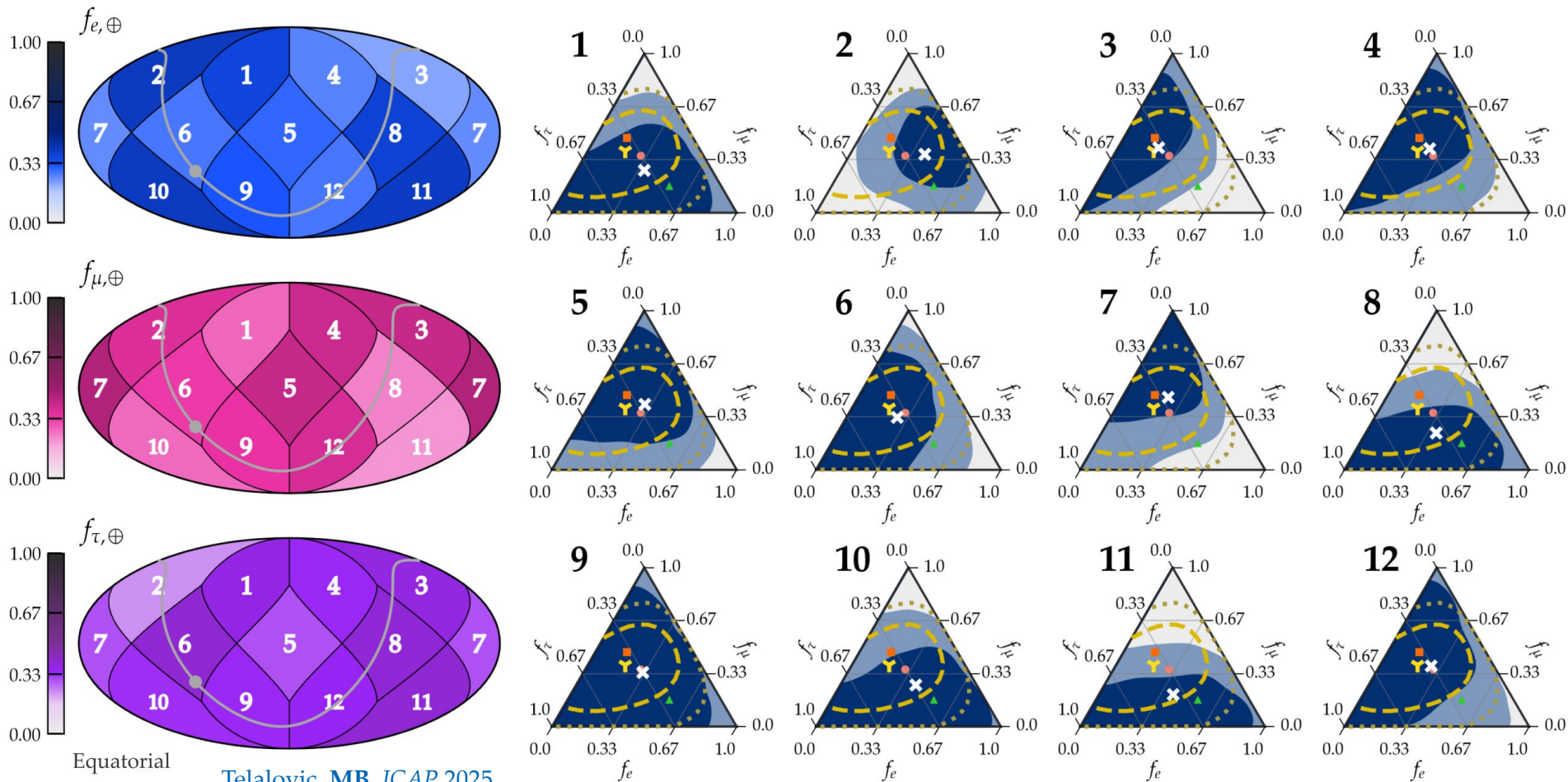
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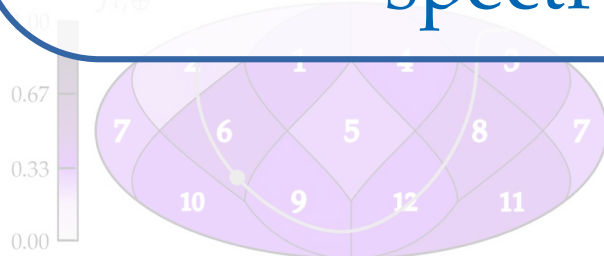
● π^\pm decay: (1:2:0)_s ■ μ -damped: (0:1:0)_s ▲ n decay: (1:0:0)_s

There is no sign of flavor anisotropy
in present-day IceCube data
(Bayes factor is ~ 1)

We place the first constraints on
the flavor neutrino angular power
spectrum *à la* CMB

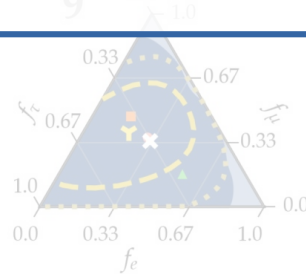


Work led by
Bernanda
Telalovic

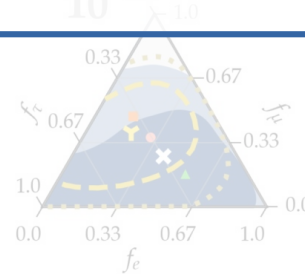


Equatorial

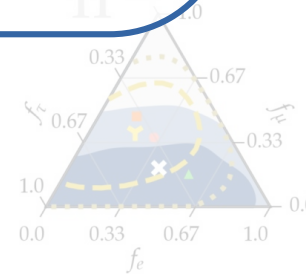
Telalovic, MB, 2310.15224



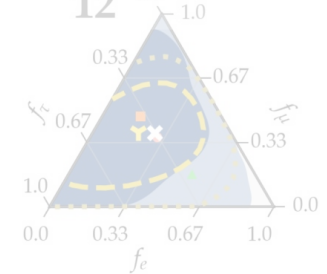
f_e



f_e



f_e



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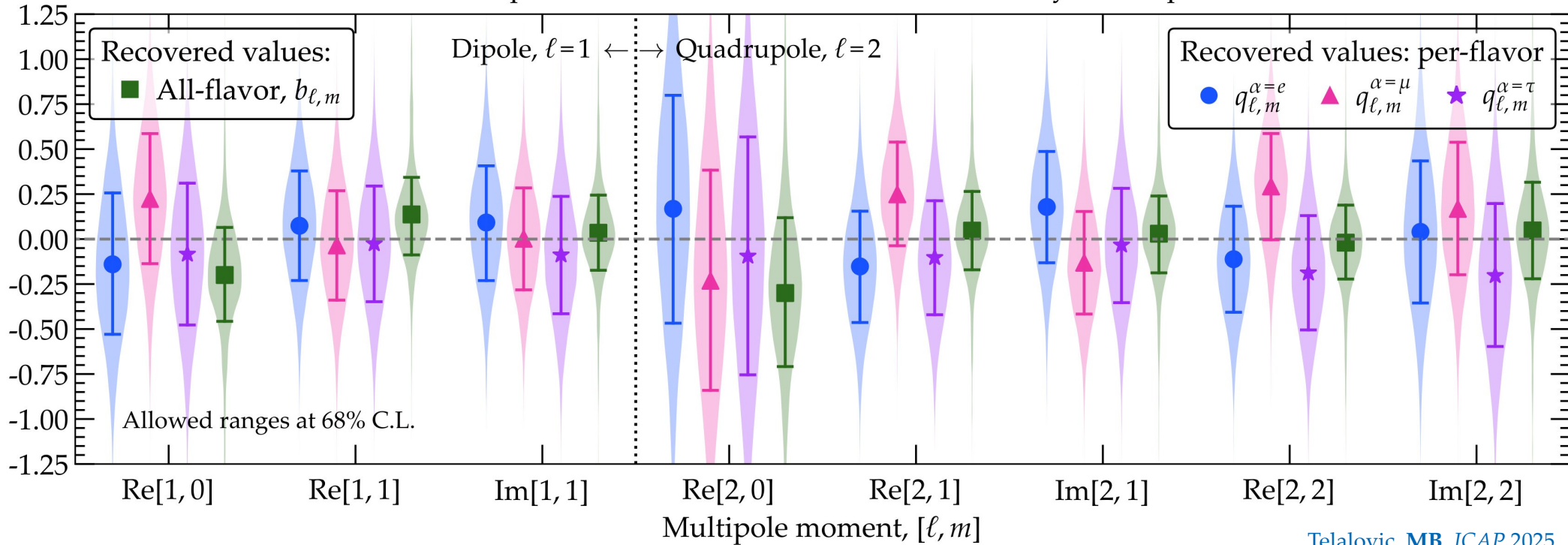
Flavor dipoles and quadrupoles in the sky?

Flavor-dependent multipole expansion

Isotropic flux

$$\Phi_{\nu\alpha}(E_\nu, \theta_z, \phi) = \Phi_0 \left(\frac{E_\nu}{100 \text{ TeV}} \right)^{-\gamma} \times \frac{1}{6} \left[1 + \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} q_{\ell,m}^\alpha Y_\ell^m(\theta_z, \phi) \right]$$

Multipole moments from the IceCube HESE 7.5-year sample



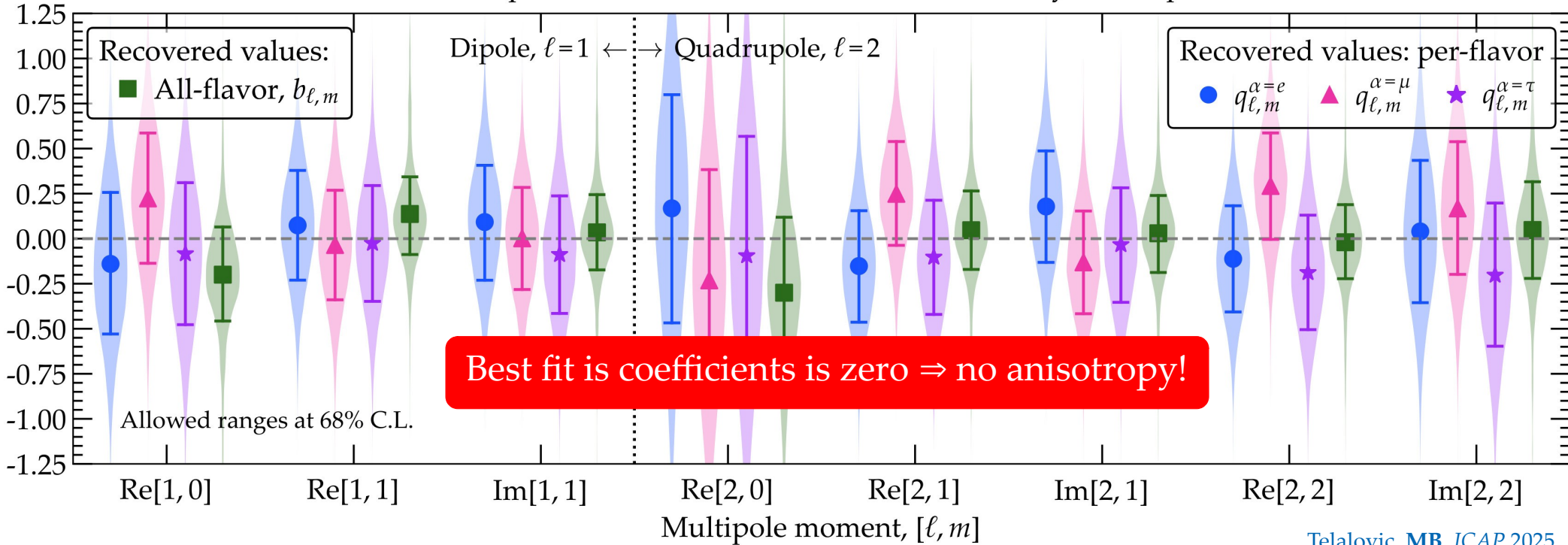
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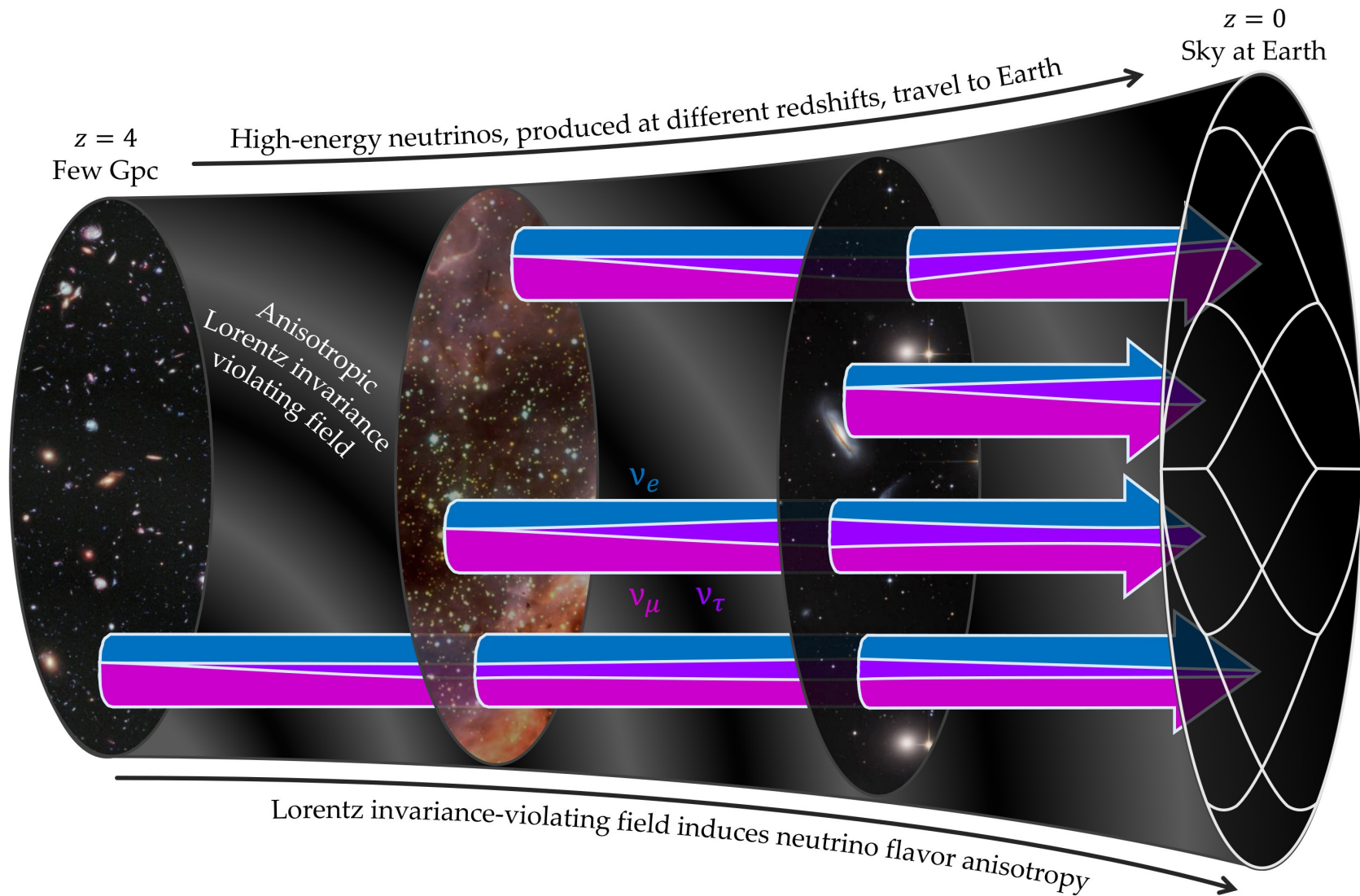
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Anisotropic Lorentz-invariance violation makes the flavor sky anisotropic:

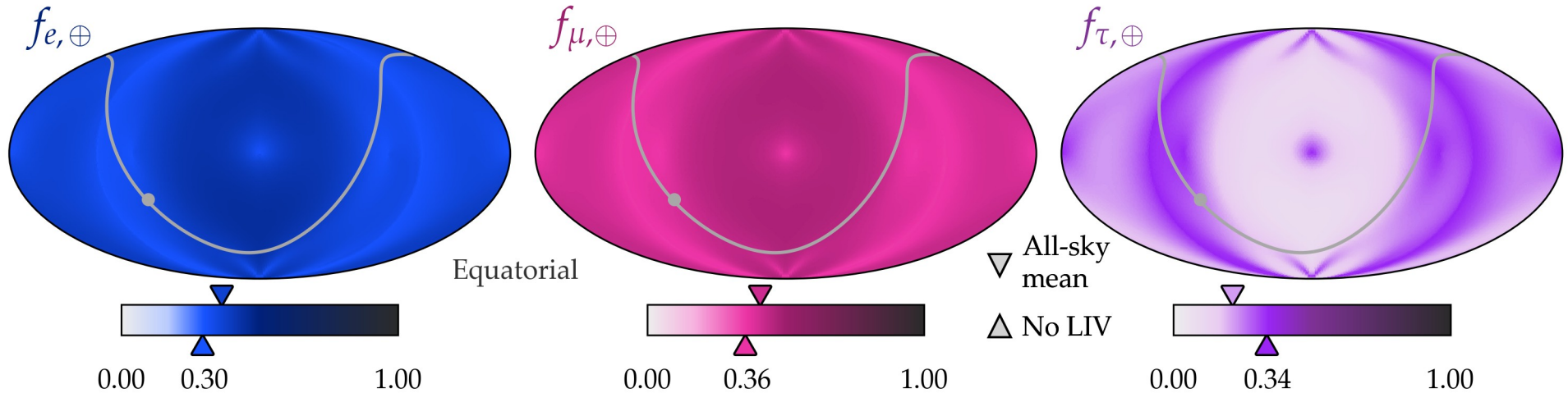
$$H_{\text{tot}} = H_{\text{vac}} + \sum_{d=2}^{\infty} H_{\text{LIV}}^{(d)} = H_{\text{vac}} + E^{d-3} \sum_{\ell=0}^{d-1} \sum_{m=-\ell}^{\ell} Y_{\ell}^m(\hat{\mathbf{p}}) (a_{\text{eff}}^{(d)})_{\ell m}^{\alpha\beta}$$

Neutrino oscillation probability becomes direction-dependent 

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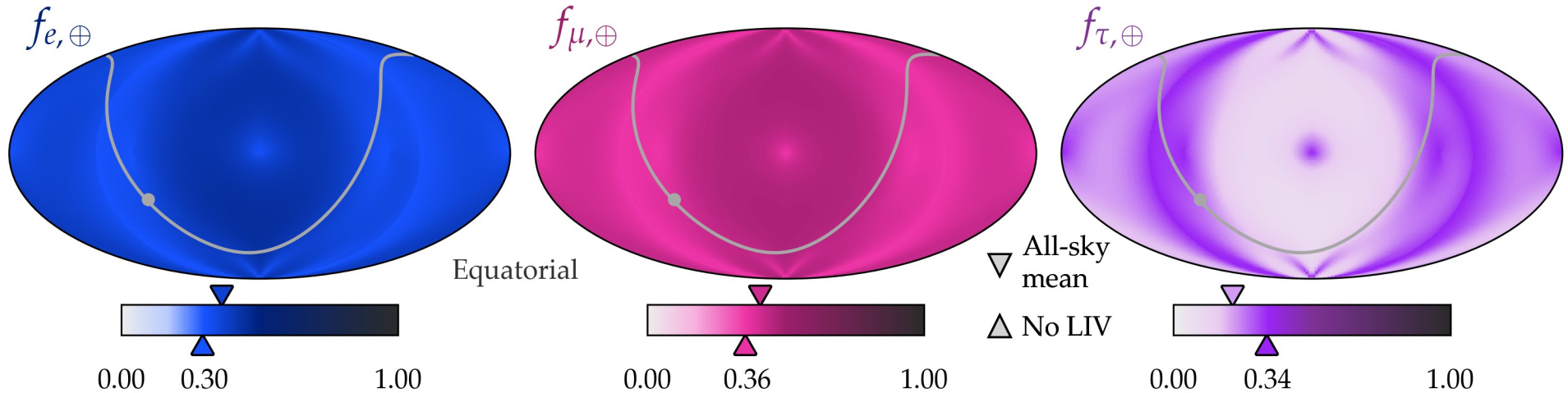
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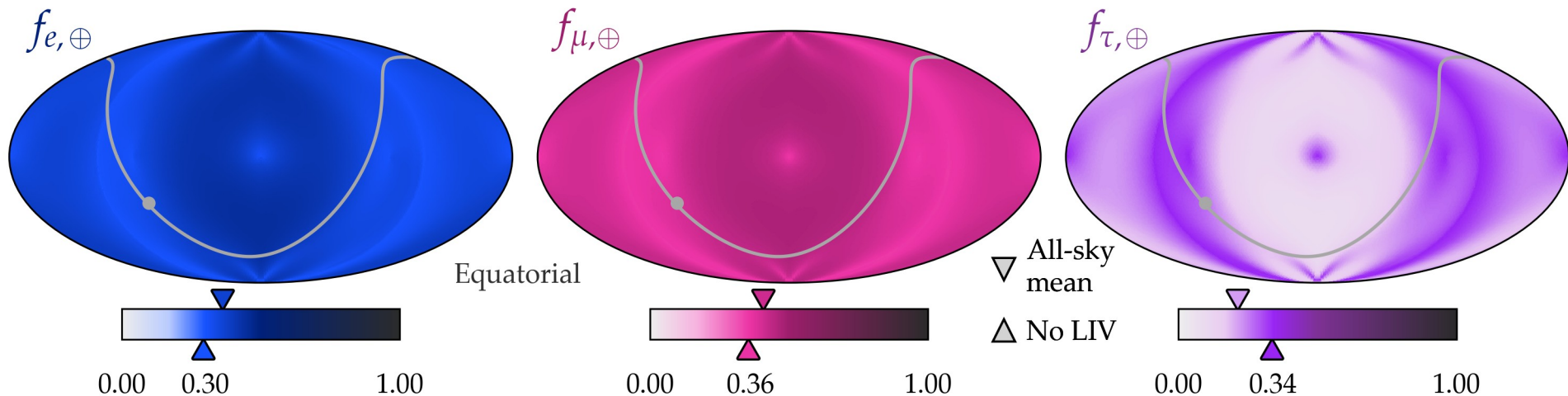
Upper limits from accelerator ν (MINOS): $< 10^{-20} - 10^{-15} \text{ GeV}^{-1}$

For dimension-5
CPT-odd LIV coefficient

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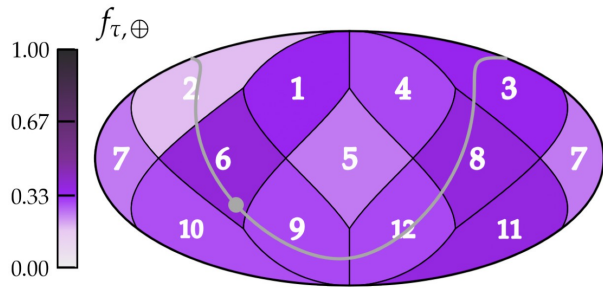
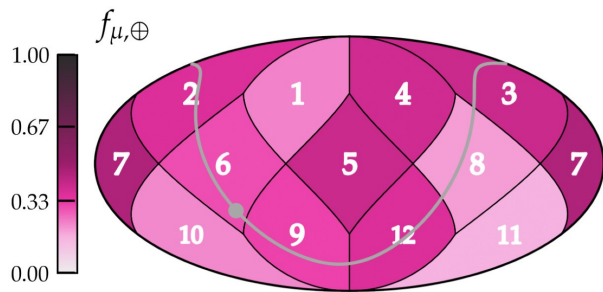
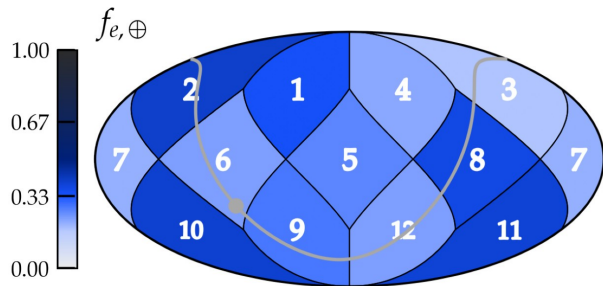
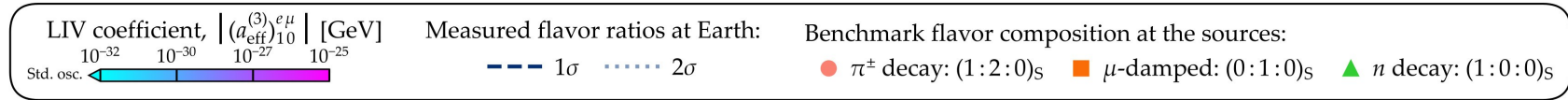


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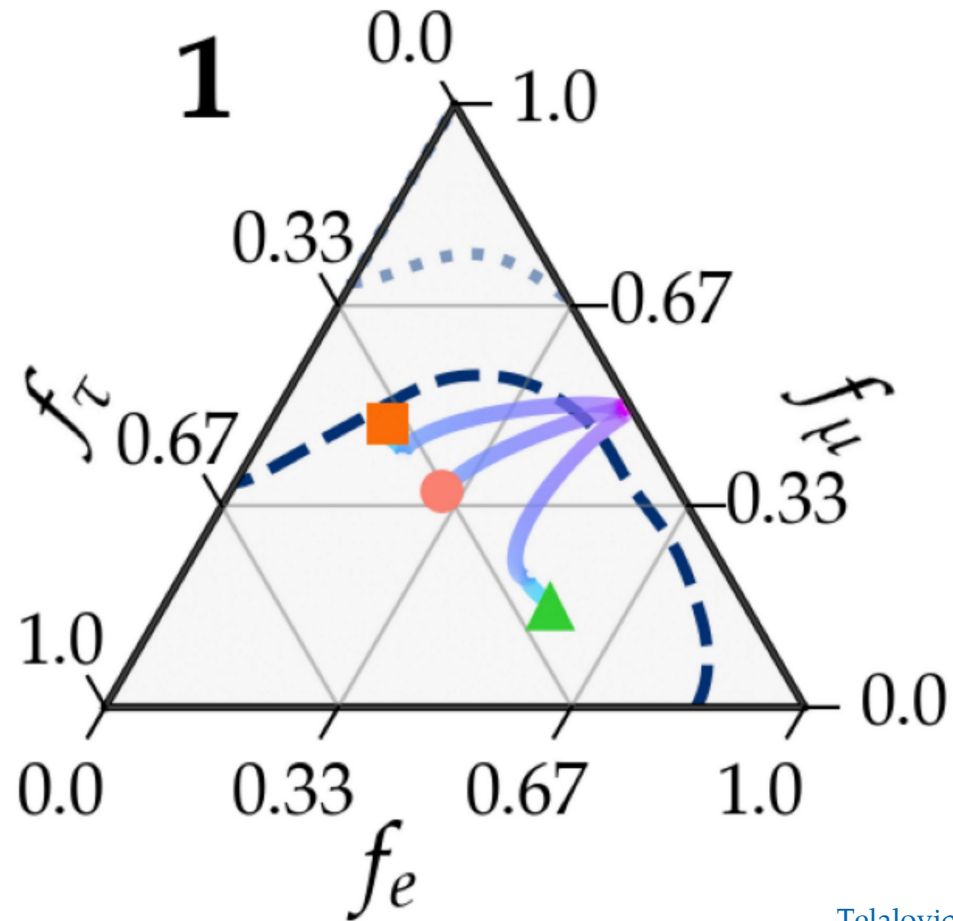
Upper limits from 7.5-year HESE: $< 10^{-34} \text{ GeV}^{-1}$

For dimension-5
CPT-odd LIV coefficient

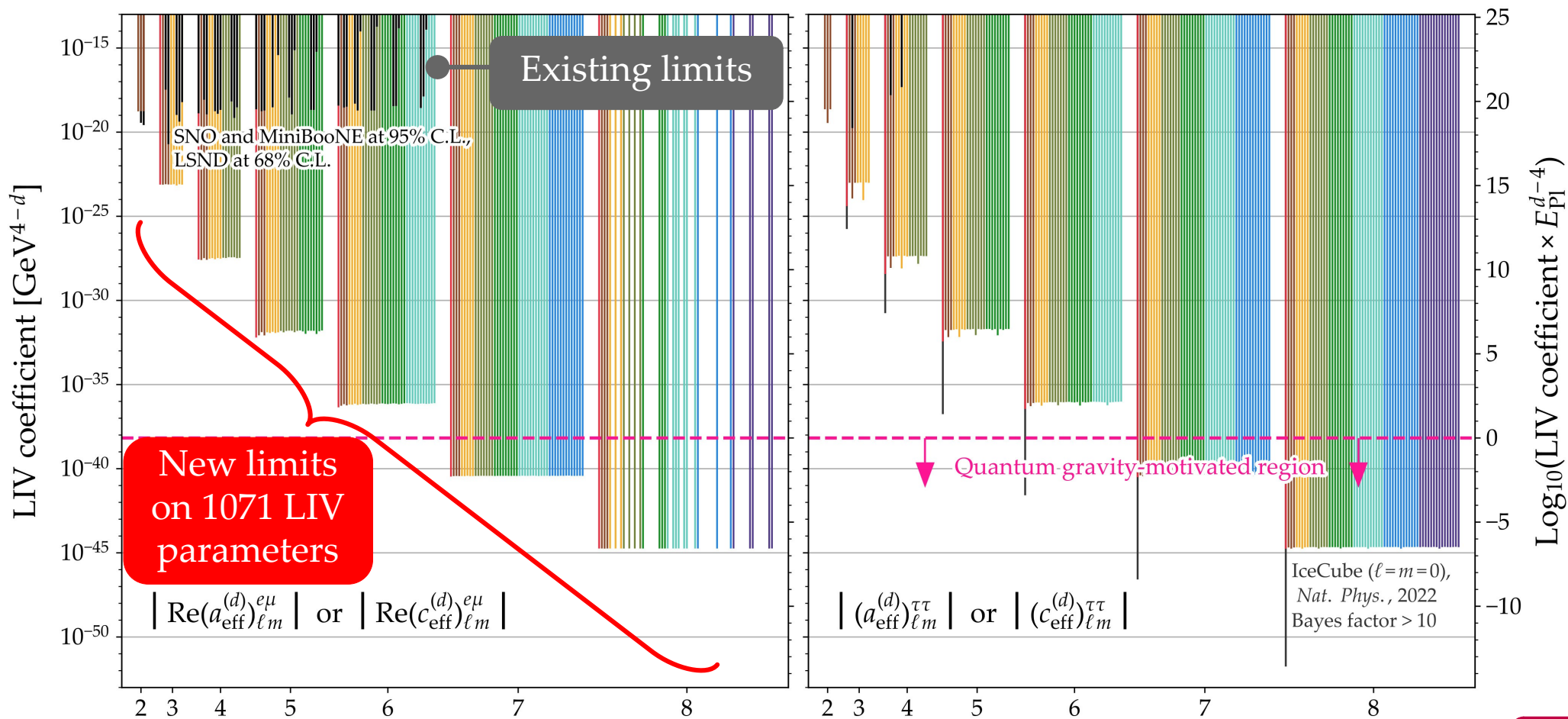
Lorentz-violating high-energy neutrino flavor anisotropy (IceCube HESE 7.5 years)



Equatorial



Disfavored at 95% C.L. from flavor isotropy (this work, using IceCube 7.5-year HESE)

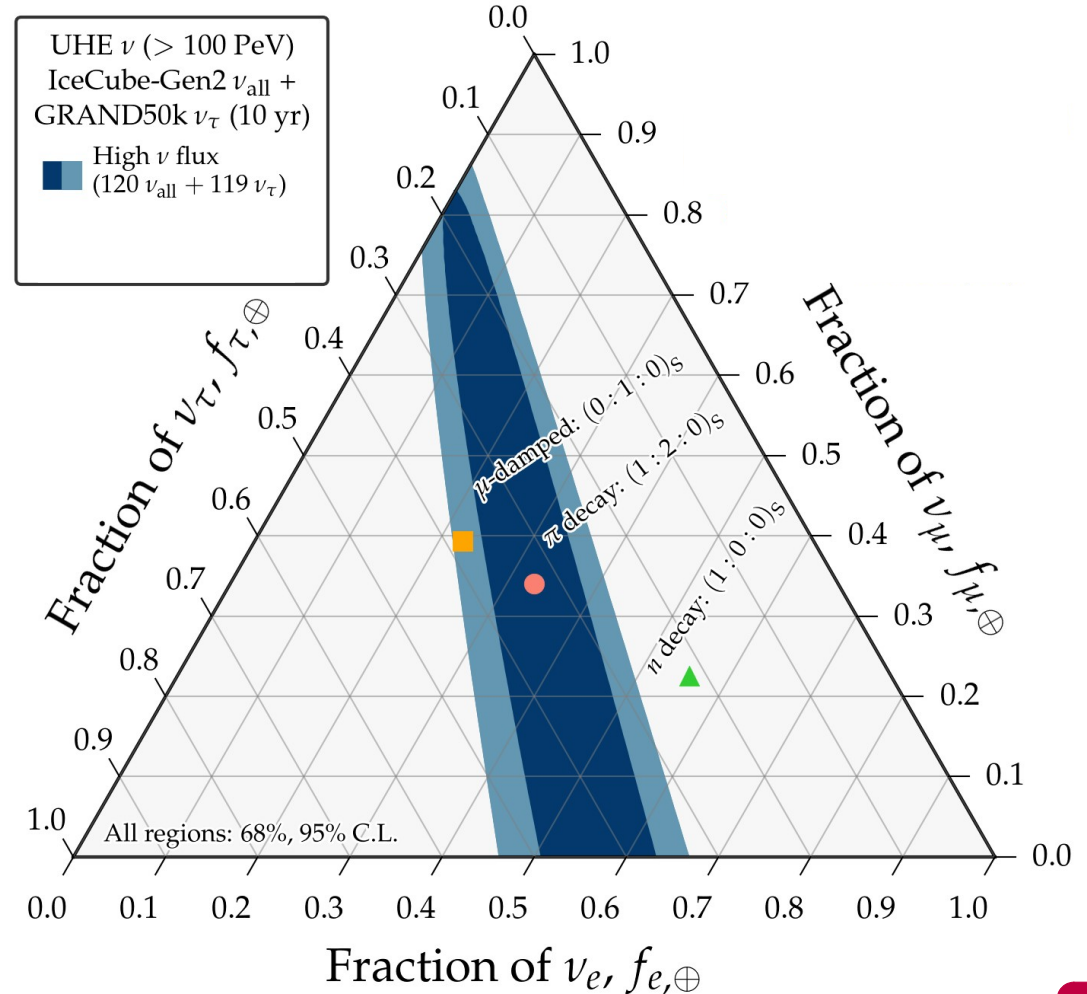


Flavor at ultra-high
energies (> 100 PeV)

Manufacturing UHE flavor sensitivity with two detectors

What if future UHE radio-detection neutrino telescopes cannot see flavor?

Then we combine two of detectors:

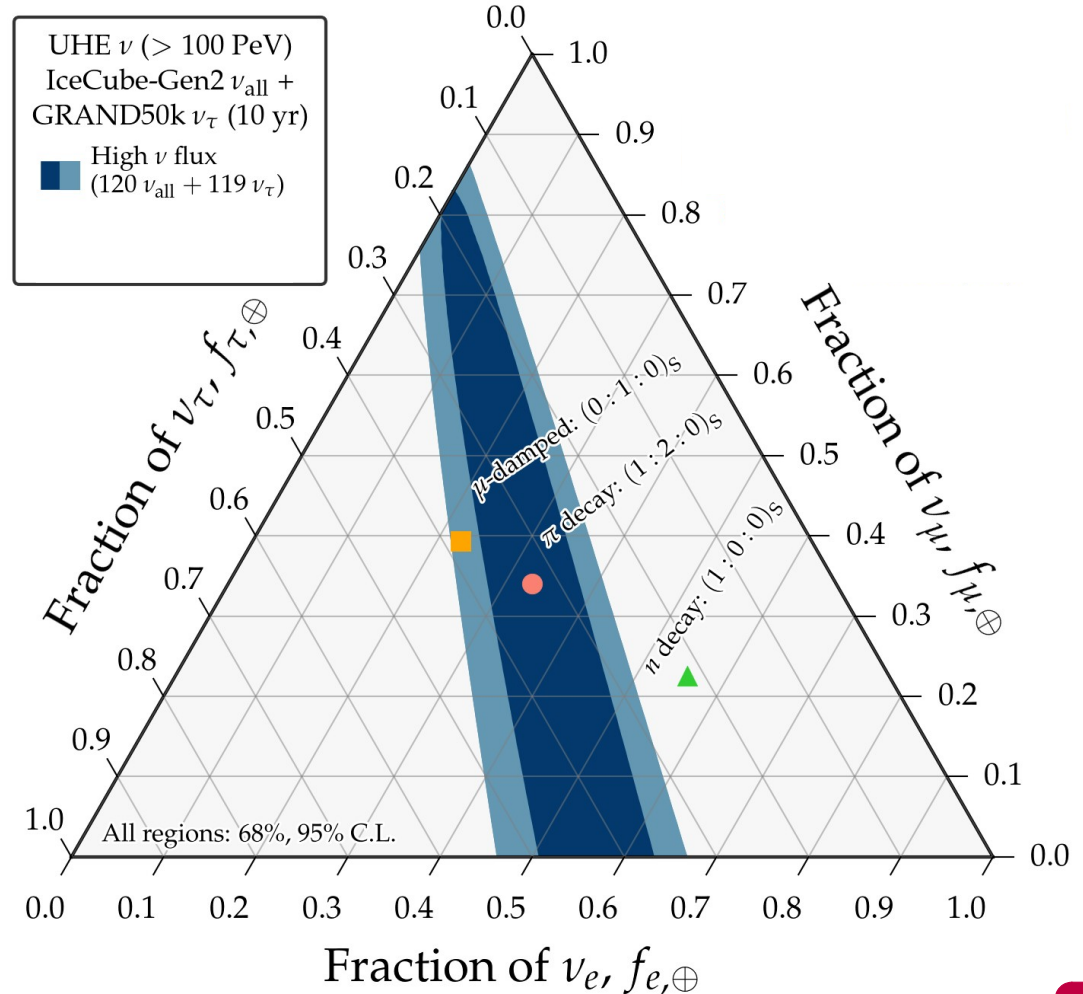


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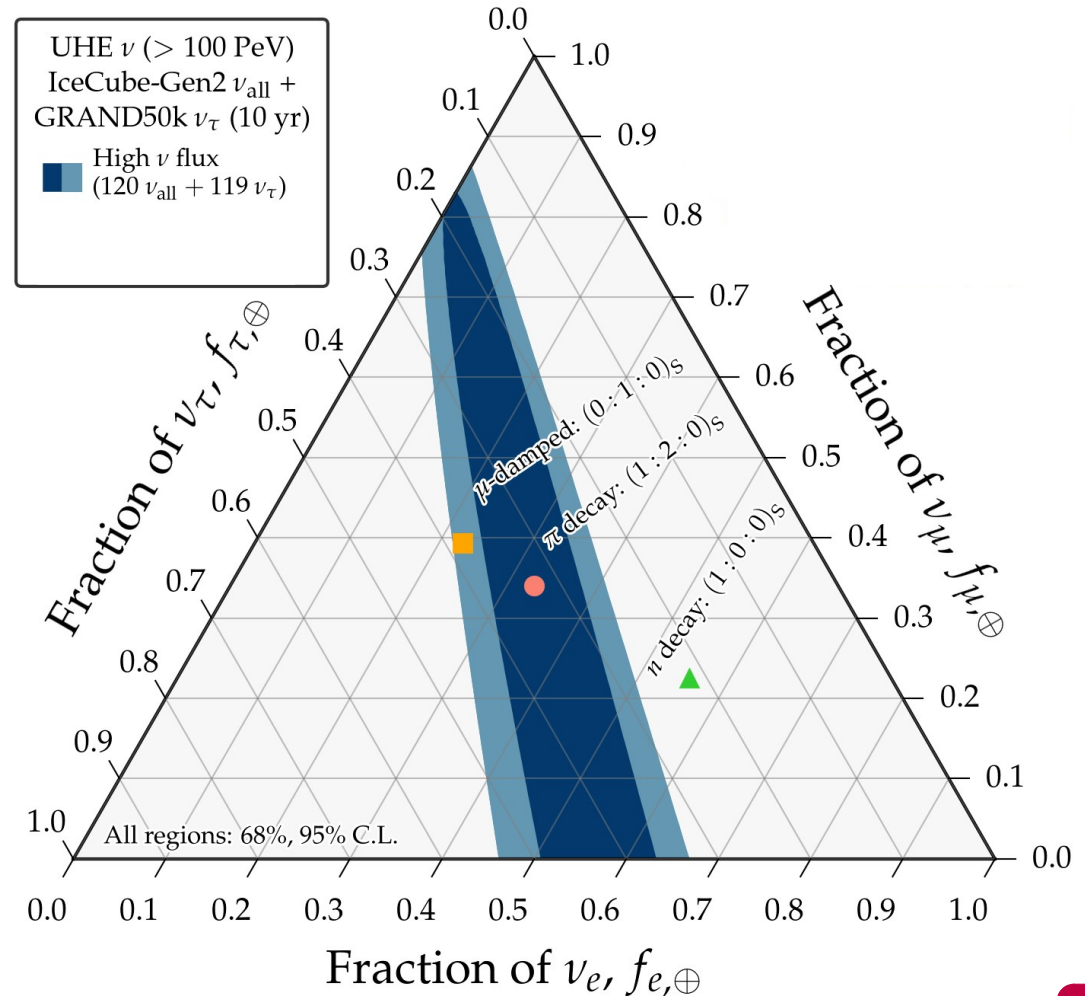
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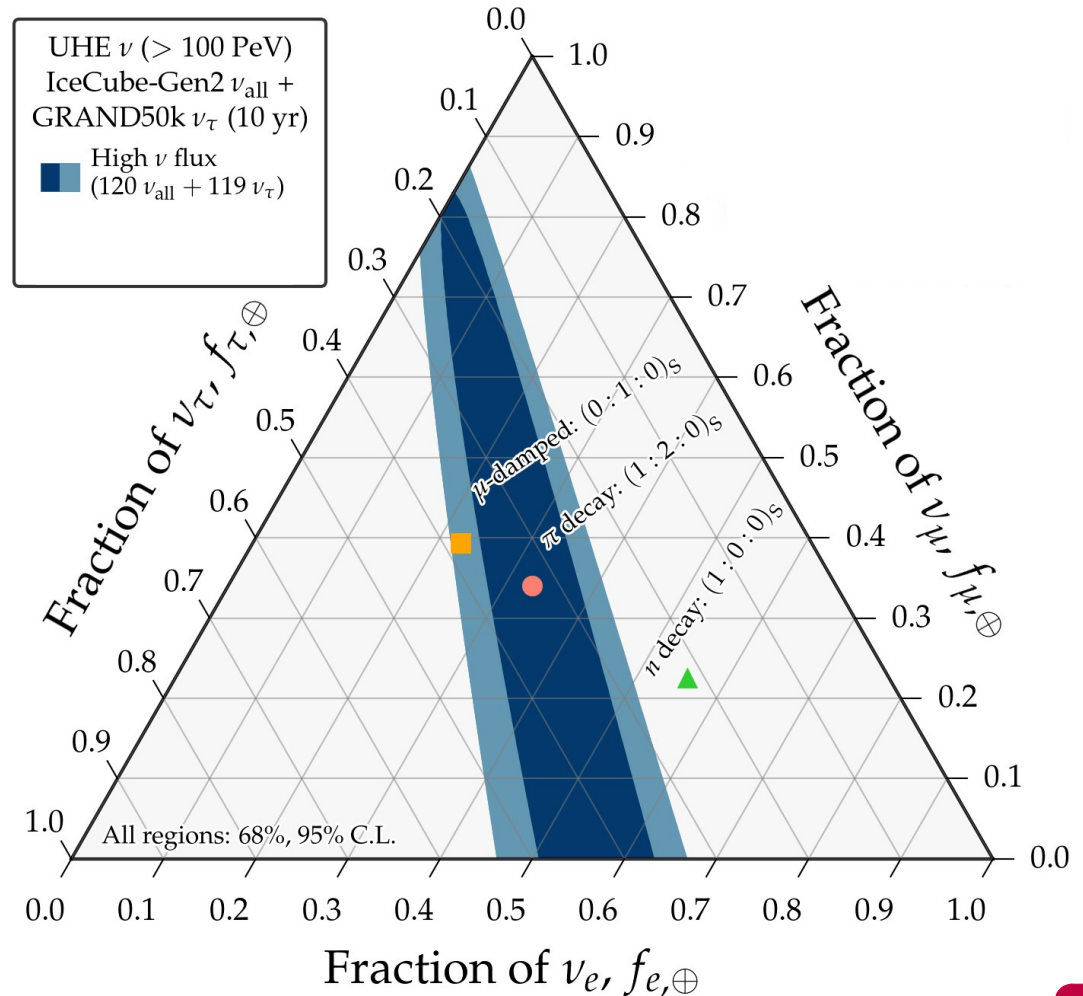
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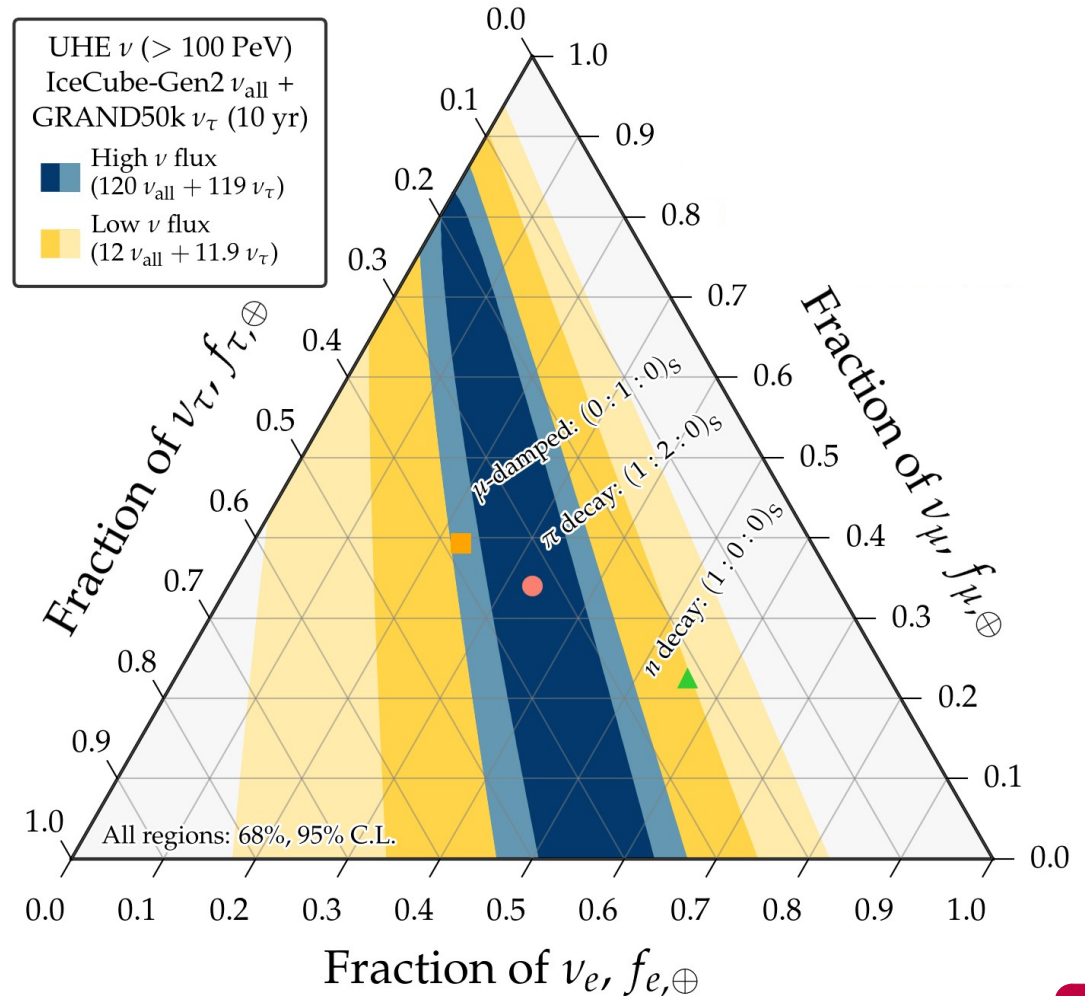
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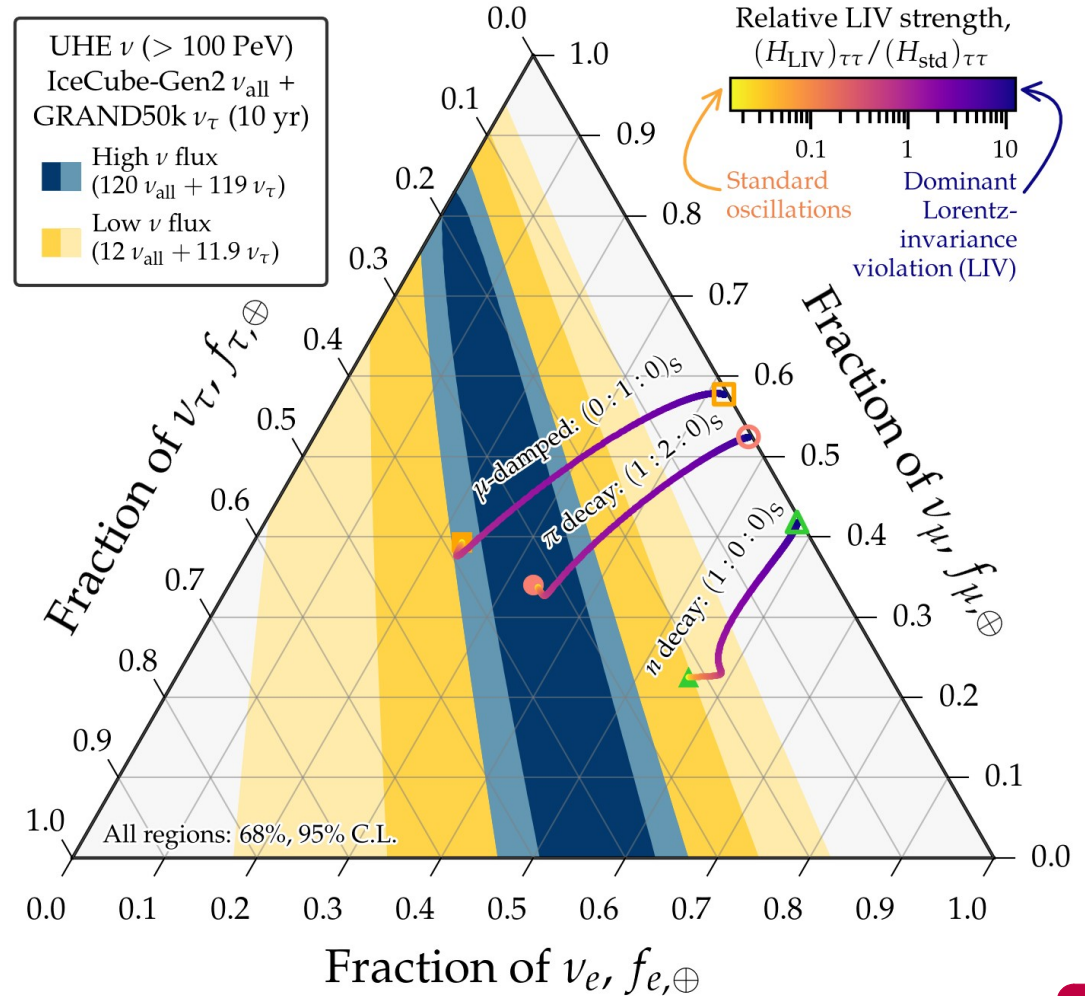
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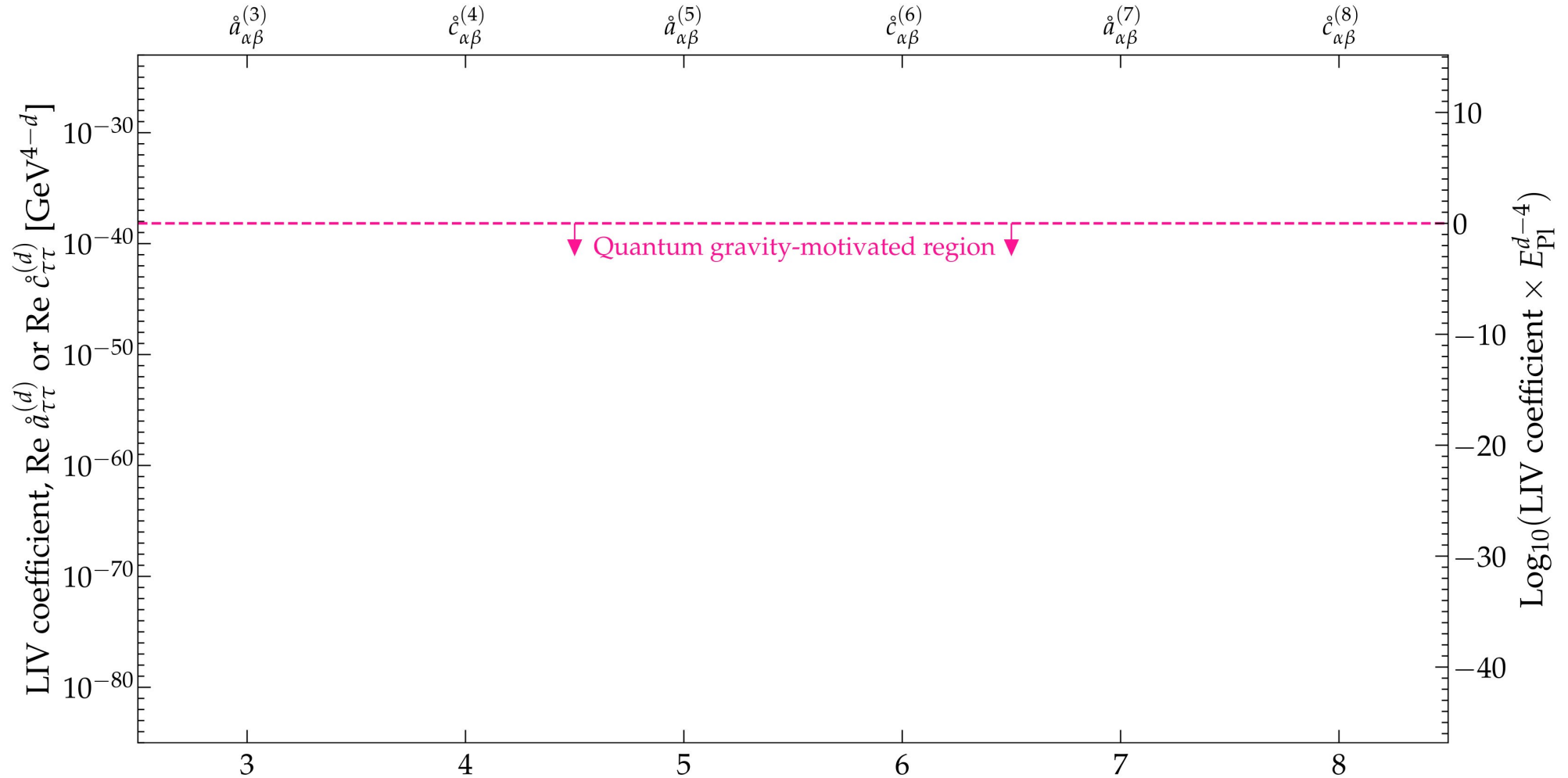
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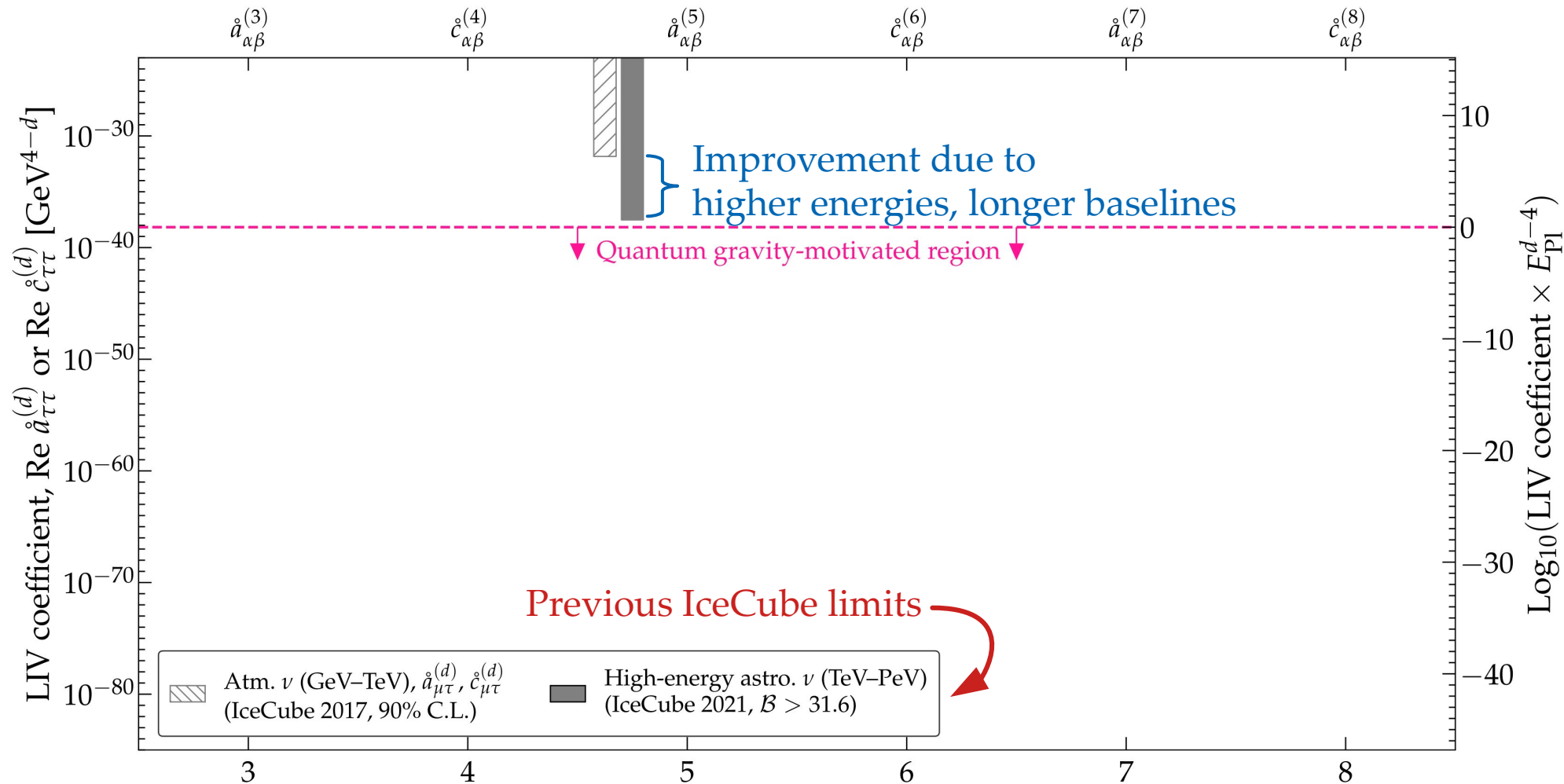
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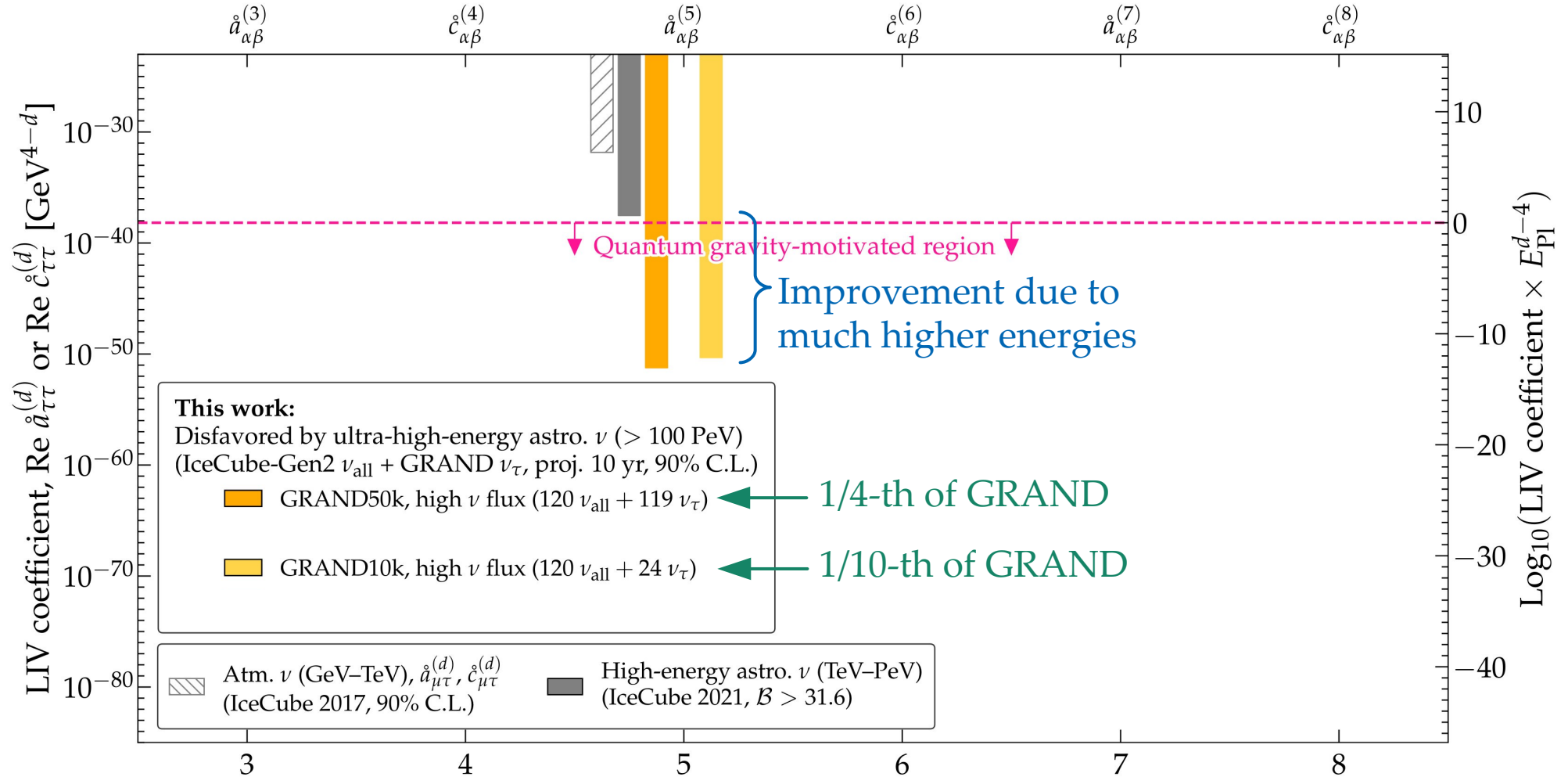
Lorentz-invariance violation at ultra-high energies



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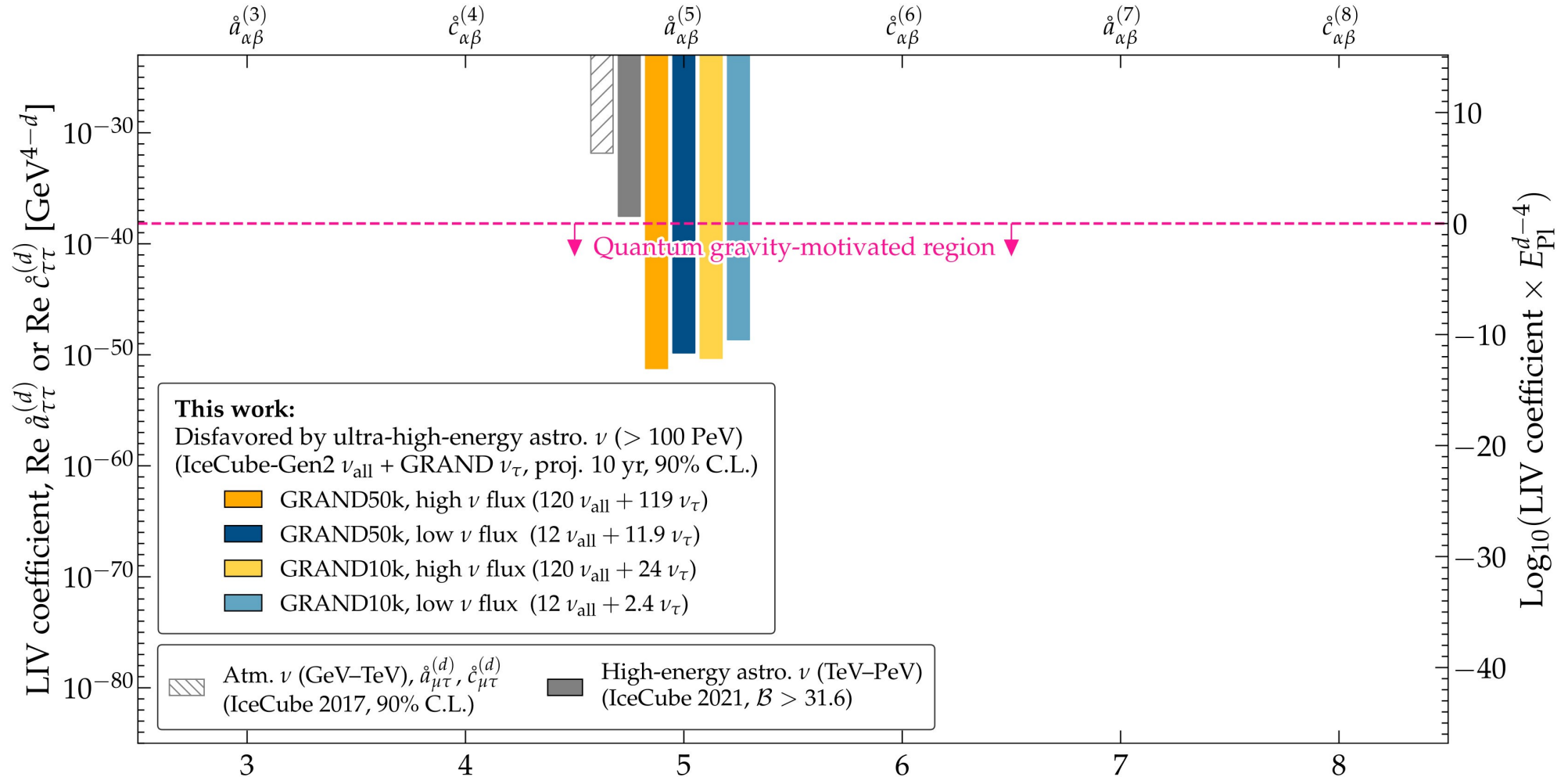


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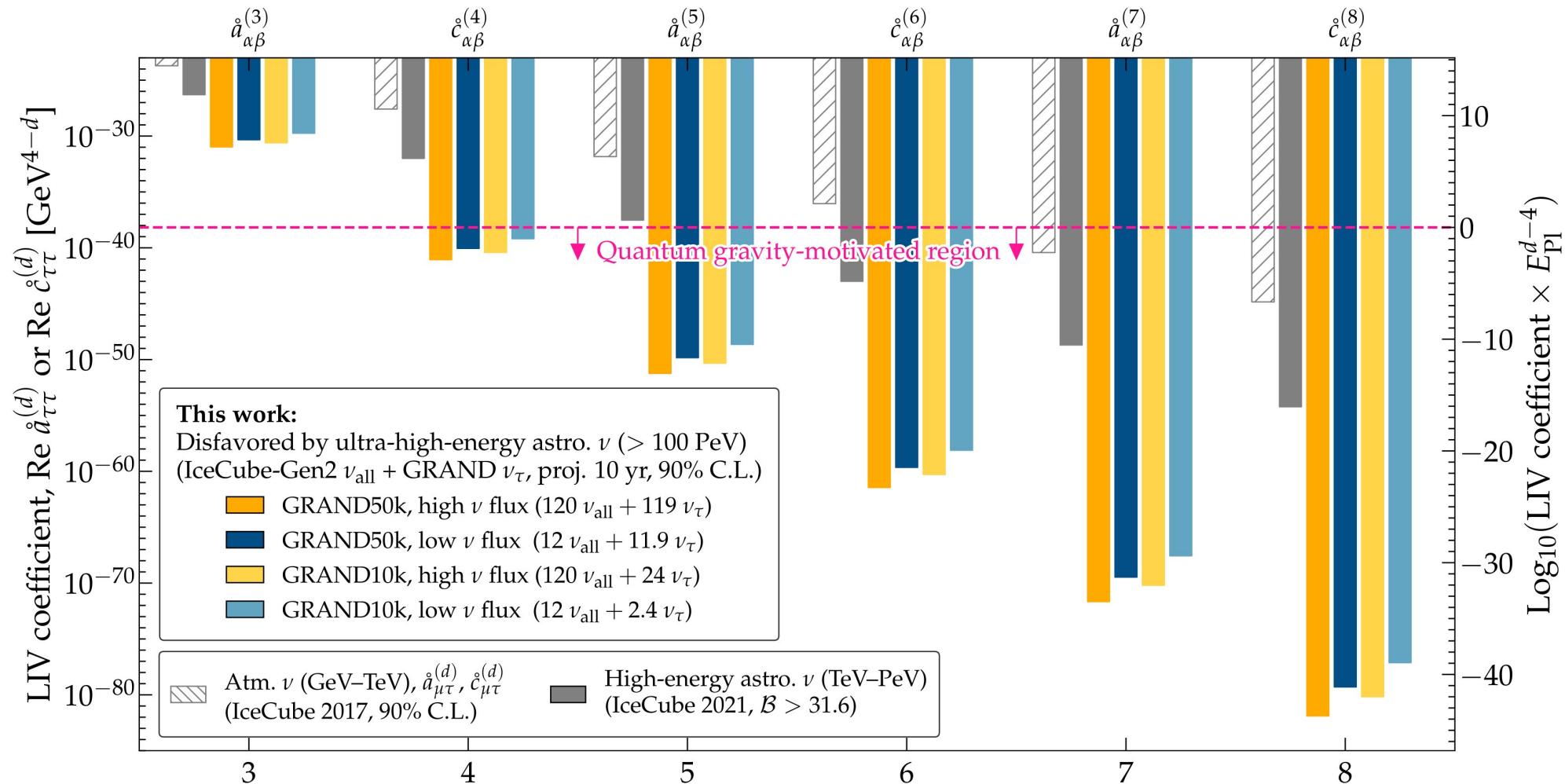


Dimension of Lorentz invariance-violation (LIV) operator, d

Lorentz-invariance violation at ultra-high energies

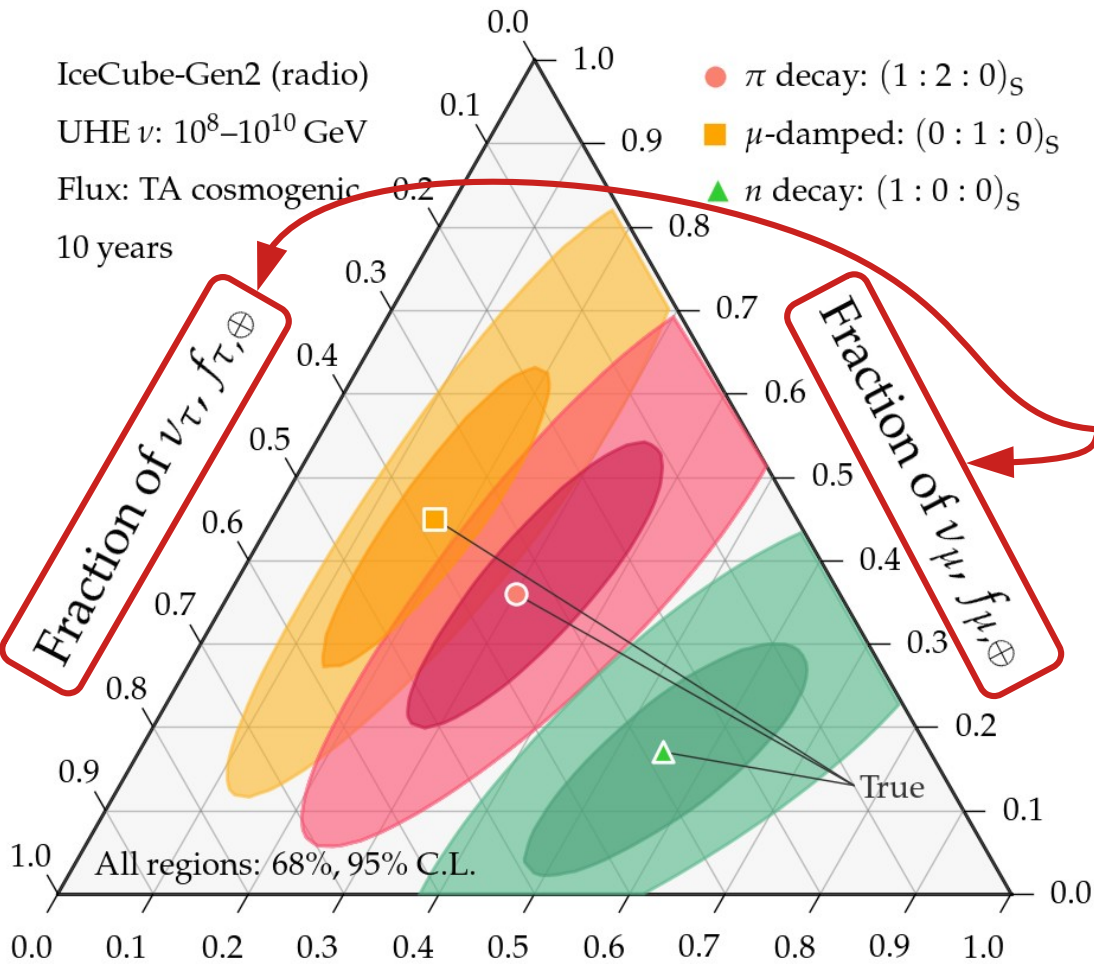


Lorentz-invariance violation at ultra-high energies



IceCube-Gen2 (radio) alone might measure flavor

Fraction of ν_e
Showers are elongated due to the LPM effect



Fraction of $\nu_\mu + \nu_\tau$
Secondary muons and tauons create showers that hit >1 radio station