Statistical methods with an application to cosmic rays

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December 18, 2012

Introduction

- We now have a working code to calculate:
  - UHECR (proton) propagation while interacting with the cosmological photon backgrounds (photohadronics + pair production)
  - cosmogenic neutrino flux from the $p\gamma$ interactions
- Extends the thoroughly verified NeuCosmA code
- Fast (not Monte Carlo) and easy to modify and add to

We want to use it to constrain models of UHECR production and propagation by comparing to experimental results

Kinematic proton transport equation

- Comoving number density of protons (GeV^{-1} cm^{-3}):

$$Y_p\left(E,z\right) = a^3\left(z\right) n_p\left(E,z\right) = n_p\left(E,z\right) / \left(1+z\right)^3 \ ,$$

with n_p the real number density

- Boltzmann transport equation (E: energy in source frame):

$$\dot{Y}_p = \underbrace{\partial_E\left(HEY_p\right)}_{\text{adiabatic losses}} + \underbrace{\partial_E\left(b_{e^+e^-}Y_p\right)}_{\text{pair production losses}} + \underbrace{\partial_E\left(b_{p\gamma}Y_p\right)}_{\text{photohadronic losses}} + \underbrace{\mathcal{L}_{\mathrm{CR}}}_{\text{CR injection from sources}}$$

Kinematic proton transport equation

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$$Y_p(E,z) = a^3(z)\, n_p(E,z) = n_p(E,z)/(1+z)^3 \ ,$$

  with n_p the real number density

- Boltzmann transport equation (E: energy in source frame):

$$\dot{Y}_p = \partial_E (HEY_p) + \partial_E (b_{e^+e^-} Y_p) + \partial_E (b_{p\gamma} Y_p) + \mathcal{L}_{\text{CR}}$$

- Cast into an equation in z by using $dz = -dt\,(1+z)\,H(z)$:

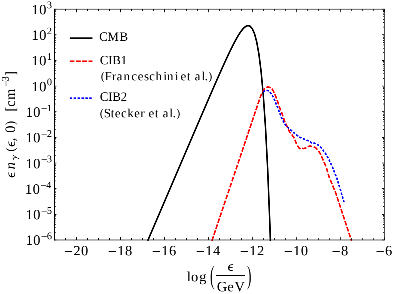
$$\begin{aligned}\partial_z Y(E,z) = \frac{-1}{(1+z)\,H(z)} \{ & \partial_E [H(z)\,EY(E,z)] \\ & + \partial_E [b_{e^+e^-}(E,z)\,Y(E,z)] + \partial_E [b_{p\gamma}(E,z)\,Y(E,z)] \\ & + \mathcal{L}_{\text{CR}}(E,z)\}\end{aligned}$$

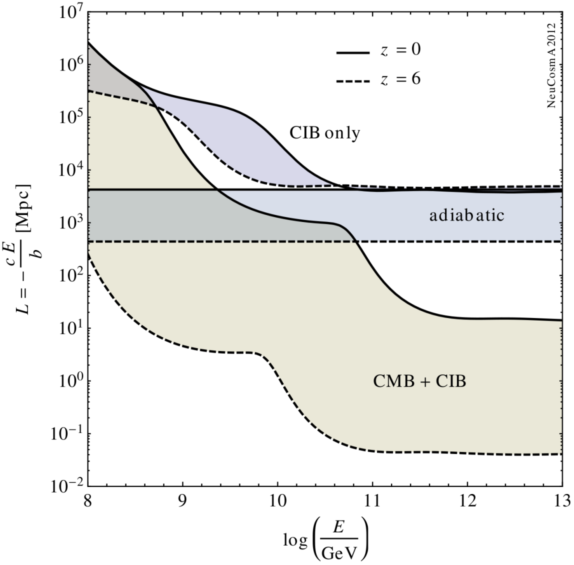
- Evolve from $z=6$ down to $z=0$

Energy loss rates

- Defined, in general, as $b \equiv dE/dt$ (GeV^{-1} cm^{-3})
- We consider two contributions:
  - Pair production: $p + \gamma_b \to e^+ + e^- + p$
  - Photohadronic processes: *e.g.*, $p + \gamma_b \to \Delta^+ \to \begin{cases} n + \pi^+ \\ p + \pi^0 \end{cases}$

- The photons come from the cosmological backgrounds:
  - Microwave background (CMB)
  - Infrared/visible background (CIB)
- Spectra evolve with z, *i.e.*, $n_\gamma(\epsilon, z)$




$z = 0$
$z = 6$
NeuCosmA 2012
CIB only
adiabatic
CMB + CIB
$L = -\frac{cE}{b}$ [Mpc]
$\log\left(\frac{E}{\mathrm{GeV}}\right)$
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
8
9
10
11
12
13

Proton injection rate

$$\mathcal{L}_{\text{CR}}(E, z) = \mathcal{H}(z)\, Q_{\text{CR}}(E, z)$$

Q_{CR}: injection rate at source (GeV^{-1} Mpc^{-3} s^{-1})
$\mathcal{H}$: cosmological evolution of sources (adimensional)

$$Q_{\text{CR}}(E, z) \propto E^{-\gamma} e^{-E/E_{p,\max}}$$

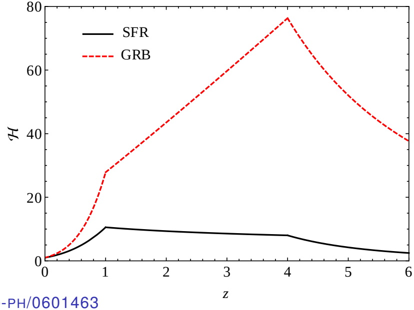
e.g., star formation rate (Hopkins & Beacom),

$$\mathcal{H}_{\text{SFR}}(z) = \begin{cases} (1+z)^{3.4} & , z < 1, \\ N_1 (1+z)^{-0.3} & , 1 \leq z < 4 \\ N_1 N_4 (1+z)^{-3.5} & , z \geq 4 \end{cases}$$

with $N_1 = 2^{3.7}$ and $N_4 = 5^{3.2}$ …

…or GRB rate (Kistler *et al.*),

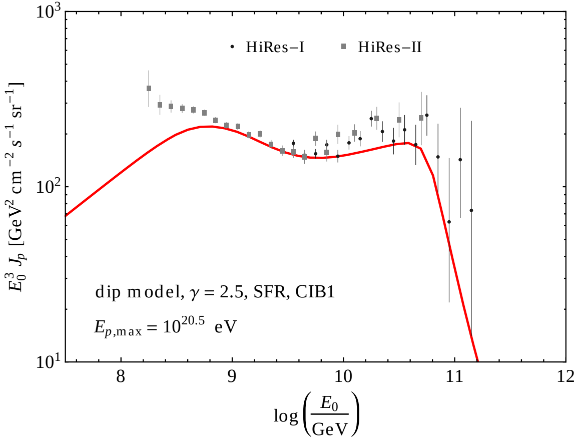
$$\mathcal{H}_{\text{GRB}}(z) = (1+z)^{1.4}\, \mathcal{H}_{\text{SFR}}(z)$$



A.M. Hopkins, J.F. Beacom, *Astrophys. J.* **651**, 142 (2006), astro-ph/0601463
M. Ahlers, M. González-García, F. Halzen, *Astropart. Phys.* **35**, 87 (2011), arXiv:1103.3421
M.D. Kistler *et al.*, *Astrophys. J.* **705**, L104 (2009), arXiv:0906.0590

Proton spectrum normalisation

Proton flux at Earth $J_p(E_0) \equiv (c/4\pi)\, n_p(E_0, 0)$ normalised to HiRes data (*Phys. Rev. Lett.* **100**, 101101 (2008), ASTRO-PH/0703099):



The electromagnetic energy density

$$p + \gamma_b \to \begin{cases} \pi^{\pm} + \ldots \to e^{\pm} + \ldots \\ p + \pi^0 \quad \to \gamma\gamma + \ldots \end{cases}$$

They interact with γ_b's to create e.m. cascades through

- pair production: $\gamma + \gamma_b \to e^+ + e^-$
- inverse Compton scattering: $e^{\pm} + \gamma_b \to e^{\pm} + \gamma$

Energy density (eV cm^{-3}) dumped into e.m. cascades:

$$\omega_{\text{cas}} = \int dt \int dE \frac{b_{\text{cas}}(E,z)}{(1+z)^4} n_p(E,z)$$

with the energy loss rate $b_{\text{cas}}(E,z) = b_{e^+e^-}(E,z) + \frac{5}{8} b_{p\gamma}(E,z)$

Fermi-LAT bound (V. Berezinksy *et al.*, *PLB* **695**, 13 (2011), arXiv:1003.1496):

$$\omega_{\text{cas}} \leq 5.8 \times 10^{-7} \text{ eV cm}^{-3}$$

The "guaranteed" neutrino flux

- UHECRs, the CMB, and CIB *have all* been observed
- $\therefore$ There is a "guaranteed" flux of UHE ν's coming from

$$p + \gamma \longrightarrow \Delta^{+}(1232) \longrightarrow n + \pi^{+}$$

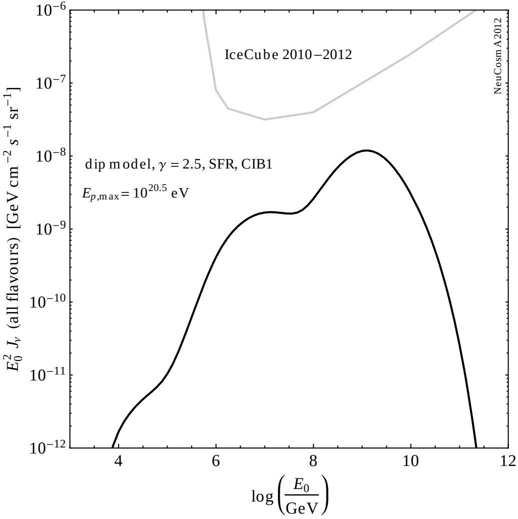
  and the decays

$$\pi^{+} \longrightarrow \mu^{+}\nu_{\mu} \longrightarrow \bar{\nu}_{\mu} e^{+} \nu_{e} \nu_{\mu}$$
$$n \longrightarrow p e^{-} \bar{\nu}_{e}$$

- Additional contributions (higher resonances, multi-pion production) implemented in NeuCosmA
- Our code calculates this cosmogenic, or GZK, ν flux given the proton injection at the sources

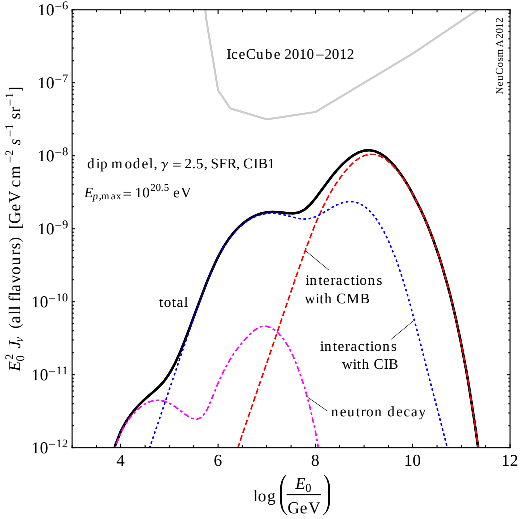
V. Berezinksy, G. Zatsepin, *Phys. Lett. B* **28**, 423 (1969)
F. Stecker, *Astrophys. J.* **228**, 919 (1979)

The cosmogenic ν flux at Earth



IceCube bound from A. Ishihara, talk at Neutrino 2012

The cosmogenic ν flux at Earth



CR parameter fitting – what has been done so far?

- Injection and propagation models constrained using
  - HiRes UHECR data; and
  - Fermi-LAT GeV-TeV photon data
- Two analyses by Ahlers *et al.*: 1005.2620, 1103.3421

$$\mathcal{L}_{\text{CR}}(E, z) = \mathcal{H}_{\text{CR}}(z)\, Q_{\text{CR}}(E)$$

$$\mathcal{H}_{\text{CR}}(z) = (1+z)^{n}\, \mathcal{H}_{\text{SFR}}(z) \;, \quad Q_{\text{CR}}(E) = \mathcal{N} E^{-\gamma} e^{-E/E_{p,\max}}$$

- Four free parameters varied:
  1. proton injection spectral index, γ
  2. normalisation of the proton injection spectrum, $\mathcal{N}$
  3. $\left\{ \begin{array}{l} \text{evolution of CR sources, } n \text{ (in 1005.2620); or} \\ \text{maximum proton energy, } E_{p,\max} \text{ (in 1103.3421)} \end{array} \right.$
  4. uncertainty in the energy scale, δ

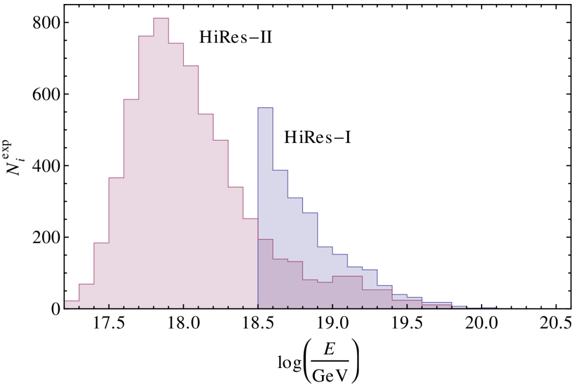
Number of expected CR events in the i-th energy bin:

$$N_i^{\text{th}}(n, \gamma, \mathcal{N}, \delta) = \underbrace{A_i}_{\text{acceptance (cm}^2\text{ s sr)}} \int_{E_i(1-\delta)-\underbrace{\Delta_i}_{\text{bin width (GeV)}}/2}^{E_i(1+\delta)+\Delta_i/2} \underbrace{J_p^{\mathcal{N},n,\gamma}(E)}_{\text{predicted } p \text{ flux at Earth (GeV}^{-1}\text{ cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1})} dE$$

Probability distribution in the i-th bin assumed Poisson:

$$P_i(n, \gamma, \mathcal{N}, \delta) = \frac{\left(N_i^{\text{exp}}\right)^{N_i^{\text{th}}(n,\gamma,\mathcal{N},\delta)} e^{-N_i^{\text{exp}}}}{N_i^{\text{th}}(n, \gamma, \mathcal{N}, \delta)!}$$

CR parameter fitting – what has been done so far?



Likelihood to observe the whole HiRes spectrum:

$$P_{\vec{N}^{\text{exp}}}(n, \gamma, \mathcal{N}, \delta) = \prod_{k=1}^{\text{bins}} P_i(n, \gamma, \mathcal{N}, \delta)$$

Detour: Bayes and likelihood parameter estimation

Problem: given some data x and a theoretical model that outputs a quantity θ based upon it, we want to determine the probability that θ has a certain value given x

Recall Bayes' theorem:

$$p(\theta|x) = \frac{p(x|\theta)\,p(\theta)}{p(x)} \dashrightarrow \text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Here:

- likelihood: calculated from model (for us, $\mathcal{L} \equiv P_{\vec{N}^{\text{exp}}}$)
- prior: user input (for us, HiRes CR spectrum and Fermi-LAT e.m. energy density bound)

Since x is fixed, $p(x)$ is just a normalisation (we can discard it)

We want to tune our model so that it reproduces the data with maximum probability, *i.e.*,

$$\theta_{\text{best}} = \arg\max_{\theta} p(\theta|x) \ ,$$

or, equivalently,

$$\theta_{\text{best}} = \arg\max_{\theta} \left(\log \mathcal{L} + \log p(\theta)\right)$$

Fitting δ and $\mathcal{N}$

The best-fit values of δ and $\mathcal{N}$ are found by maximising

$$P_{\text{exp}}(n, \gamma) = \max_{\delta,\mathcal{N}} P_{\vec{N}\text{exp}}(n, \gamma, \mathcal{N}, \delta)$$

subject to the following priors:

- For δ: a top hat of width σ_{E_s}
- For $\mathcal{N}$:
  - Bound on the e.m. energy density from Fermi-LAT:

    $$w_{\text{cas}}(\mathcal{N}, \gamma, n) \leq 5.8 \times 10^{-7} \text{ eV cm}^{-3}$$

  - Proton flux $J_p^{\mathcal{N},\gamma,n}$ should not exceed HiRes by $> 3\sigma$

We obtain δ_{best} and $\mathcal{N}_{\text{best}}$ compliant with HiRes and Fermi-LAT

Goodness of fit

The model is compatible with the experimental results at a given goodness of fit (GOF = 64%, 95%, 99%, etc.) if

$$\sum_{P_{\vec{k}} > P_{\text{exp}}(n,\gamma)} P_{\vec{k}}\left(n, \gamma, \mathcal{N}_{\text{best}}, \delta_{\text{best}}\right) \leq \text{ GOF}$$

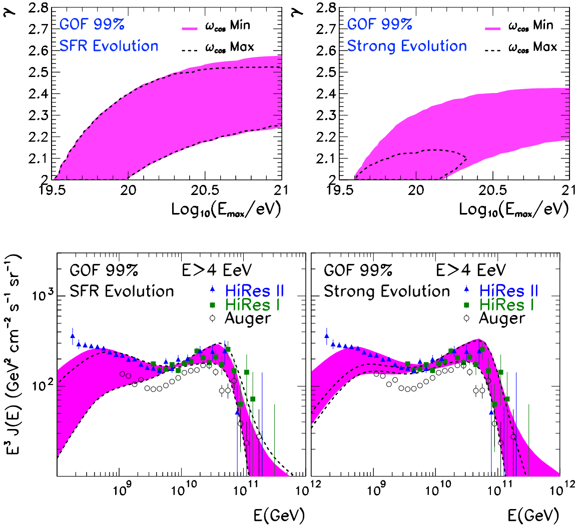
How to compute this?

1. Generate a large number of replica experiments according to the distribution $P_{\vec{k}}\left(n, \gamma, \mathcal{N}_{\text{best}}, \delta_{\text{best}}\right)$
2. Impose that a fraction $F \leq$ GOF of those satisfies $P_{\vec{k}}\left(n, \gamma, \mathcal{N}_{\text{best}}, \delta_{\text{best}}\right) > P_{\text{exp}}\left(n, \gamma\right)$

We obtain n_{best} and γ_{best}, and their confidence intervals

CR parameter fitting – what has been done so far?

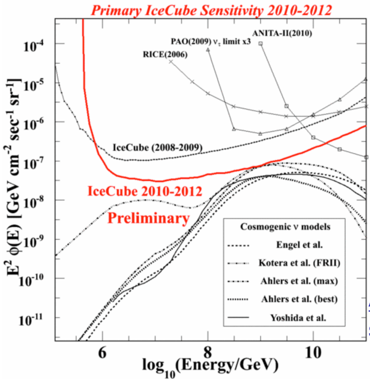
Goodness of fit



M. Ahlers, M. C. González-García, F. Halzen, *Astropart. Phys.* **35**, 87 (2011), arXiv:1103.3421

CR parameter fitting – what we want to do

Observations are starting to constrain the cosmogenic ν flux



A. Kappes,
Talk at 23rd European Cosmic Ray Symposium (2012), arXiv:1209.5855

Let us take these into account to constrain UHECR models

An updated χ^2

It is clearer to introduce the IceCube bound in a χ^2:

$$\chi^2(n,\gamma,\mathcal{N},E_{p,\max},\delta) = \sum_i^{\text{HiRes bins}} \frac{\left(N_i^{\text{th}}(n,\gamma,\mathcal{N},E_{p,\max},\delta) - N_i^{\text{exp}}\right)^2}{N_i^{\text{exp}}} + \frac{\left(w_{\text{cas}}(n,\gamma,\mathcal{N},E_{p,\max},\delta) - w_{\text{cas}}^{\text{Fermi}}\right)^2}{\sigma_w^2} + \sum_i^{\text{IC bins}} \frac{\left(N_{\nu,i}^{\text{th}}(n,\gamma,\mathcal{N},E_{p,\max},\delta) - N_{\nu,i}^{\text{exp}}\right)^2}{N_{\nu,i}^{\text{exp}}}$$

$$N_{\nu,i}^{\text{exp}} = 2\pi \int_{E_i-\Delta_i/2}^{E_i+\Delta_i/2} A_{\text{eff}}(E)\, J_\nu^{\text{IC-bound}}(E)\, dE$$

bin size: one decade in E

IceCube effective area (cm^2 s)

IceCube upper bound (GeV^{-1} cm^2 s^{-1} sr^{-1})

Alternatively, use the IceCube-40 integrated upper bound:

- < 2.35 total integrated events (90% C.L.)
- energy range $2 \times 10^6 \leq E/\text{GeV} \leq 6.33 \times 10^9$
- all-flavour bound

$$\chi^2\left(n, \gamma, \mathcal{N}, E_{p,\max}, \delta\right) = \ldots + \frac{\left(N_\nu^{\text{th}}\left(n, \gamma, \mathcal{N}, E_{p,\max}, \delta\right) - 2.35\right)^2}{2.35}$$

IceCube Collaboration, Phys. Rev. D **83**, 092003 (2011), arXiv:1103.4250

We can also use the IceCube-22 cascade bound
– 14 events with 24 TeV $\leq E \leq$ 6.6 PeV

IceCube Collaboration, Phys. Rev. D **84**, 072001 (2011), arXiv:1101.1692

Outline of strategy

Ranges of values (tentative):

- $0 \leq n \leq 4$
- $1.8 \leq \gamma \leq 3$
- $42 \leq \log \left(\mathcal{N} \left[\text{GeV}^{-1}\text{Mpc}^{-3}\text{s}^{-1} \right] \right) \leq 46$
- $10^{19} \leq E_{p,\max}/\text{GeV} \leq 10^{22}$
- δ: same as in Ahlers *et al.*

To start: fix two parameters (*e.g.*, $n = 0$, δ to Ahlers *et al.*), and generate proton and neutrino fluxes $J_{p/\nu}$ varying γ, $\mathcal{N}$, $E_{p,\max}$

– marginalise over one of them at a time and produce confidence regions for the remaining two

Backup slides

Interaction with the photon backgrounds

- Energy loss rate (GeV s^{-1}):

$$b\left(E\right) \equiv \frac{dE}{dt}$$

- For pair production $p\gamma \longrightarrow pe^{+}e^{-}$:

$$b_{e^{+}e^{-}}\left(E,z\right) = -\alpha r_0^2 \left(m_e c^2\right)^2 c \int_2^{\infty} d\xi n_\gamma \left(\frac{\xi m_e c^2}{2\gamma}, z\right) \frac{\phi\left(\xi\right)}{\xi^2}$$

  - n_γ: isotropic photon background (GeV^{-1} cm^{-3})
  - ξ: photon energy in units of $m_e c^2$
  - proton energy: $E = \gamma m_p c^2$ $(\gamma \gg 1)$
  - $\phi\left(\xi\right)$: (tabulated) integral in energy of outgoing e^{-}

G. Blumenthal, *Phys. Rev.* **D 1**, 1596 (1970)
H. Bethe, W. Heitler, *Proc. Roy. Soc.* **A146**, 83 (1934)

Interaction with the photon backgrounds (cont.)

Photohadronic interactions – $p\gamma$ interaction rate (s^{-1} per particle):

$$\Gamma_{p\gamma\to p'b}\left(E,z\right) = \frac{1}{2}\frac{m_p^2}{E^2}\int_{\frac{\epsilon_{\text{th}} m_p}{2E}}^{\infty} d\epsilon \frac{n_\gamma\left(\epsilon,z\right)}{\epsilon^2}\int_{\epsilon_{\text{th}}}^{2E\epsilon/m_p} d\epsilon_r \epsilon_r \sigma_{p\gamma\to p'b}^{\text{tot}}\left(\epsilon_r\right)$$

1. For given values of E and z, NeuCosmA calculates the cooling rate $t_{p\gamma}^{-1} \equiv -\left(1/E\right)b_{p\gamma}$ (s^{-1}) as

$$t_{p\gamma}^{-1}\left(E,z\right) = \sum_i^{\text{all channels}} \Gamma_{p\to p}^i\left(E,z\right)K^i ,$$

   with $K^i E$ the loss of energy per interaction
2. From this, we calculate back $b_{p\gamma}$ (GeV s^{-1}) ...
3. ... and the corresponding energy-loss term in the transport equation, $\partial_E\left(b_{p\gamma}Y_p\right)$.

S. Hümmer, M. Rüger, F. Spanier, W. Winter, *Astrophys. J.* **721**, 630 (2010), arXiv:1002.1310

Photon background redshift scaling – CMB

- Local CMB spectrum (GeV^{-1} cm^{-3}):

$$n_\gamma^{\text{CMB}}(\epsilon, z=0) = \frac{1}{\pi^2}\frac{1}{(\hbar c)^3}\frac{\epsilon^2}{e^{\epsilon/(k_B T)}-1}$$

- Following $\dot{Y}_\gamma = \partial_E(HEY_\gamma)$, with $Y_\gamma \propto a^3 n_\gamma$, it scales as

$$n_\gamma^{\text{CMB}}(\epsilon, z) = (1+z)^3\, n_\gamma^{\text{CMB}}(\epsilon/(1+z), z=0)$$

- This induces the scaling of the energy loss rate,

$$b(E,z) = (1+z)^2\, b((1+z)E, 0)$$

- This adiabatic scaling is exact for the CMB – not so for the CIB

Photon background redshift scaling – CIB

- The CIB spectrum has been measured up to $z = 2$
- Redshift dependence:

$$n_\gamma^{\text{CIB}}\left(\epsilon\left(1+z\right), z\right) = \left(1+z\right)^2 \int_z^\infty dz' \frac{1}{H\left(z'\right)} \mathcal{L}_{\text{CIB}}\left(\epsilon\left(1+z'\right), z'\right)$$

  $\mathcal{L}_{\text{CIB}}$: comoving injection rate of IR photons (depends on source evolution)

- Following [M. Ahlers, L.A. Anchordoqui, S. Sarkar, *PRD* **79**, 083009 (2009), arXiv: 0902.3993], we used

$$n_\gamma^{\text{CIB}}\left(\epsilon, z\right) \simeq \frac{1}{1+z} \frac{N_{\text{CIB}}\left(z\right)}{N_{\text{CIB}}\left(0\right)} n_\gamma^{\text{CIB}}\left(\epsilon / \left(1+z\right), 0\right)$$

- Other CIB scalings possible
  – the code can easily accommodate for them