Efficient computation of photohadronic interactions – an application to UHECR propagation and cosmogenic neutrinos

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Introduction

- Walter told you about NeuCosmA, which incorporates fast code to calculate photohadronic interactions
- It has been successfully used to calculate the ν flux from GRBs
- We have reused the code to calculate:
  - UHECR (proton) propagation while interacting with the cosmological photon backgrounds (photohadronics + pair production)
  - cosmogenic neutrino flux from the $p\gamma$ interactions
- Extends the thoroughly verified NeuCosmA code
- Does not use Monte Carlo methods for the propagation
- Written in C – a single external library needed: `gsl`
- Fast and easy to modify and add to

Boltzmann equation

- Comoving number density of protons (GeV^{-1} cm^{-3}):

$$Y_p\left(E,z\right) = a^3\left(z\right) n_p\left(E,z\right) = n_p\left(E,z\right) / \left(1+z\right)^3 \quad ,$$

with n_p the real number density

- Transport equation (E: energy in source frame):

$$\dot{Y}_p = \underbrace{\partial_E\left(HEY_p\right)}_{\text{adiabatic losses}} + \underbrace{\partial_E\left(b_{e^+e^-}Y_p\right)}_{\text{pair production losses}} + \underbrace{\partial_E\left(b_{p\gamma}Y_p\right)}_{\text{photohadronic losses}} + \underbrace{\mathcal{L}_{\text{CR}}}_{\text{CR injection from sources}}$$

Boltzmann equation

- Comoving number density of protons (GeV^{-1} cm^{-3}):

$$Y_p(E,z) = a^3(z)\, n_p(E,z) = n_p(E,z) / (1+z)^3 \ ,$$

with n_p the real number density

- Transport equation (E: energy in source frame):

$$\dot{Y}_p = \partial_E (HEY_p) + \partial_E (b_{e^+e^-} Y_p) + \partial_E (b_{p\gamma} Y_p) + \mathcal{L}_{\mathrm{CR}}$$

- Cast into an equation in z by using $dz = -dt\,(1+z)\,H(z)$:

$$\partial_z Y(E,z) = \frac{-1}{(1+z)\,H(z)} \{\partial_E [H(z)\, EY(E,z)] + \partial_E [b_{e^+e^-}(E,z)\, Y(E,z)] + \partial_E [b_{p\gamma}(E,z)\, Y(E,z)] + \mathcal{L}_{\mathrm{CR}}(E,z)\}$$

Interaction with the photon backgrounds

- Energy loss rate (GeV s^{-1}):

$$b(E) \equiv \frac{dE}{dt}$$

- For pair production $p\gamma \longrightarrow pe^+e^-$:

$$b_{e^+e^-}(E, z) = -\alpha r_0^2 \left(m_e c^2\right)^2 c \int_2^\infty d\xi n_\gamma \left(\frac{\xi m_e c^2}{2\gamma}, z\right) \frac{\phi(\xi)}{\xi^2}$$

  - n_γ: isotropic photon background (GeV^{-1} cm^{-3})
  - ξ: photon energy in units of $m_e c^2$
  - proton energy: $E = \gamma m_p c^2$ ($\gamma \gg 1$)
  - $\phi(\xi)$: (tabulated) integral in energy of outgoing e^-

G. Blumenthal, *Phys. Rev.* **D 1**, 1596 (1970)
H. Bethe, W. Heitler, *Proc. Roy. Soc.* **A146**, 83 (1934)

Interaction with the photon backgrounds (cont.)

Photohadronic interactions – $p\gamma$ interaction rate (s^{-1} per particle):

$$\Gamma_{p\gamma\to p'b}(E,z) = \frac{1}{2}\frac{m_p^2}{E^2}\int_{\frac{\epsilon_{\text{th}} m_p}{2E}}^{\infty} d\epsilon \frac{n_\gamma(\epsilon,z)}{\epsilon^2}\int_{\epsilon_{\text{th}}}^{2E\epsilon/m_p} d\epsilon_r \epsilon_r \sigma_{p\gamma\to p'b}^{\text{tot}}(\epsilon_r)$$

1. For given values of E and z, NeuCosmA calculates the cooling rate $t_{p\gamma}^{-1} \equiv -(1/E)\, b_{p\gamma}$ (s^{-1}) as

$$t_{p\gamma}^{-1}(E,z) = \sum_i^{\text{all channels}} \Gamma_{p\to p}^i(E,z)\, K^i \ ,$$

   with $K^i E$ the loss of energy per interaction
2. From this, we calculate back $b_{p\gamma}$ (GeV s^{-1}) ...
3. ...and the corresponding energy-loss term in the transport equation, $\partial_E(b_{p\gamma} Y_p)$.

S. Hümmer, M. Rüger, F. Spanier, W. Winter, *Astrophys. J.* **721**, 630 (2010), arXiv:1002.1310

Photon background redshift scaling – CMB

- Local CMB spectrum (GeV^{-1} cm^{-3}):

$$n_\gamma^{\text{CMB}}(\epsilon, z=0) = \frac{1}{\pi^2}\frac{1}{(\hbar c)^3}\frac{\epsilon^2}{e^{\epsilon/(k_B T)}-1}$$

- Following $\dot{Y}_\gamma = \partial_E(HEY_\gamma)$, with $Y_\gamma \propto a^3 n_\gamma$, it scales as

$$n_\gamma^{\text{CMB}}(\epsilon, z) = (1+z)^3\, n_\gamma^{\text{CMB}}(\epsilon/(1+z), z=0)$$

- This induces the scaling of the energy loss rate,

$$b(E,z) = (1+z)^2\, b((1+z)E, 0)$$

- This adiabatic scaling is exact for the CMB – not so for the CIB

Photon background redshift scaling – CIB

- The CIB spectrum has been measured up to $z = 2$
- Redshift dependence:

$$n_\gamma^{\text{CIB}}\left(\epsilon\left(1+z\right),z\right) = \left(1+z\right)^2 \int_z^\infty dz' \frac{1}{H\left(z'\right)} \mathcal{L}_{\text{CIB}}\left(\epsilon\left(1+z'\right),z'\right)$$

  $\mathcal{L}_{\text{CIB}}$: comoving injection rate of IR photons (depends on source evolution – *more on this later*)

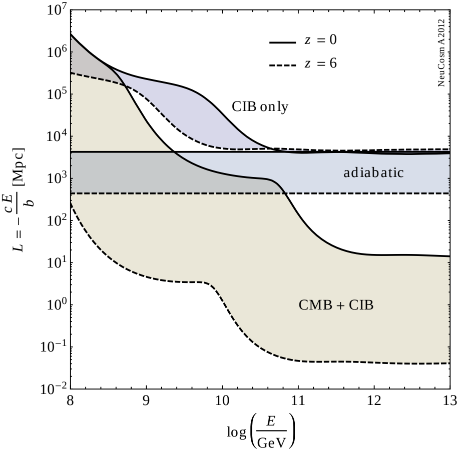
- Following [M. Ahlers, L.A. Anchordoqui, S. Sarkar, *PRD* **79**, 083009 (2009), arXiv: 0902.3993], we used

$$n_\gamma^{\text{CIB}}\left(\epsilon,z\right) \simeq \frac{1}{1+z} \frac{N_{\text{CIB}}\left(z\right)}{N_{\text{CIB}}\left(0\right)} n_\gamma^{\text{CIB}}\left(\epsilon/\left(1+z\right),0\right)$$

- Other CIB scalings possible
  – the code can easily accommodate for them

Interaction lengths

Note that $L_{\text{CIB}} \gg L_{\text{CMB}}$:



Matches, e.g., H. Takami, K. Murase, S. Nagataki, K. Sato, *Astropart. Phys.* **31**, 201 (2009), arXiv:0704.0979

Proton injection rate

$$\mathcal{L}_{\mathrm{CR}}(E,z) = \mathcal{H}(z)\, Q_{\mathrm{CR}}(E,z)$$

Q_{CR}: injection rate at source (GeV^{-1} Mpc^{-3} s^{-1})
$\mathcal{H}$: cosmological evolution of sources (adimensional)

$$Q_{\mathrm{CR}}(E,z) \propto E^{-\gamma} e^{-E/E_{p,\max}}$$

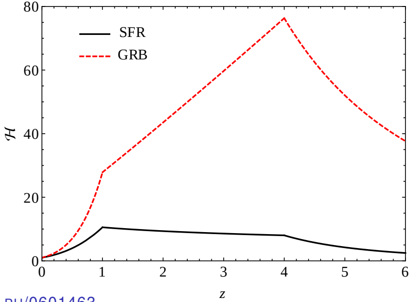
e.g., star formation rate (HOPKINS & BEACOM),

$$\mathcal{H}_{\mathrm{SFR}}(z) = \begin{cases} (1+z)^{3.4} & , z < 1, \\ N_1 (1+z)^{-0.3} & , 1 \leq z < 4 \\ N_1 N_4 (1+z)^{-3.5} & , z \geq 4 \end{cases}$$

with $N_1 = 2^{3.7}$ and $N_4 = 5^{3.2}$ …

…or GRB rate (KISTLER *et al.*),

$$\mathcal{H}_{\mathrm{GRB}}(z) = (1+z)^{1.4}\, \mathcal{H}_{\mathrm{SFR}}(z)$$



A.M. HOPKINS, J.F. BEACOM, *Astrophys. J.* **651**, 142 (2006), ASTRO-PH/0601463
M. AHLERS, M. GONZÁLEZ-GARCÍA, F. HALZEN, *Astropart. Phys.* **35**, 87 (2011), ARXIV:1103.3421
M.D. KISTLER *et al.*, *Astrophys. J.* **705**, L104 (2009), ARXIV:0906.0590

Discretised Boltzmann equation

- Use $x \equiv \log_{10}(E)$
- Discrete variable $Y_i(x) \equiv Y(z_i, x)$

$$\frac{Y_{i+1}(x) - Y_i(x)}{\Delta z} = -\frac{1}{(1+z_i) H(z_i)} \left\{ \frac{\log_{10} e}{E} \left[\partial_x \left(H(z) E Y_i(x) \right) + \partial_x \left(b_{e^+e^-}(x, z_i) Y_i(x) \right) + \partial_x \left(b_{p\gamma}(x, z_i) Y_i(x) \right) \right] + \mathcal{L}_{\mathsf{CR}}(x, z_i) \right\}$$

- Spectrum evolved from $z = 6$ to $z = 0$ in steps $\Delta z = 5 \times 10^{-5}$
- $Y_i(x)$ calculated at $N_x = 60$ values at each z_i – then interpolated
- Protons injected down to $z = 10^{-3}$ (inhomogeneity sets in)
- Typical running time (including CMB + CIB interactions, and cosmogenic ν production): $\sim$ 120 s (on an i7 processor)

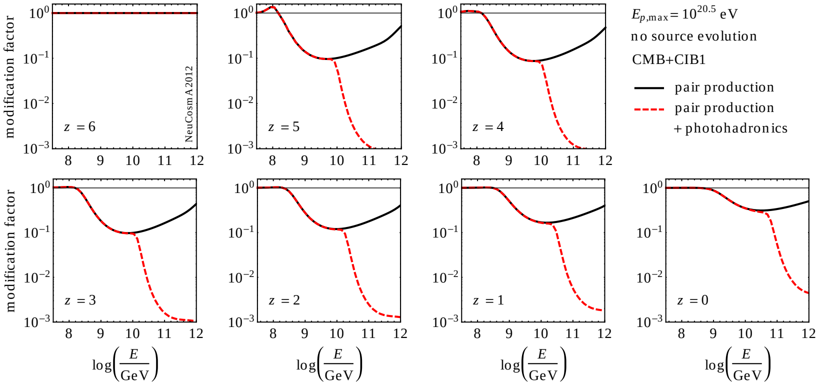
A few extra tricks for speed-up

- NeuCosmA already calculates the photohadronics fast
- Protons and neutrons are treated as the same species:
  – valid between 10^9 GeV and 4×10^{11} GeV
  - below: diffusion effects due to magnetic fields
  - above: neutron interaction time < decay time

  T. Stanev, eConf C040802, L020 (2004), astro-ph/0411113
- Local $(z = 0)$ $b_{e^+e^-}$ and $b_{p\gamma}$ on the CMB calculated only once, and then scaled to different z
  - $\sim$ 120 s (w/ scaling) vs. $\sim$ 40 min (w/o scaling)
- CIB interactions calculated only every 500 redshift steps
  - since CIB interaction length $\gg$ CMB length
- Cosmogenic neutrino flux calculated only every 1000 steps, for the same reason

Modification factor

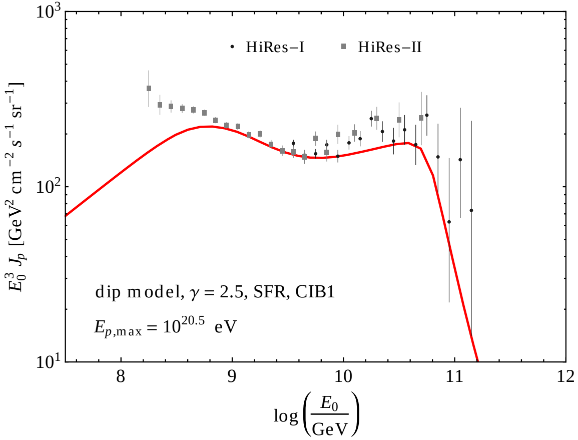
$$\text{modification factor} = \frac{Y_p\ (\text{adiabatic+CMB+CIB losses})}{Y_p\ (\text{adiabatic losses only})}$$



Matches the results from R. ALOISIO *et al.*, *Astropart. Phys.* **27**, 76 (2007), ASTRO-PH/0608219

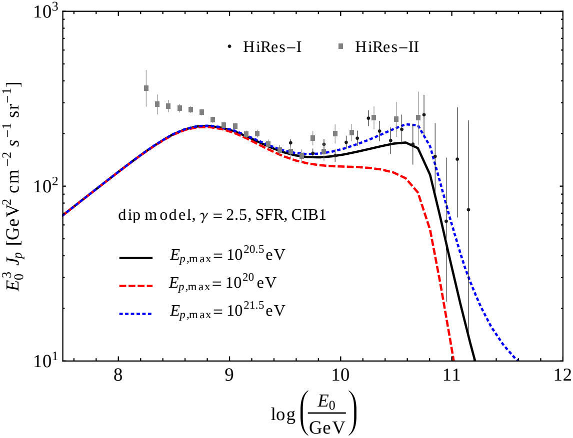
Proton spectrum normalisation

Proton flux at Earth $J_p(E_0) \equiv (c/4\pi)\, n_p(E_0, 0)$ normalised to HiRes data (*Phys. Rev. Lett.* **100**, 101101 (2008), ASTRO-PH/0703099):



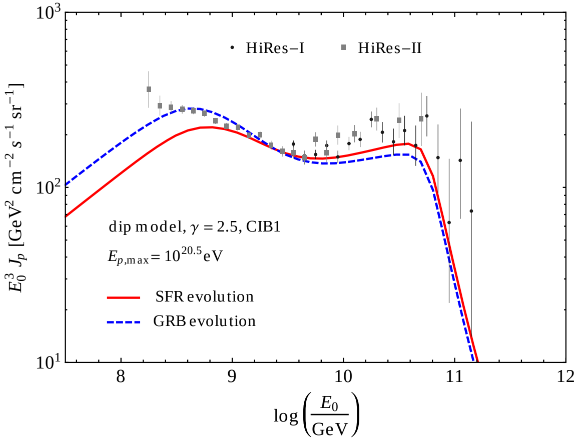
Varying $E_{p,\max}$

The flux grows with $E_{p,\max}$:



Source evolution

SFR vs. GRB evolution:



The “guaranteed” neutrino flux

- We *have* measured UHECRs *and* the CMB and CIB
- ∴ There is a “guaranteed” flux of UHE ν’s coming from

$$p + \gamma \longrightarrow \Delta^+ (1232) \longrightarrow n + \pi^+$$

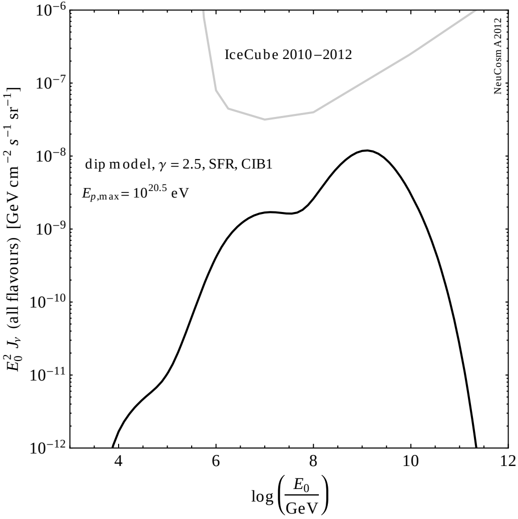
and the decays

$$\pi^+ \longrightarrow \mu^+ \nu_\mu \longrightarrow \bar{\nu}_\mu e^+ \nu_e \nu_\mu$$
$$n \longrightarrow p e^- \bar{\nu}_e$$

- Additional contributions (higher resonances, multi-pion production) implemented in NeuCosmA
- Our code calculates this cosmogenic, or GZK, ν flux given the proton injection at the sources

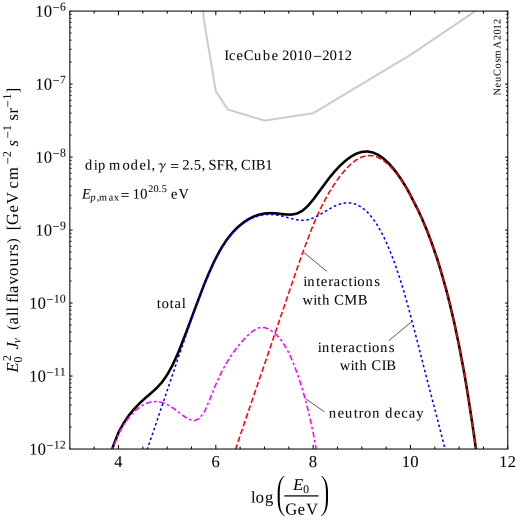
V. Berezinksy, G. Zatsepin, *Phys. Lett. B* **28**, 423 (1969)
F. Stecker, *Astrophys. J.* **228**, 919 (1979)

The flux at Earth



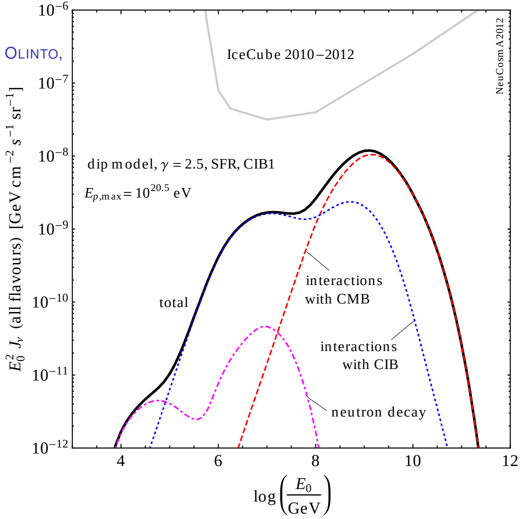
IceCube bound from
A. Ishihara,
talk at Neutrino 2012

The flux at Earth



The flux at Earth

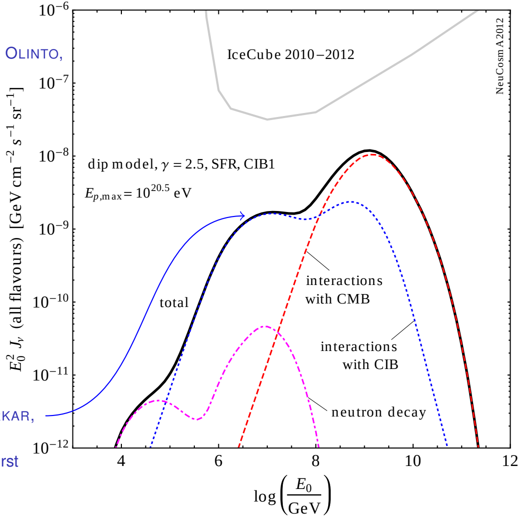
matches
K. KOTERA, D. ALLARD, A.V. OLINTO,
JCAP **1010**, 013 (2010),
ARXIV:1009.1382



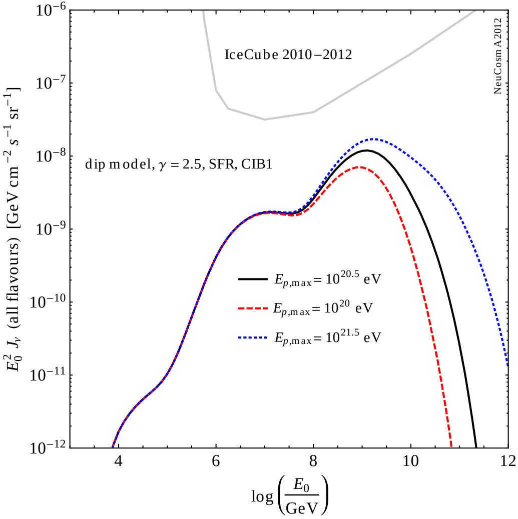
The flux at Earth

matches
K. KOTERA, D. ALLARD, A.V. OLINTO,
JCAP **1010**, 013 (2010),
ARXIV:1009.1382

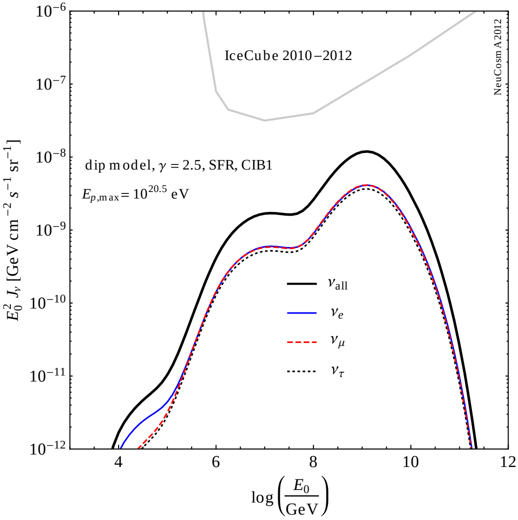
we use a different CIB parametrisation (from M. AHLERS, L.A. ANCHORDOQUI, S. SARKAR, *PRD* **79**, 083009 (2009), ARXIV:0902.3993), so our first CIB bump is lower



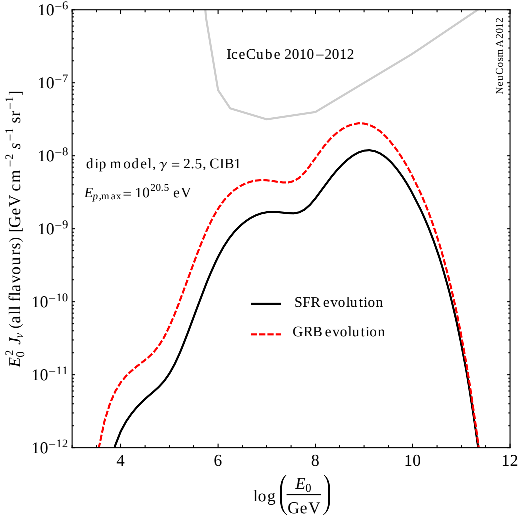
Varying $E_{p,\max}$



Per flavour


IceCube 2010−2012
NeuCosmA 2012
dip model, $\gamma = 2.5$, SFR, CIB1
$E_{p,\max} = 10^{20.5}$ eV
ν_{all}
ν_e
ν_μ
ν_τ
$E_0^2\ J_\nu$ [GeV cm^{-2} s^{-1} sr^{-1}]
$\log\left(\frac{E_0}{\mathrm{GeV}}\right)$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
4
6
8
10
12

SFR vs. GRB evolution



To do

To implement:

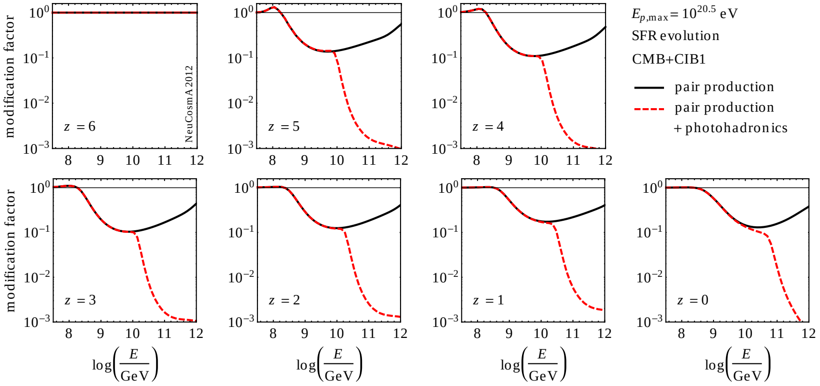
- universal radio background
- probably still room for optimisation

To explore:

- different choices for CIB redshift evolution
- calculate the UHECR and cosmogenic ν flux for different models of CR injection at sources – e.g., we can test different forms of cut-off
- (a bit further down the line) add calculation of photon flux from $p\gamma$ interactions with photon backgrounds

Backup slides

Modification factor with SFR source evolution



Cosmogenic $\nu_{e,\mu,\tau}$ flux with GRB source evolution